

Optimal and Distributed Scheduling For Multicell Capacity Maximization

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Abstract

We address the problem of multicell co-channel scheduling in view of mitigating interference in a wireless data network with full spectrum reuse. The centralized joint multicell scheduling optimization problem, based on the complete co-channel gain information, has so far been justly considered impractical due to complexity and real-time cell-to-cell signaling overhead. However, we expose here the following remarkable result for a large network with a standard power control policy: The capacity maximizing joint multicell scheduling problem admits a simple and fully distributed solution! This result is proved analytically for an idealized network. From the constructive proof, we propose a practical algorithm that is shown to achieve near maximum capacity for realistic cases of simulated networks of even small sizes.

Index Terms

Multicell, Co-channel Scheduling, Network Capacity, Full spectrum reuse

I. INTRODUCTION

High data rate requirement for beyond 3G services directly translates into a heavy demand for expensive and precious spectral resources. It is well known that full reuse of spectrum, in any of the dimensions allowed by the multiple access scheme (time or frequency slots, codes etc.) is the key to achieving much greater capacity in wireless data networks. In practice however, aggressive reuse of the spectral resource leads to an increased, sometimes unbearable, level of interference throughout the network. Traditionally, interference control is performed through the

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use of resource management techniques which, combined with power control algorithms, allow the network to operate under a satisfactory carrier to interference level (C/I), that is compatible with the receiver's sensitivity at the access points (base stations) and the user terminals. This is achieved by maintaining a sufficient spatial separation of most co-channel links, based on standard path loss and fading models. In addition to inter-cell interference mitigation, recently developed dynamic resource management techniques aim at better utilization of the spectrum inside each cell by encouraging channel access for users temporarily experiencing better (than others) propagation conditions, giving rise to the so-called multi-user diversity gain [1]. Clearly multi-user diversity is gained at the expense of throughput fairness, which may be restored by modifying the scheduling criteria in one of several possible manners [2]. A fundamental point of this paper is that the joint multicell user scheduling problem offers an enormous number of degrees of freedom (governed by the number of cells times the number of user times the number of possible scheduling slots) that can be potentially used to maximize the *network capacity* in an interference-limited setting.

Notably, a number of recent channel allocation schemes have been proposed to mitigate co-channel interference in the particular case of fixed wireless data networks [3] with aggressive spectral reuse. *Staggered Resource Allocation* (SRA) and variants [4] exploit directional antennae, user classification and ordering of users within sub-frames to obtain gains when traffic load is low. *Time-Slot Resource Partitioning* (TSRP) [5] turns off BS sector beams according to a determined sequence, which permits users to measure the interference received and then tell their respective BS their preferred sub-frame for reception. *Power-Shaped Advanced Resource Assignment* (PSARA) [6] allows the BS to transmit with different powers in different portions of the frame and users are allotted slots according to the amount of interference tolerated. In a similar vein, base-station coordination is achieved in [7], by exchanging information between the dominant interfering set of sectors and then making transmissions orthogonal in time for these BS. Such schemes can be extended to mobile networks, at the cost of increased overhead in signaling. The authors observe capacity gains associated with interference avoidance scheduling in interference-limited networks. These clever resource planning schemes are interesting as they offer some (limited) flexibility in mitigating interference. Nevertheless, they are far from fully exploiting the degrees of freedom provided by the joint multicell scheduling problem as they do not attempt to find the optimal scheduling rule for simultaneous transmission in all co-channel

cells.

Unfortunately, the study of such optimal schemes faces two great challenges. One is complexity and the other, even more problematic, is the need for the joint processing of traffic and channel gain parameters for all network users. The latter requires a central control unit, which makes global network coordination hard to realize in practice, especially in mobile settings where the scheduler ought to track fast-fading channels. These issues remain problematic despite some interesting results such as [8], where a centralized heuristic algorithm works by inserting co-channel users one by one, as long as the channel throughput increases. Or that of [9] which provides a useful theoretical quantification of inter-cell coordination in terms of user queue stability regions for various network topologies.

This paper takes a closer look at the challenging yet interesting multicell scheduling problem in view of network capacity maximization. We consider resource-fair schedulers under backlogged traffic for all users and initially ignore issues related to throughput fairness. We begin by formulating the capacity maximization problem for an arbitrary (realistic) network, given knowledge of the complete multicell channel gain information for a standard power control rule (gain inversion-based power control). Next, we define the concept of *interference-ideal* networks which we use to approximate large regular networks. Focusing on simplification in the case of interference-ideal networks, we obtain the following striking result:

- For interference-ideal networks, maximum network capacity can be reached by using a low-complexity *fully distributed* scheduling protocol, based on local channel gains. This result admits a theoretical constructive proof which we further exploit to propose a multicell scheduling algorithm for realistic (non-ideal) networks.
- For fast-fading, the algorithm is a generalization of the single cell maximum capacity scheduler [1] to the multicell case. As a result, per-cell throughput maximization and multicell interference avoidance are shown to go hand in hand and multi-user diversity scheduling can also be throughput optimal in a multicellular scenario.

From the analysis above we derive a practical co-channel scheduling algorithm, called Power Matched Scheduling (PMS), that can trade-off resource fairness for system capacity. These results have applications in cellular/ad-hoc networks with interference-limited transmission. In this paper we test the algorithms over finite-size non-ideal cellular-type networks and show the throughput gains over a non-coordinated co-channel scheduler in the presence of interference.

This paper is organized as follows: The network model is described in Section II. The capacity maximization co-channel user scheduling problem based on interference avoidance is formulated in Section III. In Section IV, the interference-ideal network concept is introduced and a fully distributed optimal co-channel scheduling policy is obtained. We discuss issues related to multi-user diversity and fairness in Section V. Finally, numerical results for capacity evaluation are presented in Section VI.

II. NETWORK MODEL

Consider a multicell system with N access points (AP) communicating with U user terminals (UT) in each cell. We are particularly interested in the downlink in which the AP sends data to the UT, but the results presented in this paper can be generalized to the uplink situation. The system employs the *same spectral resource* in each cell giving rise to an interference-limited system (fig. 1), although interference limitation is not a requirement for our approach. We also assume power control is used in the network in an effort to preserve power and limit interference and fading effects.

A. Resource Fair Partitioning

Within each cell, we consider a multiple access scheme in which an orthogonally divided resource (e.g. codes, time, frequency etc.) is used to separate the transmissions to the cell users. Each cell user is allocated a portion of the resource called a resource slot (fig. 2). A “frame” consists of a set of K slots. We enforce K -th order resource fairness, where $1 \leq K \leq U$. This means that a scheduling frame consists of K slots assigned to K distinct users per cell. Note that this does not necessarily yield throughput fairness, even with $K = U$, as users may not enjoy an equal throughput due to local channel conditions. Moreover, because of concurrent transmissions in all cells in any one slot, an assigned user “sees” interference from all co-channel cells.

B. Signal Model

To preserve light notation we focus on the single antenna case. For a user u_n in cell n , the downlink is a typical interference channel [10], the received signal for which is given by

$$Y_{u_n} = \sqrt{G_{u_n \leftarrow n}} X_{u_n} + \sum_{i \neq n}^N \sqrt{G_{u_n \leftarrow i}} X_{u_i} + Z_{u_n},$$

where X_{u_n} is the signal from the serving AP n and Z_{u_n} is additive white Gaussian noise. The signal to interference-plus-noise ratio (SINR), Γ is given by,

$$\Gamma_{u_n} = \frac{G_{u_n \leftarrow n} \mathbb{E}|X_{u_n}|^2}{\mathbb{E}|Z_{u_n}|^2 + \sum_{i \neq n}^N G_{u_n \leftarrow i} \mathbb{E}|X_{u_i}|^2}.$$

If transmit power used by an AP to serve u_n is P_{u_n} , we have $\mathbb{E}|X_{u_n}|^2 = P_{u_n}$. Note that $G_{u_n \leftarrow m} \in \mathbb{R}^+$ reflects the composite channel gain between user u_n and AP m possibly including fast-fading.

C. Power Control

As is seen later, power control plays a key role in enabling the gains of network coordination. Typical power control strategies aim at adjusting the transmitter power to reduce co-channel interference experienced at the receivers. Power control policies may target a given signal-to-interference ratio (SIR) or a certain received signal power level. In [11], a distributed iterative algorithm is proposed for attaining the best possible common SIR and this is extended to an “if at all achievable” target SIR in [12]. Received signal-level based power control is studied in [13], [14] and also shown to contribute to mitigating co-channel interference although the performance of optimal interference balancing is slightly better than received signal-level power control [13]. Combining power control with cell diversity was subsequently shown to increase the number of supported users in the uplink [15]. For an overview on power control issues refer to [16].

The power control effect can be formulated simply in the following way: Assuming each AP has a peak transmission power constraint P_{MAX} , a multiplicative power control factor $0 < \rho \leq 1$ is used to adjust the transmitted power of the AP, such that we have for user u_n

$$P_{u_n} = \rho_{u_n} P_{MAX}$$

Using $R_{u_n \leftarrow u_i} = G_{u_n \leftarrow i} P_{u_i}$ to express the received power at user u_n (which is served by AP n) from the AP of cell i when it transmits to its user u_i , the SINR can be expressed as

$$\Gamma_{u_n} = \frac{R_{u_n \leftarrow u_n}}{\eta + \sum_{i \neq n}^N R_{u_n \leftarrow u_i}} \quad (1)$$

where $R_{u_n \leftarrow u_n}$ is the received power from the *-serving* AP of user u_n and η is the thermal noise power assumed the same for all users. $\sum_{i \neq n}^N R_{u_n \leftarrow u_i}$ is the total interference received by user u_n from other APs when they transmit to their respective scheduled users.

The value of ρ_{u_n} depends on the adopted power control policy. We assume the same received signal-level based power control policy throughout the paper as this is the most practical scheme and has already been implemented in many systems. We draw the reader's attention however to the fact that the optimal scheduling policy should ultimately be jointly optimized with the power control policy. Such issues are however beyond the scope of this paper and will be addressed in a later paper.

We define R^* as the target received power and assume that each user is able to measure and communicate back the power received from the serving AP so that the transmit power may be adjusted. The power control factor can then be obtained via:

$$\begin{aligned} G_{u_n \leftarrow n} \rho_{u_n} P_{MAX} &= R^* \\ \rho_{u_n} &= \frac{R^*}{G_{u_n \leftarrow n} P_{MAX}} \end{aligned}$$

But since there is a power constraint P_{MAX} , ρ is upper bounded by one:

$$\rho_{u_n} = \min \left\{ \frac{R^*}{G_{u_n \leftarrow n} P_{MAX}}, 1 \right\} \quad (2)$$

Power control scenarios: Depending on the value of the R^* and the channel gain, a user will be receiving in full ($\rho = 1$) or reduced ($\rho < 1$) power mode. We consider three network scenarios. (1) *fully power controlled* (FPC) network: all users achieve R^* after power control. (2) *mixed power controlled* (MPC) network: Only a fraction of users achieve R^* . (3) *no power controlled* (NPC) network: all users use $\rho = 1$. As we will see shortly, different optimal multicell scheduling policies will arise in each network scenario.

III. THE CO-CHANNEL USER MATCHING PROBLEM

We assume that channel gains do not vary over the scheduling frame duration which is sized in accordance with the coherence period of the channel. Under the K -th order resource fairness constraint, the co-channel user matching problem consists in selecting K users in each cell and assigning these users to K slots so as to optimize the system utility function (joint capacity). To facilitate the formulation of the problem, we state the following definitions:

Definition 1: A **scheduling policy** φ is a bijective mapping of the subset \mathcal{U}_n , consisting of K users chosen from the set of all users in cell n , onto \mathcal{K} the set of slots, $\varphi_n : \mathcal{U}_n \mapsto \mathcal{K}$.

Definition 2: A **scheduling vector** $\mathcal{J}^{(k)}$ contains the set of users scheduled in slot k across all cells (based on φ):

$$\mathcal{J}^{(k)} = \left[u_1^{(k)} u_2^{(k)} \dots u_n^{(k)} \dots u_N^{(k)} \right]^T \in [1, K]^N$$

where $[\mathcal{J}^{(k)}]_n = u_n^{(k)}$ is the user scheduled during slot k in cell n . Note that because φ is a bijection, scheduling vectors are element-wise disjoint, $\mathcal{J}^{(a)} \cap \mathcal{J}^{(b)} = \emptyset \forall a \neq b$. The scheduling vector is the ensemble of users which interfere with each other and thus it determines the sum capacity for slot k .

Definition 3: A **scheduling matrix** \mathbf{S} is a K -column matrix composed of scheduling vectors given by the scheduling policy φ .

$$\mathbf{S} = [\mathcal{J}^{(1)} \mathcal{J}^{(2)} \dots \mathcal{J}^{(K)}]$$

This matrix describes the complete ordering of all users during one frame. For example, considering the scheduling matrix given in fig. 3, users 2 and 5 of cell 1 are scheduled with users 3 and 1 of cell 2, respectively.

A. System Performance

The SINR for users scheduled in slot k will depend on the scheduling vector $\mathcal{J}^{(k)}$. We can express the SINR during slot k in cell n as

$$\begin{aligned} \Gamma(\mathcal{J}^{(k)}, n) &= \frac{R_{[\mathcal{J}^{(k)}]_n \leftarrow [\mathcal{J}^{(k)}]_n}}{N} \\ &= \frac{\eta + \sum_{i \neq n} R_{[\mathcal{J}^{(k)}]_n \leftarrow [\mathcal{J}^{(k)}]_i}}{N} \\ &= \frac{G_{u_n^{(k)} \leftarrow n} \rho_{u_n^{(k)}} P_{MAX}}{N} \\ &= \frac{\eta + \sum_{i \neq n} G_{u_n^{(k)} \leftarrow i} \rho_{u_i^{(k)}} P_{MAX}}{N}, \end{aligned} \quad (3)$$

where $u_i^{(k)} = [\mathcal{J}^{(k)}]_i \forall i$ is the user scheduled during slot k in cell i . Assuming an ideal link adaptation protocol, the per cell capacity in slot k can be expressed in bits/sec/Hz/cell using the Shannon capacity,

$$C(\mathcal{J}^{(k)}) = \frac{1}{N} \sum_{n=1}^N \log \left(1 + \Gamma(\mathcal{J}^{(k)}, n) \right) \quad (4)$$

By averaging the per ce;; capacity over the total number of slots, we obtain the network capacity,

$$\begin{aligned} \mathcal{C}(\mathbf{S}) &\triangleq \frac{1}{K} \sum_{k=1}^K C(\mathcal{I}^{(k)}) \\ &\triangleq \frac{1}{NK} \sum_{k=1}^K \sum_{n=1}^N \log \left(1 + \frac{G_{u_n^{(k)} \leftarrow n} \rho_{u_n^{(k)}} P_{MAX}}{\eta + \sum_{i \neq n} G_{u_n^{(k)} \leftarrow i} \rho_{u_i^{(k)}} P_{MAX}} \right), \end{aligned} \quad (5)$$

which is a function of the scheduling matrix \mathbf{S} .

B. Round Robin Scheduling

A standard approach for resource fair scheduling is round robin (RR) in which users are given slots turn by turn in each frame and thus, every possible permutation of a scheduling matrix is equiprobable. Letting \mathbb{S} be the set of all scheduling matrices, the network capacity for RR will be the expectation over all scheduling matrix permutations given by,

$$C_{RR} \triangleq \mathbb{E}_{(\mathbf{S} \in \mathbb{S})} \{ \mathcal{C}(\mathbf{S}) \}. \quad (6)$$

C. Optimal Co-channel Scheduling

On the other hand, the scheduling policy for optimum network capacity (5) can be stated as

$$\mathbf{S}^* = \underbrace{\operatorname{argmax}}_{\mathbf{S} \in \mathbb{S}} \{ \mathcal{C}(\mathbf{S}) \} \quad (7)$$

Notice that finding the optimal scheduling policy φ^* is equivalent to finding the optimal scheduling matrix \mathbf{S}^* . As \mathbf{S}^* gives the optimal network capacity, we have in general:

$$\mathcal{C}(\mathbf{S}^*) \geq C_{RR},$$

where inequality will be strict in most cases, thus showing the gain of coordinated networks over uncoordinated ones.

D. Multicell Scheduling Gains vs. Power Control Scenarios

It is easy to see that some scenarios will result in no gain at all as shown below:

Lemma 1: For a no power control (NPC) network, the network capacity gain associated with multicell scheduling is zero.

Proof: With no power control $\rho_{u_n} = 1 \forall u_n$, and thus all BS transmit at same (maximum) power. Substituting this in (3) we obtain

$$\Gamma(\mathcal{S}^{(k)}, n) = \frac{G_{u_n^{(k)} \leftarrow n} P_{MAX}}{\eta + \sum_{i \neq n}^N G_{u_n^{(k)} \leftarrow i} P_{MAX}}, \quad (8)$$

which is independent of the choice of co-channel users in other cells. It follows that the capacity will be the same no matter which users are scheduled with each other. ■

This result indicates that the gain can be intuitively expected to depend much on the degree of *variability* of channel *and* power control coefficients across the network users, as well as on the number of cells and users. We now turn on to the issue of *finding* the optimal \mathbb{S} .

IV. OPTIMUM SUM CAPACITY SCHEDULING

A. Exhaustive Search Approach

As \mathbb{S} is a discrete finite set, clearly (7) is a non-linear *combinatorial optimization problem* for which, finding optimal solutions is NP-hard (Non-deterministic Polynomial-time hard).

Lemma 2: For $K = U$, the cardinality of the search space for the optimization problem in \mathbb{S} can be shown to be given by

$$|\mathbb{S}| = (U!)^{N-1}. \quad (9)$$

Proof: The system has N frames each consisting of K slots. The problem is finding all possible permutations of size K from a set of U elements, N times. This is given by

$$\left(\frac{U!}{(U-K)!} \right)^N. \quad (10)$$

Notice that (10) gives all possible permutations of scheduling matrices including those of the same scheduling vectors ordered in different ways inside a scheduling matrix. Clearly, column-wise permutations of the same scheduling vectors give the same network capacity. By taking into account that a set of K scheduling vectors can be ordered in $K!$ ways, we obtain

$$|\mathbb{S}| = \frac{1}{K!} \left(\frac{U!}{(U-K)!} \right)^N,$$

and substituting $K = U$ gives (9). ■

Exhaustive search thus has factorial complexity in the number of users and exponential complexity in the number of cells. Even for a small network with $N = 7$ cells and $U = 5$ users, the complexity of this method remains prohibitive: $|\mathcal{S}| = (5!)^{7-1} \approx 2.9 \times 10^{12}$. Alternatively, *heuristic methods* offer sub-optimal solutions at reasonable computational cost and have been applied to the classical channel assignment problem [17], [18]. However, there is no guarantee on consistency and how close a heuristic solution is to the optimum [19].

Finally, another challenge of implementing the exhaustive search or greedy approaches is the need of a central control unit that collects all path gain information, processes it to find \mathbf{S} , then broadcast the result to all APs within a time of much less than the coherence time of the channel. The delay and signaling overhead necessary for this approach makes it very hard to implement in practice.

We now proceed to find a *distributed* multicell scheduling algorithm instead. To this end, we introduce a simplified model for network capacity used to later approximate the actual capacity. The idealized network model serves as a tool to first establish our theoretical result, then construct a practical algorithm for a non-idealized (practical) setting.

B. Interference-Ideal Networks

Full spectral reuse has benefits in terms of increased spectral efficiency, but excess interference diminishes the gain associated with increasing reuse. Fortunately, full reuse networks lend themselves to simpler modeling of the total interference experienced by the user, due mostly to the large number of interference sources averaging themselves out at the receiver.

To obtain this model we define the concept of an *interference-ideal network* as one in which, *the total interference received by any cell user is independent of its location in the cell*. Though not rigorously true in practice, this model proves remarkably useful for certain large networks as shown below.

Definition: A network is interference-ideal if, for any user u_n in cell n :

$$\sum_{i \neq n}^N G_{u_n \leftarrow i} \rho_{u_i} P_{MAX} = G \sum_{i \neq n}^N \rho_{u_i} P_{MAX}, \quad (11)$$

where G is a constant which does not depend on the location of u_n , but depends on pathloss and link budget parameters.

Justification in Large Random Networks: Fortunately, the interference-ideal network is a good model for a full reuse network with a large number of cells:

$$\sum_{i \neq n}^N G_{u_n \leftarrow i} \rho_{u_i} P_{MAX} = (N-1) \underbrace{\left(\frac{1}{N-1} \sum_{i \neq n}^N G_{u_n \leftarrow i} \rho_{u_i} P_{MAX} \right)}_{\approx \mathbb{E}\{G_{u_n \leftarrow i} \rho_{u_i} P_{MAX}\} \text{ (for large } N)}$$

and as inter-cell channel gains and power control factors are uncorrelated

$$\begin{aligned} \sum_{i \neq n}^N G_{u_n \leftarrow i} \rho_{u_i} P_{MAX} &\approx (N-1) \mathbb{E}\{G_{u_n \leftarrow i}\} \mathbb{E}\{\rho_{u_i} P_{MAX}\} \\ &\approx \mathbb{E}\{G_{u_n \leftarrow i}\} (N-1) \frac{1}{N-1} \sum_{i \neq n}^N \rho_{u_i} P_{MAX} \\ &\approx \mathbb{E}\{G_{u_n \leftarrow i}\} \sum_{i \neq n}^N \rho_{u_i} P_{MAX}. \end{aligned} \quad (12)$$

We denote the expectation of the inter-cell channel gain as follows:

$$\mathbb{E}\{G_{u_n \leftarrow i}\} = G(r),$$

where r is the distance of a user u_n from the cell center. By considering a random network model restricted to a disc of radius D and APs at i.i.d. locations (fig 4), it can be shown that $G(r)$ is given by [20],

$$\begin{aligned} G(r) = & \frac{e^{(\frac{\sigma \ln 10}{10})^2/2}}{(\pi D^2 - 4\pi R^2)(-\xi + 2)} \int_0^{2\pi} \left(D \left(1 - \frac{r^2}{D^2} \cos^2 \theta \right)^{\frac{1}{2}} - r \sin \theta \right)^{-\xi+2} \\ & - \left(2R \left(1 - \frac{r^2}{4R^2} \cos^2 \theta \right)^{\frac{1}{2}} - r \sin \theta \right)^{-\xi+2} d\theta. \end{aligned} \quad (13)$$

R is the radius of a cell under consideration present at the center of the disc and thus $0 \leq r \leq R$. As we are modeling a cellular network, the closest interferers (AP) are at a distance of at least $2R$ from the center of the cell under consideration and are randomly present over the interference region according to a uniform distribution. The variance in dB of the lognormal shadowing and the pathloss exponent are given by σ^2 and ξ respectively. Details of the derivation can be found in [20]. We note that $G(r)$ is a monotonically increasing function of r and independent of the azimuth angle due to the symmetry of the network. Thus, we can show that in a large random network, interference increases monotonically, but slowly, from the cell center to the

cell boundary (fig 5). Given this result we can model the best case or worst case interference by selecting $G = G(0)$ or $G(R)$. However, we will see later that the numerical value of G plays no role in the final multicell scheduling algorithm.

C. Optimum Scheduling in Interference-Ideal Networks

Armed with the idealized network model above, we proceed to present the main result of this paper. We characterize the solution to the optimal network scheduling problem in an interference-ideal network and a *fully power controlled scenario*. Using (11) and (2) we can rewrite (3) as

$$\Gamma(\mathcal{S}^{(k)}, n) = \frac{R^*}{\eta + GR^* \sum_{i \neq n}^N \frac{1}{G_{u_i^{(k)} \leftarrow i}}} \quad (14)$$

The network capacity will be given by

$$\mathcal{C} = \frac{1}{NK} \sum_{k=1}^K \sum_{n=1}^N \log \left(1 + \frac{R^*}{\eta + GR^* \sum_{i \neq n}^N \frac{1}{G_{u_i^{(k)} \leftarrow i}}} \right). \quad (15)$$

Next, we define a vector $\mathcal{U}_n \downarrow$, containing the K users of \mathcal{U}_n ordered in descending order of intra-cell channel gains,

$$\mathcal{U}_n \downarrow = [u_{1,n} \dots u_{j,n} \dots u_{K,n}]^T$$

where,

$$G_{u_{1,n} \leftarrow n} \geq \dots \geq G_{u_{j,n} \leftarrow n} \geq \dots \geq G_{u_{K,n} \leftarrow n}$$

We now present the following result:

Theorem 1: Let $\mathbf{S} \downarrow = [\mathcal{U}_1 \downarrow \dots \mathcal{U}_n \downarrow \dots \mathcal{U}_N \downarrow]^T$, then

$$\mathbf{S} \downarrow = \begin{pmatrix} u_{1,1} & u_{2,1} & \dots & u_{k,1} & \dots & u_{K,1} \\ u_{1,2} & u_{2,2} & \dots & u_{k,2} & \dots & u_{K,2} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ u_{1,n} & u_{2,n} & \dots & u_{k,n} & \dots & u_{K,n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ u_{1,N} & u_{2,N} & \dots & u_{k,N} & \dots & u_{K,N} \end{pmatrix} \quad (16)$$

Letting $\pi(\mathbf{S} \downarrow)$ be the scheduling matrix obtained by applying any column-wise permutation on $\mathbf{S} \downarrow$. Then, for an interference-ideal network, $\pi(\mathbf{S} \downarrow)$ is an optimal scheduling matrix, \mathbf{S}^* for the problem (7).

Proof: See Appendix. ■

Based on Theorem 1 an optimal scheduling policy is for each cell to rank its users by (say decreasing) order of channel gain and assign the best K users to the K available slots, regardless of the channel gains in other cells. As co-channel users are matched based on the rank of their channel gain, we call this scheduling policy *Power Matched Scheduling* (PMS). As local channel gain is the only scheduling criteria, PMS is completely distributed. Note that a side-effect of the policy is to group users with similar channel quality levels, possibly creating unfair service across resource slots.

D. Validity for Uplink

If the interference-ideal network model is valid for the uplink as well, the scheduling policy proposed in the previous section will also be optimal for the uplink. This is due the fact that uplink intra-cell orthogonal transmission and the gain-inversion power control policy result in network scenarios equivalent to the downlink. Based on a similar model for the uplink as described in Section IV-B, we obtain the following expression for G [20]:

$$G = e^{(\frac{\sigma \ln 10}{10})^2 / 2} \frac{2(D^{-\xi+2} - R^{-\xi+2})}{(-\xi + 2)(D^2 - R^2)}. \quad (17)$$

We see that this depends only on link parameters and is a constant with regard to user positions. Thus the interference-ideal model is valid for the uplink as well. This leads us to conclude that power matched scheduling also provides gain in the uplink as well.

V. MULTI-USER DIVERSITY AND FAIRNESS

An interesting result from the earlier study is the conclusion that scheduling based on multi-user diversity is also optimal in a multicellular scenario.

A. Multi-user Diversity

Lemma 3: Throughput optimal multi-user scheduling in a single cell case is also throughput optimal in the multicell case if received signal-level power control is used.

Proof: This can be easily seen by considering the frame size $K = 1$. Theorem 1 will result in the following scheduling matrix for $K = 1$

$$\mathbf{S} \downarrow^{N \times 1} = \begin{pmatrix} u_{1,1} \\ u_{1,2} \\ \vdots \\ u_{1,N} \end{pmatrix}$$

The users with the best channel gains in each cell are scheduled, which is also throughput optimal in the single cell case [1] as it maximizes the so called multi-user diversity in each and every cell. ■

B. Fairness

We have shown that the optimal multicell scheduling rule corresponds to grouping good users together and bad users together. Thus, network capacity is optimized at the expense of throughput fairness since weaker users will see their channel conditions worsened by the addition of the worst possible interference. This is demonstrated in fig. 6. Note, however, that resource fairness can be guaranteed by choosing $K = U$, but not throughput fairness. The capacity maximization vs. fairness tradeoff is not surprising since it gives an intuitive generalization of results derived previously for the single cell scenario [1], [21]. Notice that the value of K also has an effect on performance, where $K = 1$ gives only multi-user diversity gain without regard for fairness, while $K = U$ provides full resource fairness at the cost of capacity.

As in single cell scheduling, throughput fairness can be restored in several ways. One strategy is to use a clever admission control policy. An outage percentage can be imagined where a minimum SINR, Γ_{min} is guaranteed to $(100 - \Delta)\%$ of the users. The $\Delta\%$ of the users which are not able to achieve Γ_{min} can be compensated in a number of ways. One way is to increase access time for underprivileged users where slot duration is prolonged to increase throughput. In another way, these users can be put on an inter-cell orthogonal resource so that they see less interference. Yet another way is to provide protection in dedicated slots by keeping some cells silent similar to TSRP [5], thereby improving SINR. The amount of protection can range from providing "exclusive access" to a cell user or removing cells from a slot turn by turn until the required SINR is achieved. The degree of protection will obviously depend on the degree of degradation as well as the number of users needing compensation. We point out however, that

implementing these kinds of schemes will require global knowledge of the system and will result in a loss of capacity as compared to the full power matching scheduling algorithm.

VI. NUMERICAL RESULTS

The performance of Power Matched Scheduling (PMS) is compared with RR in terms of network capacity based on Monte Carlo simulations under a full resource fairness constraint ($K = U$). A hexagonal cellular system functioning at 1800 MHz is considered, consisting of 1 km. radius cells with users randomly spread according to a uniform distribution. Channel gains for *both inter-cell and intra-cell* AP-UT links are based on a COST-231 path loss model [22] including log-normal shadowing plus fast-fading. Log-normal shadowing is a zero mean Gaussian distributed random variable in dB with a standard deviation of 10 dB. Fast-fading is modeled by i.i.d. random variables $h_{u_n,i} \sim \mathcal{CN}(0, 1)$. R^* corresponds to an SNR target of 30 dB and $P_{MAX} = 1W$. These network settings result in a mixed power control (MPC) system which serves to test the robustness of PMS in a realistic scenario.

A. PMS vs. Optimal Scheduler

We first compare PMS with an optimal scheduler which in theory performs an exhaustive search over all possible scheduling matrices to find the optimal solution. In practice, this would amount to a centralized entity collecting information about all AP-UT links in the network in order to compute the system capacity for every scheduling matrix. For PMS, users are scheduled according to Theorem 1. As mentioned earlier the exhaustive search approach entails significant computational complexity and thus we consider a network with $N = 12$ and $U = 2$. Figure 7 demonstrates the performance of PMS compared to that of exhaustive search where we trace the frame network capacity for both schemes. Mean network capacity is then obtained by averaging over the total number of frames. We see that the difference in performance between PMS and exhaustive search is quite small, showing that even for a modest network size, the interference-ideal model allows us to conveniently obtain a distributed scheduling solution.

B. PMS vs. Round Robin

In accordance with (6), round robin (RR) is modeled by selecting a random permutation of the scheduling matrix for each frame. For this comparison, we assume that there are 30 users/cell.

We first show traces of network capacity obtained using RR and PMS with $N = 19$ (fig. 8) and we see that PMS provides substantial gain over RR. The proposed scheme is robust even for a small network size of $N = 3$ (fig. 9). We observe that as the number of cells increases, interference averaging reduces variation in network capacity and yields an increase in gain. The relative performance of the two scheduling policies is represented by the Network Capacity Gain τ , of PMS over RR, which is given by

$$\tau = \frac{\mathcal{C}(\mathbf{S}^*)}{\mathcal{C}_{RR}}.$$

Figure 10 shows the variation of network capacity gain with the size of the network. We notice that the gain is greater in the presence of both shadowing and fast-fading leading to the conclusion that greater channel variation improves performance and mobile environments will also benefit from this scheduling policy. The PMS scheme outperforms RR in all cases and moreover, the gain increases with system size.

VII. CONCLUSION

In this paper we address the problem of multi-user multicell scheduling for wireless networks. An optimal scheduler is proposed for asymptotically large networks. We show that large gains are obtained from inter-cell coordination thanks to the inter-cell channel gain variability which stems from power control and fading. In the optimal scheduler each cell ranks its users according to decreasing channel gains. As *local* channel gains are used the optimal scheduler can be efficiently approximated by a fully distributed multicell scheduler. The multi-cell scheduler is also consistent with maximizing the capacity of each cell independently through multi-user diversity. Simulations on a realistic network show substantial gains over un-coordinated scheduling and these gains increase with the size of the network.

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APPENDIX

PROOF OF THEOREM 1

We prove the optimality of $\mathbf{S} \downarrow$ by first showing that it is valid for N cells and two slots. This is then extended to K slots.

Lemma 4: For an arbitrary number of cells N and two slots, let

$$\mathbf{S} \downarrow^{N \times 2} = \begin{pmatrix} u_{1,1} & u_{2,1} \\ u_{1,2} & u_{2,2} \\ \vdots & \vdots \\ u_{1,n} & u_{2,n} \\ \vdots & \vdots \\ u_{1,N} & u_{2,N} \end{pmatrix}$$

The optimal scheduling matrix for (7), $\mathbf{S}^* = \mathbf{S} \downarrow^{N \times 2}$.

Proof: We show that interchanging users in $M < N$ cells will result in either no change or a decrease in network capacity ($M = N$ will result in same capacity). Without loss of generality let these be the first M cells. We employ lighter notation by letting $G_{k \leftarrow n}$ represent the channel gain between user scheduled in slot $k = 1, 2$ and it's serving AP n . Capacity before the swapping is given by

$$\begin{aligned} \mathcal{C}^* &= \sum_{n=1}^N \log \left(1 + \frac{R^*}{\eta + GR^* \left[\sum_{\substack{i=1 \\ i \neq n}}^M \frac{1}{G_{1 \leftarrow i}} + \sum_{\substack{j=M+1 \\ j \neq n}}^N \frac{1}{G_{1 \leftarrow j}} \right]} \right) \\ &+ \sum_{n=1}^N \log \left(1 + \frac{R^*}{\eta + GR^* \left[\sum_{\substack{i=1 \\ i \neq n}}^M \frac{1}{G_{2 \leftarrow i}} + \sum_{\substack{j=M+1 \\ j \neq n}}^N \frac{1}{G_{2 \leftarrow j}} \right]} \right) \end{aligned}$$

and after the swap,

$$\begin{aligned} \mathcal{C}' &= \sum_{n=1}^N \log \left(1 + \frac{R^*}{\eta + GR^* \left[\sum_{\substack{i=1 \\ i \neq n}}^M \frac{1}{G_{2 \leftarrow i}} + \sum_{\substack{j=M+1 \\ j \neq n}}^N \frac{1}{G_{1 \leftarrow j}} \right]} \right) \\ &+ \sum_{n=1}^N \log \left(1 + \frac{R^*}{\eta + GR^* \left[\sum_{\substack{i=1 \\ i \neq n}}^M \frac{1}{G_{1 \leftarrow i}} + \sum_{\substack{j=M+1 \\ j \neq n}}^N \frac{1}{G_{2 \leftarrow j}} \right]} \right). \end{aligned}$$

As $G_{1 \leftarrow n} \geq G_{2 \leftarrow n} \forall n$, we declare

$$\begin{aligned} \left(\beta_{1,n} = \sum_{\substack{i=1 \\ i \neq n}}^M \frac{1}{G_{1 \leftarrow i}} \right) &\leq \left(\beta_{2,n} = \sum_{\substack{i=1 \\ i \neq n}}^M \frac{1}{G_{2 \leftarrow i}} \right) \\ \left(\alpha_{1,n} = \sum_{\substack{j=M+1 \\ j \neq n}}^N \frac{1}{G_{1 \leftarrow j}} \right) &\leq \left(\alpha_{2,n} = \sum_{\substack{j=M+1 \\ j \neq n}}^N \frac{1}{G_{2 \leftarrow j}} \right) \end{aligned}$$

$$\begin{aligned} \mathcal{C}^* - \mathcal{C}' &= \sum_{n=1}^N \left[\left(\log \left(1 + \frac{R^*}{\eta + GR^*(\alpha_{1,n} + \beta_{1,n})} \right) - \log \left(1 + \frac{R^*}{\eta + GR^*(\alpha_{1,n} + \beta_{2,n})} \right) \right) \right. \\ &\quad \left. - \left(\log \left(1 + \frac{R^*}{\eta + GR^*(\alpha_{2,n} + \beta_{1,n})} \right) - \log \left(1 + \frac{R^*}{\eta + GR^*(\alpha_{2,n} + \beta_{2,n})} \right) \right) \right] \end{aligned}$$

Letting

$$g_n(x) = \log \left(1 + \frac{R^*}{\eta + GR^*(x + \beta_{1,n})} \right) - \log \left(1 + \frac{R^*}{\eta + GR^*(x + \beta_{2,n})} \right)$$

then we need to show

$$\mathcal{C}^* - \mathcal{C}' = \sum_{n=1}^N \left(g_n(\alpha_{1,n}) - g_n(\alpha_{2,n}) \right) \geq 0 \quad \forall \alpha_{1,n} \leq \alpha_{2,n}$$

Differentiating $g_n(x)$,

$$\begin{aligned} \frac{dg_n(x)}{dx} &= \frac{-R^*GR^*}{\ln(2) \left(1 + \frac{R^*}{\eta + GR^*(x + \beta_{1,n})} \right) (\eta + GR^*(x + \beta_{1,n}))^2} \\ &+ \frac{R^*GR^*}{\ln(2) \left(1 + \frac{R^*}{\eta + GR^*(x + \beta_{2,n})} \right) (\eta + GR^*(x + \beta_{2,n}))^2} \end{aligned}$$

Letting

$$\left(d_2 = \eta + GR^*(x + \beta_{2,n}) \right) \geq \left(d_1 = \eta + GR^*(x + \beta_{1,n}) \right)$$

we have

$$\begin{aligned} \frac{dg_n(x)}{dx} &= \frac{-R^*GR^*}{\ln(2)(1 + \frac{R^*}{d_1})d_1^2} + \frac{R^*GR^*}{\ln(2)(1 + \frac{R^*}{d_2})d_2^2} \\ &= -\frac{R^*GR^*}{\ln(2)} \left(\frac{1}{d_1^2 + R^*d_1} - \frac{1}{d_2^2 + R^*d_2} \right) \end{aligned} \quad (18)$$

As $d_2 \geq d_1$, $\frac{dg_n(x)}{dx} \leq 0$ and $g_n(x)$ is a decreasing function. Thus $\mathcal{C}^* - \mathcal{C}' \geq 0$. This proves that $\mathbf{S}^* = \mathbf{S} \downarrow^{N \times 2}$. ■

Next, we define an operator $\mathcal{Q}_{l,k}(\mathbf{S})$ which orders the users in columns (slots) l and k of the scheduling matrix in decreasing order of channel gain.

$$\mathcal{Q}_{l,k}(\mathbf{S}) = \begin{bmatrix} \mathcal{I}^{(1)} & \mathcal{I}^{(2)} & \dots & \mathcal{I}^{(l-1)} & \zeta(\mathcal{I}^{(l)}, \mathcal{I}^{(k)})_{:,1} & \mathcal{I}^{(l+1)} & \dots \\ & & & & \mathcal{I}^{(k-1)} & \zeta(\mathcal{I}^{(l)}, \mathcal{I}^{(k)})_{:,2} & \mathcal{I}^{(k+1)} & \dots & \mathcal{I}^{(K)} \end{bmatrix}$$

where $\zeta(u, v) \in \mathbb{N}^{N \times 2}$ obtained through

$$\begin{aligned} \zeta(u, v)_{i,1} &= \max(G_{u_i \leftarrow i}, G_{v_i \leftarrow i}) \\ \zeta(u, v)_{i,2} &= \min(G_{u_i \leftarrow i}, G_{v_i \leftarrow i}) \end{aligned}$$

Lemma 5: For an arbitrary scheduling matrix \mathbf{S} , $\mathcal{C}(\mathcal{Q}_{l,k}(\mathbf{S})) \geq \mathcal{C}(\mathbf{S})$

Proof: As only columns l and k are manipulated, the capacity due to other columns remains unchanged. From Lemma 4, the capacity of two slots arranged in decreasing order of channel gains will be more than when they are arranged in any other fashion. Thus, $\mathcal{C}(\mathcal{Q}_{l,k}(\mathbf{S})) \geq \mathcal{C}(\mathbf{S})$. ■

Lemma 6: For an arbitrary scheduling matrix \mathbf{S}

$$\mathcal{Q}_{K-1,K} \cdots \mathcal{Q}_{2,K} \cdots \mathcal{Q}_{2,3} \mathcal{Q}_{1,K} \cdots \mathcal{Q}_{1,3} (\mathcal{Q}_{1,2}(\mathbf{S})) = \mathbf{S} \downarrow$$

Proof: From Lemma 5, the capacity of the scheduling matrix after each \mathcal{Q} operation will be greater than the previous. The successive $\frac{K(K-1)}{2}$ \mathcal{Q} operations will result in the perfectly ordered matrix $\mathbf{S} \downarrow$. ■

Since there is an increase in capacity at every step, $\mathcal{C}(\mathbf{S} \downarrow) \geq \mathcal{C}(\mathbf{S})$. This concludes the proof.

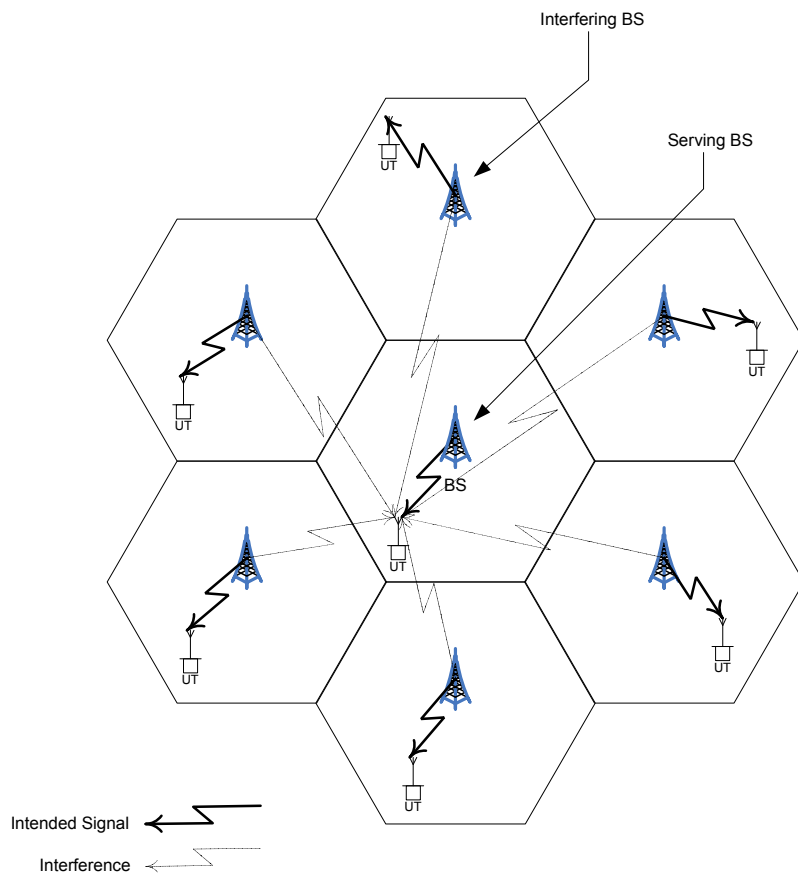


Fig. 1. An interference limited cellular system employing full resource reuse

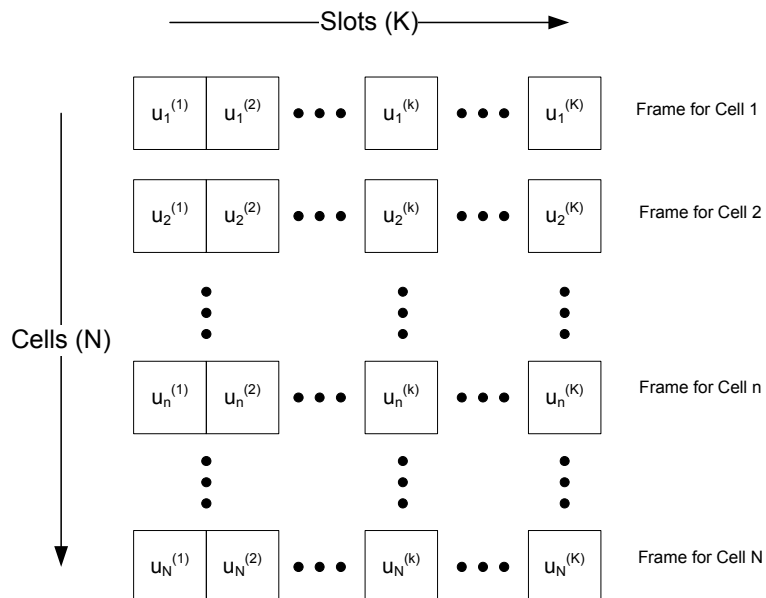


Fig. 2. Frame structure and resource fair scheduling matrix for N co-channel cells with K orthogonal slots. User $u_n^{(k)}$ is the user scheduled in cell n during slot k . Dimension K can be sub-frequencies, orthogonal codes or time-slots.

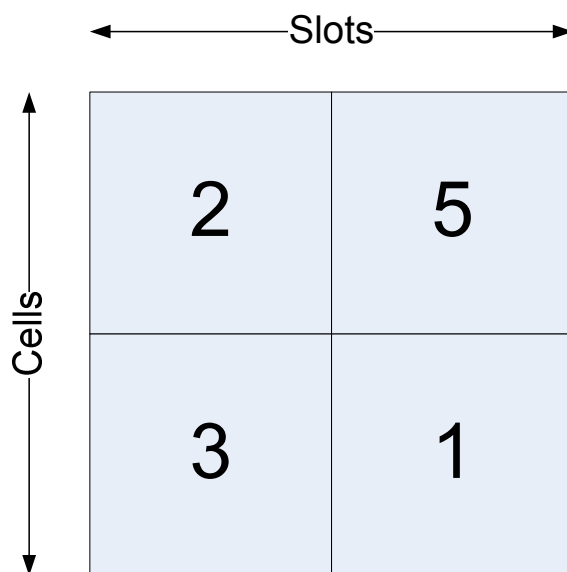


Fig. 3. Example of Scheduling Matrix for $N = K = 2$.

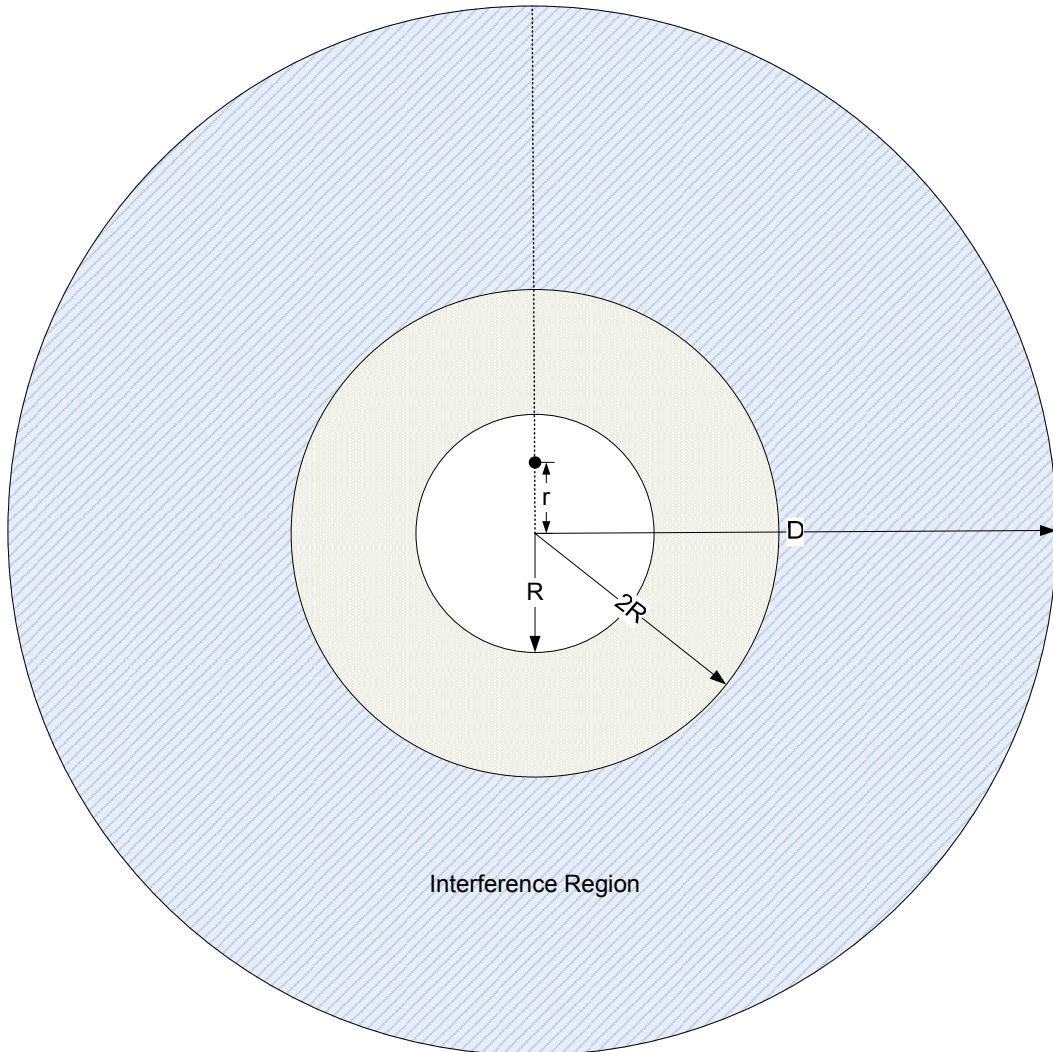


Fig. 4. Disc model for the downlink of a large wireless network. Interferers (APs) randomly spread over the blue shaded Interference Region. Protection region shaded gray in which no interferer can lie.

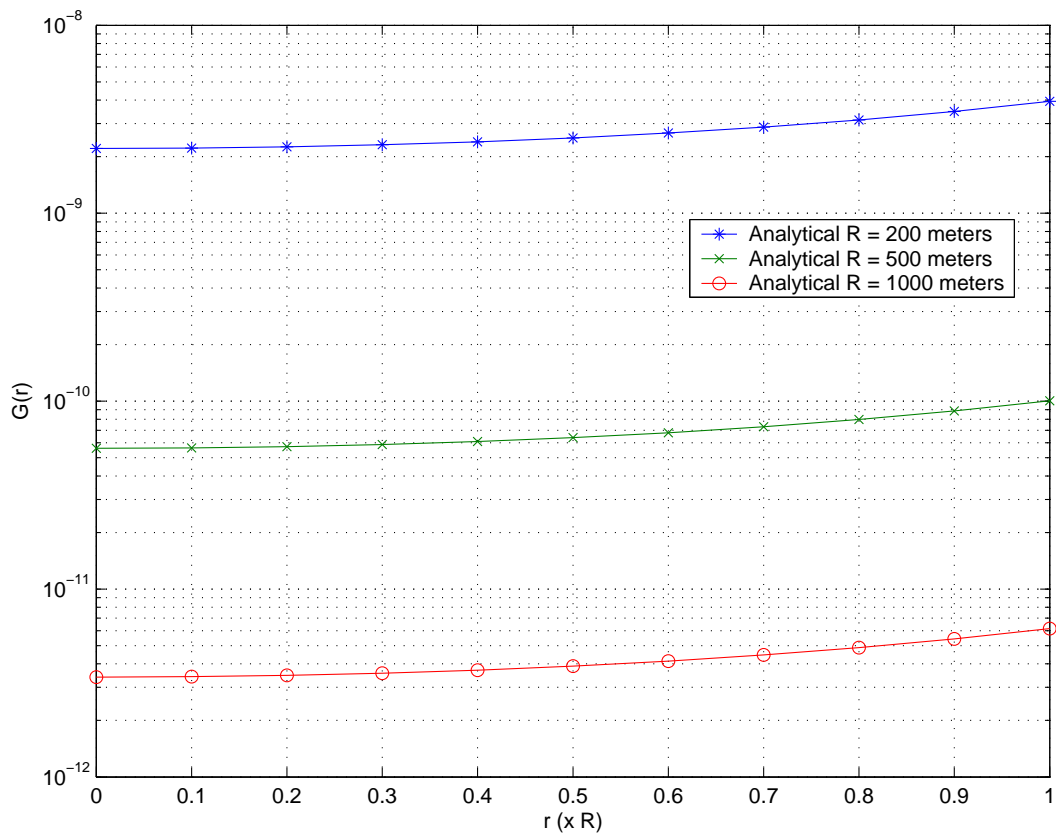


Fig. 5. Variation of expected interference from cell center to cell boundary for various cell sizes in a multicell wireless network. The interference gain increases monotonically, but slowly, from cell center to cell boundary.

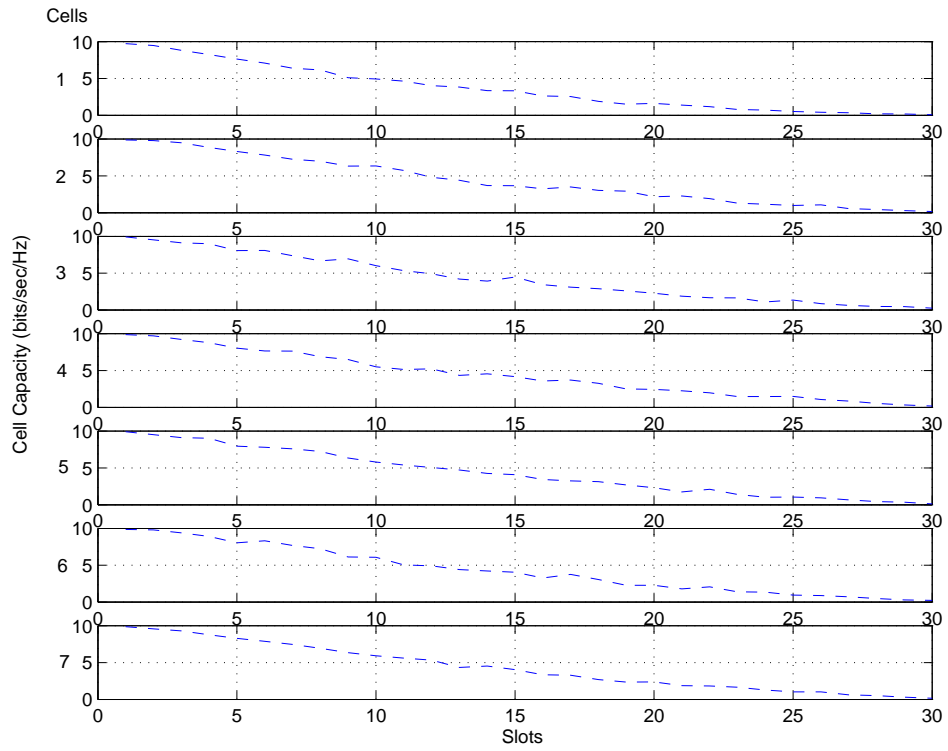


Fig. 6. Slot capacities for $N = 7$ cells, each with $K = 30$ slots. The capacities are highest in the first slots and lowest in the last slots due to the coupled effect of lower channel gain and higher level of interference. As expected, optimal network capacity scheduling gives rise to greater lack of fairness.

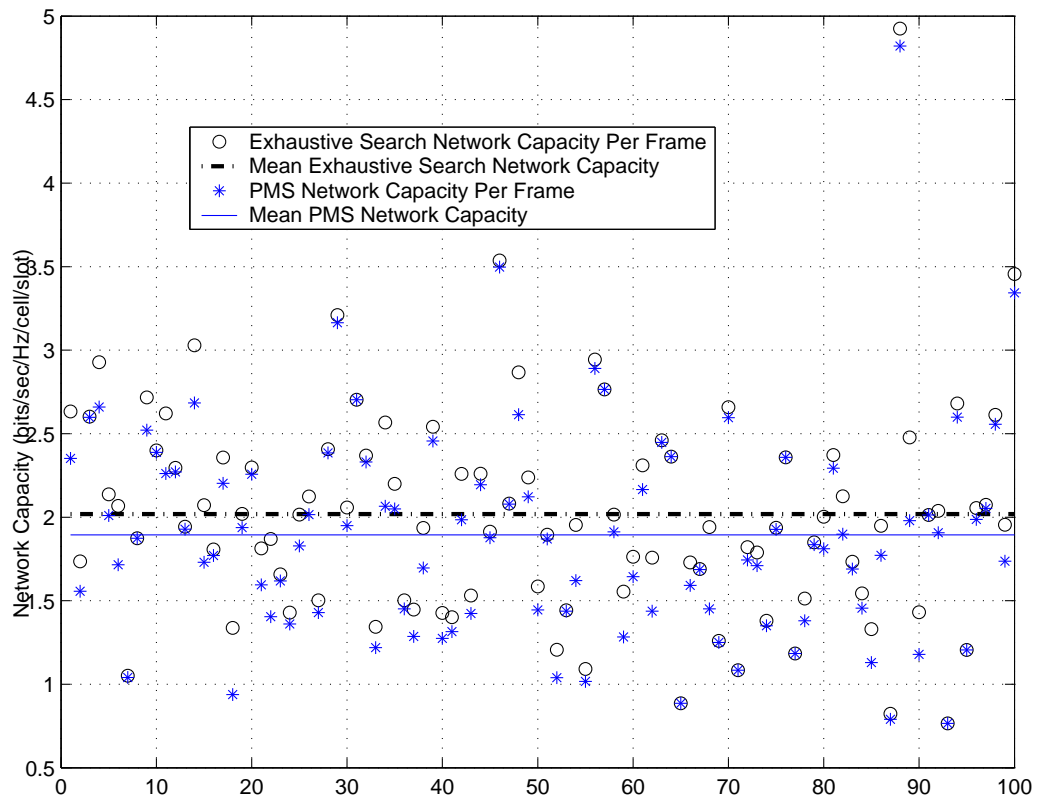


Fig. 7. Trace of network capacity values for 12 cells and 2 users per cell comparing Power Matched Scheduling PMS with the optimal scheduler based on exhaustive search. Independent channel realizations are generated on a frame by frame basis. The performance gap between PMS and the optimal scheduler is quite small.

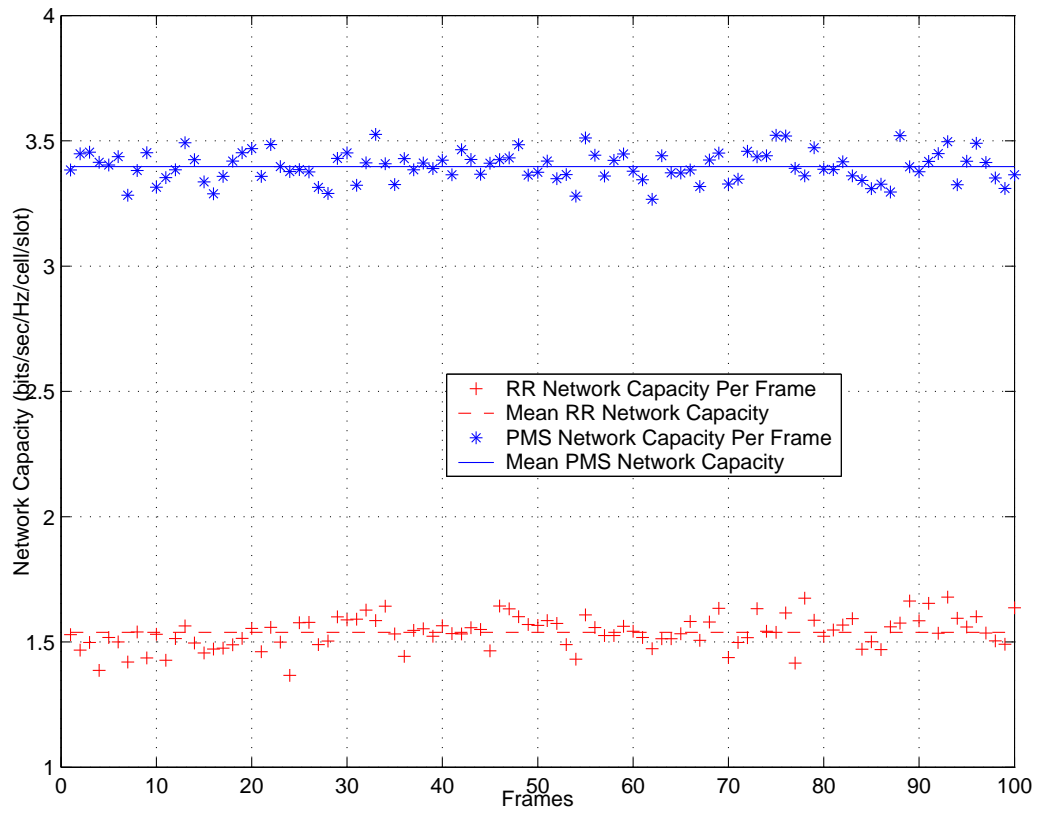


Fig. 8. Trace of network capacity values for 19 cells and 30 users per cell. Independent channel realizations are generated on a frame by frame basis. Power Matched Scheduling (PMS) provides substantial improvement as compared to Round Robin (RR) for large network sizes

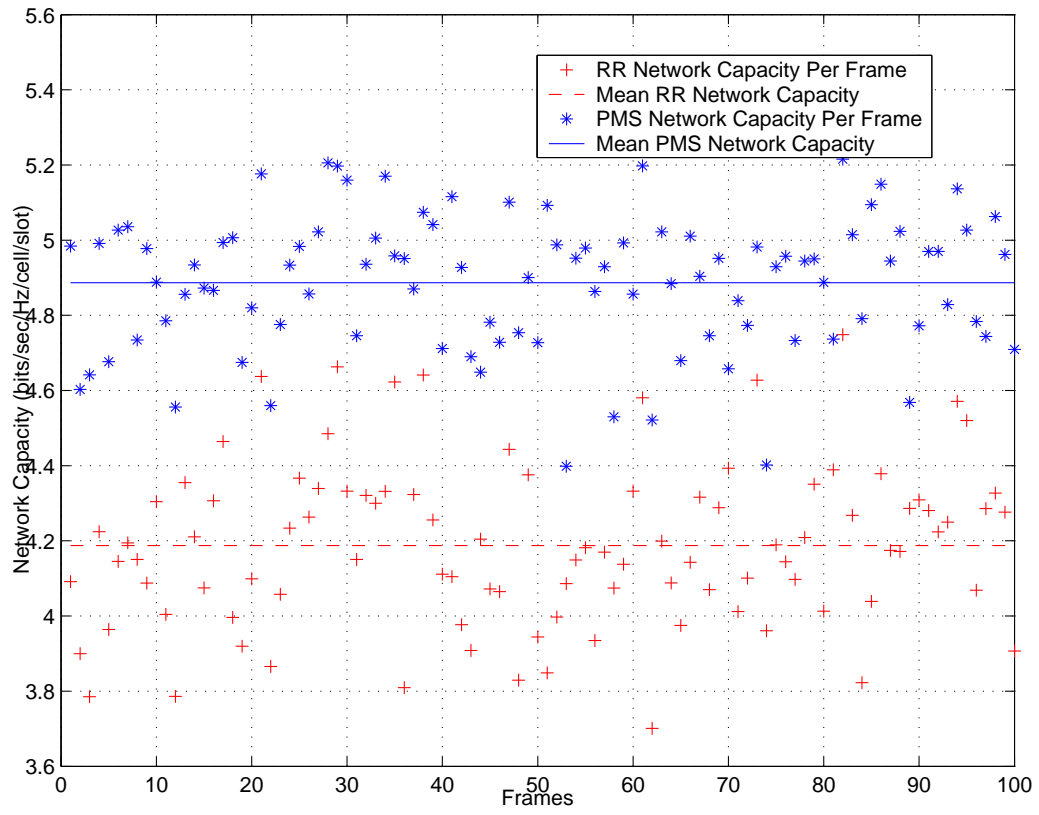


Fig. 9. Trace of network capacity values for 3 cells and 30 users per cell. Independent channel realizations are generated on a frame by frame basis. PMS provides better multicell capacity gain than RR even for small network sizes.

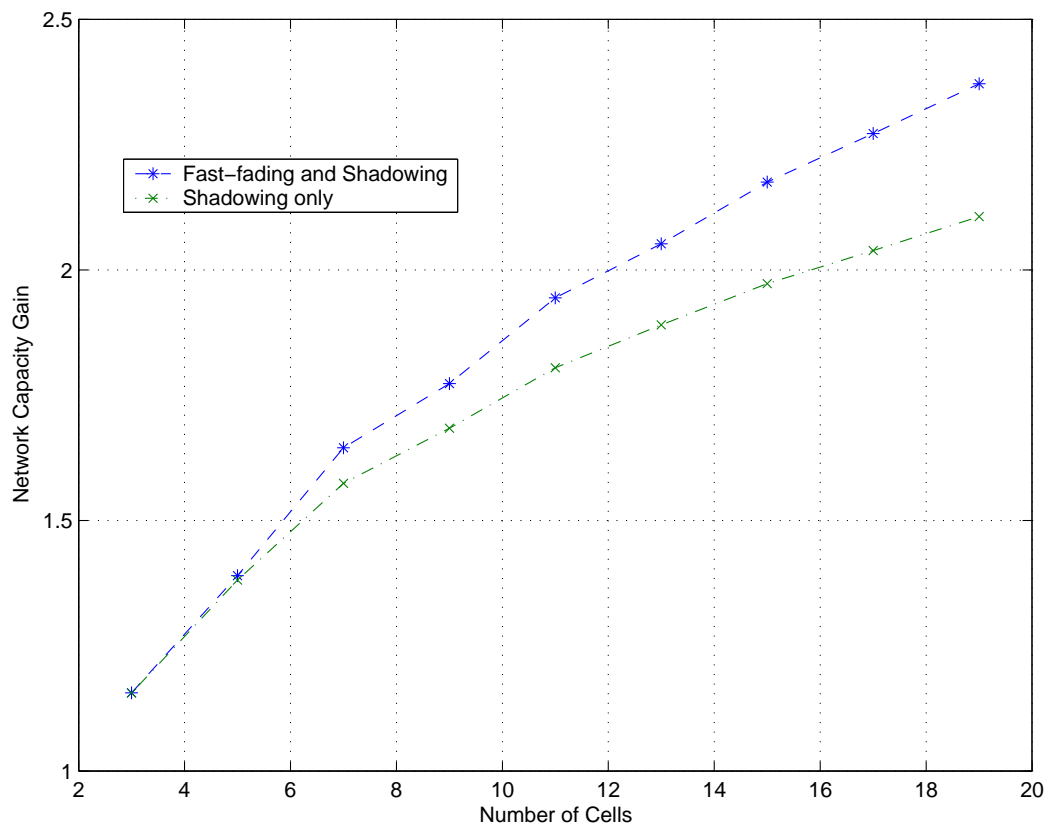


Fig. 10. Network capacity gain versus number of cells for different propagation scenarios. Network capacity gain is the ratio given by PMS network capacity upon RR network capacity. Gain increases with system size as optimization space increases. Greater channel variation increases performance gap between the two scheduling policies thereby increasing gain.