A DETERMINISTIC BLIND RECEIVER FOR MIMO OFDM SYSTEMS

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ABSTRACT

In this paper a deterministic PARAllel FACtor (PARAFAC) receiver is proposed, for Multiple Input Multiple Output (MIMO) Orthogonal Frequency Division Multiplexing (OFDM) systems. We show that the received signal forms a 4-way tensor whose dimensions are space, time, and frequency, and can be written as a sum of tensor products (over paths and users) of four tensors representing the channel (for two of them), the symbols, and the modulation. Parameters of this model are identified via an Alternating Least Squares (ALS) algorithm, called DEBRE, whose identifiability conditions are pointed out.

1. INTRODUCTION

The use of multiple antennas at both transmitter and receiver sides has gained great interest, since it increases the spectral efficiency in wireless transmissions. Some of the well known MIMO techniques are STBC and STTC [1] [2], that improve power efficiency by maximizing spatial diversity, the V-BLAST system proposed by Foschini et al. in [3], or SVD-based methods, just to name a few.

OFDM has become very popular in MIMO systems as it transforms a frequency-selective channel to a range of non-selective (flat fading) channels, hence reducing the Inter Symbol Interference (ISI) and improving the robustness against large delay spreads. The frequency spacing between subcarriers is chosen in such a way that the orthogonality of their corresponding time domain waveforms is maintained. Then, at the frequency where the received signal is evaluated, all other signals are zero.

OFDM is a block modulation scheme where F block symbols are transmitted over F orthogonal subcarriers. A Cyclic Prefix (CP) of length D is added to the beginning of each transmitted sequence to remove ISI caused by the channel time spread. The addition of the CP converts the linear convolution to a circular one. The channel length should be smaller than the CP, otherwise the orthogonality between subcarriers is affected, which causes Inter Carrier Interference (ICI).

OFDM modulation is adopted in three normalization standards of Local Area Network (LAN) systems: ETSI BRAN HIPERLAN12, IEEE 802.11a, and ARIB MMAC. The IEEE 802.11a LAN standard, for example, operates at raw data rates up to 54 Mb/s with a 20 Mhz channel spacing, thus yielding a bandwidth efficiency of 2.7 b/s/Hz.

Most of the existing MIMO OFDM models rely on matrix representations, with the two dimensions generally being space and time. Here, we fully exploit the multidimensional structure of the received signal and form a 4-way tensor whose dimensions are space, time, and frequency. The obtained 4-way tensor can be decomposed using PARAFAC decomposition, which was independently introduced by Harshman [4] and Caroll and Chang [5] in 1970. The PARAFAC model is very popular in Psychometrics and Chemometrics where it was first used along with its extension to higher orders [4] [5] [6]. It also finds applications in Signal Processing [7] [8] [9].

While the 2-way model suffers a rotational indeterminacy that yields an infinite set of solutions, the PARAFAC model enjoys a uniqueness property under simple conditions summarized in the Kruskal theorem [10], hence its importance. Moreover, the model being deterministic, smaller data blocks can be used without affecting the performance of the solution.

Some PARAFAC receivers for OFDM systems have been proposed in the literature. In [11], the authors proposed a blind receiver for SIMO OFDM systems, that estimates the Carrier Frequency Offset (CFO) in order to restore the orthogonality between subcarriers, and recovers the transmitted symbols. A model was also proposed for the MIMO case but simulations were not carried out. In [12], a unified tensor modeling for wireless communication systems was proposed. The general Parafac model particularly applies to MIMO OFDM systems by making appropriate choices in the structure of its matrix components, but once again, simulations were not carried out for the MIMO OFDM case.

Here, a tensor modeling is derived for MIMO OFDM systems. The DEterministic Blind REceiver (DEBRE) is computed via an ALS algorithm, which differs from the usual PARAFAC because of coupling between terms in the decomposition.

Other results in the same spirit have been presented in the case of CDMA systems [13] [14]. Those systems are similar to MIMO OFDM ones. In fact, the spreading code replaces the frequency in the third dimension; coupling is however different.

The paper is organized as follows: Section 2 presents the MIMO OFDM model along with the notation used through-

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out the paper. Parafac and DEBRE algorithms are presented in Section 3. Identifiability conditions are stated in section 4; a bound on the model parameters is given, and induces identifiability necessary conditions. Computer experiments eventually show that the latter necessary conditions are not always sufficient.

2. MODEL AND NOTATION

For the sake of simplicity we start with a Single Input Multiple Output (SIMO) OFDM system. The MIMO case will be addressed at the end of this section. Noise is ignored in the remaining, except when running computer experiments.

As stated in the introduction, OFDM is a block modulation scheme, where every sequence of F symbols $\boldsymbol{s}[m,n] = [s_{F-1}[m,n], ..., s_1[m,n], s_0[m,n]]^\intercal$, with m representing the number of the time interval (of size NF) ($m = 1, ..., M$), and n representing the OFDM block number ($n = 1, ..., N$), is converted from serial to parallel before being transmitted over F orthogonal subcarriers as shown in figure 1.

The symbols $\boldsymbol{s}[m,n]$ are modulated via IFFT (Inverse Fast Fourier Transform) to produce $\boldsymbol{x}[m,n] = [x_{F-1}[m,n], ..., x_1[m,n], x_0[m,n]]^\intercal$, and a CP, built from the D last samples of $\boldsymbol{x}[m,n]$, is added to the beginning of $\boldsymbol{x}[m,n]$. Then, the transmitted sequence is: $\boldsymbol{x}^{CP}[m,n] = [x_{F-1}[m,n], ..., x_0[m,n], x_{F-1}[m,n], ..., x_{F-D}[m,n]]^\intercal$.

In the present framework, the channel is assumed narrowband, non stationary due to the mobile motion, and have P resolvable paths. Hence, the sequence $\boldsymbol{y}_k[m,n]$ received on the k^{th} antenna, and obtained from $\boldsymbol{y}_k^{CP}[m,n] = [y_{k,F-1}[m,n], ..., y_{k,0}[m,n], y_{k,F-1}[m,n], ..., y_{k,F-D}[m,n]]^\intercal$ by ignoring the D first samples in order to remove ISI, has the following form :

$$\boldsymbol{y}_k[m,n] = \boldsymbol{h}_{k,m}\boldsymbol{x}^{CP}[m,n] \quad (1)$$

with: (for space reasons we denote by i the pair k,m, e.g. $h_{i,p} = h_{k,m,p}$)

$$\boldsymbol{h}_i = \begin{pmatrix} h_{i,0} & \dots & h_{i,P-1} & 0 & \dots & \\ 0 & h_{i,0} & \dots & h_{i,P-1} & 0 & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \boldsymbol{0}_{D-P+1} \\ 0 & \dots & h_{i,0} & \dots & h_{i,P-1} & \vdots \end{pmatrix}$$

$$h_{k,m,p} = \rho_{k,p} exp(-\frac{j\omega_f}{c}\boldsymbol{v}_{k,m}{}^\intercal\boldsymbol{u}_{k,p}) exp(-j\omega_f\tau_p)$$

where:

- $\rho_{k,p}$ denotes the p^{th} path gain to receive antenna k. In most applications, the antenna spacing Δ is considered to be less than half the wavelength, such that $\rho_{k,p}$ does not depend on the antenna ($\rho_{k,p} = \rho_p$)
- ω_f is the f^{th} subcarrier pulsation. As the channel is assumed narrowband, $\omega_f \approx \omega_c$, where ω_c is the central carrier pulsation. Hence, $h_{k,m,p}$ does not depend on ω_f

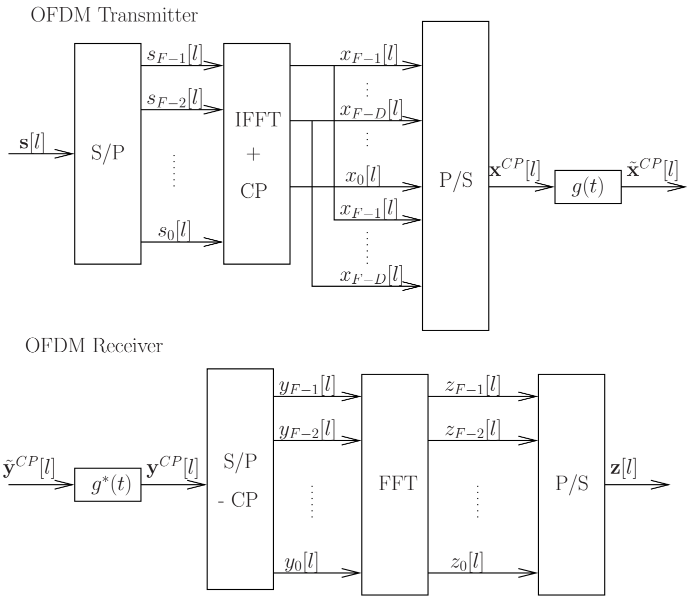

Fig. 1: OFDM Modulation. Index $[l] = [m,n]$

- $\boldsymbol{v}_{k,m}$ is the k^{th} antenna position at epoch m. Its value is : $v_{k,m} = v_k + \Delta d = v_k + v\Delta t = v_k + v(m-1)(F+D)N$, where v_k is the initial mobile position, v is the mobile speed that causes the change in distance Δd over time interval $\Delta t = (m-1)(F+D)N$
- $\boldsymbol{u}_{k,p}$ denotes the direction of arrival of the p^{th} path to antenna k
- τ_p is the delay due to the distance traveled along the p^{th} path

Expression (1) can alternatively be written as :

$$\boldsymbol{y}_k[m,n] = \begin{pmatrix} h_{i,0} & \dots & h_{i,P-1} & 0 & \dots \\ 0 & h_{i,0} & \dots & h_{i,P-1} & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ h_{i,1} & \dots & h_{i,P-1} & 0_{F-P} & h_{i,0} \end{pmatrix} \boldsymbol{x}[m,n]$$

After IFFT/FFT transformations, we obtain :

$$\boldsymbol{z}_k[m,n] = \begin{pmatrix} H_{F-1,i} & 0 & \dots & 0 \\ 0 & H_{F-2,i} & 0 & \vdots \\ \vdots & 0 & \ddots & \vdots \\ \vdots & \vdots & 0 & H_{0,i} \end{pmatrix} \boldsymbol{s}[m,n]$$

where $\boldsymbol{H}_{f,k,m} = \sum_{p=0}^{P-1} h_{k,m,p} e^{-\frac{j2\pi fp}{F}}$ is the overall channel frequency response for the f^{th} subcarrier and the k^{th} receive antenna at epoch m. With $r_{k,p} = \rho_{k,p}$, $t_{k,m,p} = exp(-\frac{j\omega_c}{c}\boldsymbol{v}_{k,m}{}^\intercal\boldsymbol{u}_{k,p})$, and $a_{f,p} = exp(-j\omega_c\tau_p)exp(-\frac{j2\pi fp}{F})$, the received signal forms a 4-way tensor, whose $(f,k,m,n)^{th}$ element is :

$$z_{f,k,m,n} = H_{f,k,m}\, s_{f,n} = \sum_{p=0}^{P-1} r_{k,p}\, t_{k,m,p}\, a_{f,p}\, s_{f,m,n}$$

Now, consider the case of a MIMO OFDM system with Q users. Then, the n^{th} symbol of the m^{th} epoch, received on the k^{th} antenna, at the subcarrier frequency f is:

$$\begin{aligned} z_{f,k,m,n} &= \sum_{q=1}^{Q} H_{f,k,m,q}\, s_{f,m,n,q} \\ z_{f,k,m,n} &= \sum_{q=1}^{Q}\sum_{p=0}^{P-1} r_{k,p,q}\, t_{k,m,p,q}\, a_{f,p,q}\, s_{f,m,n,q} \quad (2) \end{aligned}$$

We are interested in estimating, from the receiver outputs $z_{f,k,m,n}$, the above parameters under the constraints :

- **C1** the path gain $r_{k,p,q}$ is real positive
- **C2** the phase delay $t_{k,m,p,q}$ is of unit modulus, and depends on the antenna geometry
- **C3** the time delay $a_{f,p,q}$ is of unit modulus
- **C4** the transmitted symbols $s_{f,m,n,q}$ belong to a finite alphabet

3. PARAFAC DECOMPOSITION

For any pair of indices (p,q), with $1 \le p \le P$ and $1 \le q \le Q$, define index $[p,q]$ ranging from 1 to PQ by :

$$[p,q] = (p-1)Q + q$$

With this definition, the $[p,q]^{th}$ entry of a vector $\boldsymbol{u} \otimes \boldsymbol{v}$, where $\boldsymbol{u}$ is $1 \times P$ and $\boldsymbol{v}$ is $1 \times Q$, is $u_p v_q$, $\otimes$ being the Kronecker product. Equivalently, for any triplet of indices (m,p,q), with $1 \le m \le M$, define index $[m,p,q]$ ranging from 1 to MPQ by :

$$[m,p,q] = (m-1)PQ + (p-1)Q + q$$

Equation (2) is almost a 4-way PARAFAC model, except for the fact that, both $r_{k,p,q}$ and $t_{k,m,p,q}$ depend on k, both $t_{k,m,p,q}$ and $s_{f,m,n,q}$ depend on m, and both $a_{f,p,q}$ and $s_{f,m,n,q}$ depend on f. The most popular algorithm to fit the PARAFAC model is the ALS algorithm. It consists of estimating, in the Least Squares (LS) sense, one of the four loading matrices, e.g. $\boldsymbol{S}$, with the three remaining matrices, e.g. $\boldsymbol{R}$, $\boldsymbol{T}$, and $\boldsymbol{A}$, being fixed to previously obtained values. The same steps are repeated until a convergence criterion is met. $\boldsymbol{R}$ (resp. $\boldsymbol{T}$) is the $K \times PQ$ (resp. $KM \times PQ$) matrix whose $(k,[p,q])^{th}$ (resp. $([k,m],[p,q])^{th}$) entry is $r_{k,p,q}$ (resp. $t_{k,m,p,q}$). Entries of the $F \times PQ$ matrix $\boldsymbol{A}$ are denoted by $a_{f,p,q}$, and those of the $MFN \times Q$ matrix $\boldsymbol{S}$ are denoted by $s_{f,m,n,q}$.

The DEBRE receiver proposed is based on an ALS algorithm, but differs from the usual PARAFAC since matrix $\boldsymbol{S}$ depends on f and m, and $\boldsymbol{T}$ depends on k. With matrices $\boldsymbol{T}$, $\boldsymbol{A}$, and $\boldsymbol{S}$ set to random initial values, DEBRE performs the four following steps at every iteration.

1. **Estimate $\boldsymbol{R}$:** For every fixed k, $1 \le k \le K$, the matrix unfolding of the $K \times M \times F \times N$ tensor $\boldsymbol{\mathcal{Z}}$ in the first mode is :

$$\boldsymbol{Z}^{1\times FMN}(k) = [r_{k,1,1}, ..., r_{k,P,Q}]\boldsymbol{B}(k)$$

where $\boldsymbol{B}(k)$ is a $PQ \times FMN$ matrix for every k, whose $([p,q],[f,m,n])^{th}$ entry is $b_{[p,q],[f,m,n]}(k) = t_{k,m,p,q} a_{f,p,q} s_{f,m,n,q}$. The estimate $\hat{\boldsymbol{R}}(k,:)$ of the k^{th} line of $\boldsymbol{R}$ is obtained by minimizing in the LS sense the error :

$$\Upsilon = \|\boldsymbol{Z}^{1\times FMN}(k) - [r_{k,1,1}, ..., r_{k,P,Q}]\boldsymbol{B}(k)\|_F^2$$

where $\|\bullet\|_F$ is the Frobenius norm. With $\boldsymbol{B}(k)$ fixed, $\hat{\boldsymbol{R}}(k,:)$ is given by :

$$\hat{\boldsymbol{R}}(k,:) = \boldsymbol{Z}^{1\times FMN}(k)\boldsymbol{B}^{+}(k)$$

where $(^{+})$ denotes the Moore-Penrose pseudo-inverse. We perform the same operations for every value of k to obtain successive lines of $\hat{\boldsymbol{R}}$. In order to achieve uniqueness of $\hat{\boldsymbol{R}}$, $\boldsymbol{B}(k)$ should be fat for every value of k, hence $PQ \le FMN$.

2. **Estimate $\boldsymbol{T}$:** For every fixed k and m, $1 \le m \le M$, unfold tensor $\boldsymbol{\mathcal{Z}}$ in the second mode :

$$\boldsymbol{Z}^{1\times FN}(k,m) = [t_{k,m,1,1}, ..., t_{k,m,P,Q}]\boldsymbol{C}(k,m)$$

where $\boldsymbol{C}(k,m)$ is a $PQ \times FN$ matrix for every k and m, whose $([p,q],[f,n])^{th}$ entry is $c_{[p,q],[f,n]}(k,m) = r_{k,p,q} a_{f,p,q} s_{f,m,n,q}$. We use updated value of $\boldsymbol{R}$ obtained from the previous step. The estimate $\hat{\boldsymbol{T}}([k,m],[:,:])$ of the $((k-1)M+m)^{th}$ line of $\boldsymbol{T}$, in the LS sense, is then given by :

$$\hat{\boldsymbol{T}}([k,m],[:,:]) = \boldsymbol{Z}^{1\times FN}(k,m)\boldsymbol{C}(k,m)^{+}$$

The same operations are performed for every values of k and m in order to obtain successive lines of $\boldsymbol{T}$. In order to achieve uniqueness, we should have $PQ \le FN$.

3. **Estimate $\boldsymbol{A}$:** For every fixed f, $1 \le f \le F$, the matrix unfolding of tensor $\boldsymbol{\mathcal{Z}}$ in the third mode is :

$$\boldsymbol{Z}^{1\times KMN}(f) = [a_{f,1,1}, ..., a_{f,P,Q}]\,\boldsymbol{D}(f)$$

where $\boldsymbol{D}(f)$ is a $PQ \times KMN$ matrix for every f, whose $([p,q],[k,m,n])^{th}$ entry is $d_{[p,q],[k,m,n]}(f) = r_{k,p,q} t_{k,m,p,q} s_{f,m,n,q}$. We use updated values of $\boldsymbol{R}$ and $\boldsymbol{T}$, obtained from the previous steps. The estimate $\hat{\boldsymbol{A}}(f,:)$ of the f^{th} line of $\boldsymbol{A}$, in the LS sense, is then given by :

$$\hat{\boldsymbol{A}}(f,:) = \boldsymbol{Z}^{1\times KMN}(f)\boldsymbol{D}^{+}(f)$$

We perform the same operations for every value of f to obtain successive lines of $\hat{\boldsymbol{A}}$. To achieve uniqueness, we should have $PQ \le KMN$.

4. **Estimate $\boldsymbol{S}$:** For every fixed f and m, unfold tensor $\boldsymbol{\mathcal{Z}}$ in the fourth mode :

$$\boldsymbol{Z}^{N\times K}(f,m) = \begin{bmatrix} s_{f,m,1,1} & \cdots & s_{f,m,1,Q} \\ \vdots & \vdots & \vdots \\ s_{f,m,N,1} & \cdots & s_{f,m,N,Q} \end{bmatrix} \boldsymbol{E}(f,m)$$

where $\boldsymbol{E}(f,m)$ is a $Q \times K$ matrix whose $(q,k)^{th}$ entry is $e_{q,k}(f,m) = \sum_p r_{k,p,q} t_{k,m,p,q} a_{f,p,q}$. We use the updated values of $\boldsymbol{R}$, $\boldsymbol{T}$, and $\boldsymbol{A}$, from the previous steps. The estimate $\hat{\boldsymbol{S}}([f,m,:],:)$ of the $((f-1)M+m)^{th}$ block of size $N \times Q$ of $\boldsymbol{S}$ is given by :

$$\hat{\boldsymbol{S}}([f,m,:],:) = \boldsymbol{Z}^{N\times K}(f,m)\boldsymbol{E}^{+}(f,m)$$

The same operations are performed for every values of f and m in order to obtain successive $N \times Q$ blocks of $\boldsymbol{S}$. To achieve uniqueness, Q should verify $Q \leq K$.

Once we update the four loading matrices $\boldsymbol{R}$, $\boldsymbol{T}$, $\boldsymbol{A}$, and $\boldsymbol{S}$, we check for convergence by comparing the change in the error, $|\Upsilon^{it} - \Upsilon^{it-1}|$, to a predefined threshold.

4. IDENTIFIABILITY

4.1. Necessary conditions

The previous ALS algorithm may deliver a unique solution at every iteration if the following necessary conditions

$$PQ \leq \min(FMN, KMN) \quad (3)$$
$$Q \leq K \quad (4)$$

are satisfied. Inequality (3) can be easily satisfied by taking large number F of subcarriers, or large time interval N. However, (4) is more constrained as it imposes more sensors than users.

Another necessary condition for the ALS algorithm is that the number of parameters to be identified in model (2) does not exceed the number of degrees of freedom of the data tensor, which yields:

$$KMFN \geq KPQ + KMPQ + FPQ + MFNQ - 2P - 2Q$$

For identifiability of the DEBRE model, let's first consider the 3-way PARAFAC model: $z_{i,j,k} = \sum_{r=1}^{R} h_{i,r}\, a_{j,r}\, s_{k,r}$. Kruskal showed in [10] that the model is essentially unique if (sufficient condition): $k_H + k_A + k_S \geq 2R + 2$, where k_A is the Kruskal rank of matrix $\boldsymbol{A}$ defined in [10].

Essential uniqueness is understood up to a multiplication by a permutation and a diagonal matrix, which means that, under Kruskal condition, any other decomposition $(\bar{\boldsymbol{H}}, \bar{\boldsymbol{A}}, \bar{\boldsymbol{S}})$ of $\boldsymbol{\mathcal{Z}}$ verifies : $\bar{\boldsymbol{H}} = \boldsymbol{H}\boldsymbol{\Pi}\boldsymbol{\Lambda}_H$, $\bar{\boldsymbol{A}} = \boldsymbol{A}\boldsymbol{\Pi}\boldsymbol{\Lambda}_A$, and $\bar{\boldsymbol{S}} = \boldsymbol{S}\boldsymbol{\Pi}\boldsymbol{\Lambda}_S$, where $\boldsymbol{\Pi}$ is a permutation matrix, and $\boldsymbol{\Lambda}_H$, $\boldsymbol{\Lambda}_A$, and $\boldsymbol{\Lambda}_S$ are $R \times R$ diagonal matrices with elements λ_r^H, λ_r^A, and λ_r^S $(r = 1, ..., R)$ respectively in their diagonal, and verifying : $\boldsymbol{\Lambda}_H\boldsymbol{\Lambda}_A\boldsymbol{\Lambda}_S = \boldsymbol{I}$ ($\boldsymbol{I}$ is the $R \times R$ identity matrix). $\boldsymbol{\Lambda}_H$, $\boldsymbol{\Lambda}_A$, and $\boldsymbol{\Lambda}_S$ introduce a scale factor ambiguity on the columns of matrices $\boldsymbol{H}$, $\boldsymbol{A}$, and $\boldsymbol{S}$ respectively, which can be written as :

$$z_{i,j,k} = \sum_{r=1}^{R} \left(\frac{h_{i,r}}{\lambda_r^H}\right)\left(\frac{a_{j,r}}{\lambda_r^A}\right)(s_{k,r}\underbrace{\lambda_r^H \lambda_r^A}_{=1/\lambda_r^S})$$

If both $\boldsymbol{A}$ and $\boldsymbol{S}$ depend on j, for example :

$$z_{i,j,k} = \sum_{r=1}^{R} h_{i,r} a_{j,r} s_{j,k,r} \quad (5)$$

we have an additional indeterminacy, which is a scale factor on lines of $\boldsymbol{A}$ and $\boldsymbol{S}$. In fact, expression (5) can be written as :

$$z_{i,j,k} = \sum_{r=1}^{R} \left(\frac{h_{i,r}}{\lambda_r^H}\right)\left(\frac{a_{j,r}}{\gamma_j^A \lambda_r^A}\right)(s_{k,r}\,\gamma_j^A \lambda_r^H \lambda_r^A)$$

Hence, any other triplet $(\bar{\boldsymbol{H}}, \bar{\boldsymbol{A}}, \bar{\boldsymbol{S}})$, such that :

$$\bar{h}_{i,r} = \frac{h_{i,r}}{\lambda_r^H}\ ,\ \bar{a}_{j,r} = \frac{a_{j,r}}{\gamma_j^A \lambda_r^A}\ ,\ \bar{s}_{j,k,r} = \frac{s_{j,k,r}}{\gamma_r^S \lambda_r^S}$$

with $\left(\gamma_r^S = \frac{1}{\gamma_r^A}\ ,\ \lambda_r^S = \frac{1}{\lambda_r^H \lambda_r^A}\right)$ is also a decomposition of tensor $\boldsymbol{\mathcal{Z}}$.

Now, by generalizing to the model of expression (2), we have the following indeterminacies :

$$z_{k,m,f,n} = \sum_{p=0}^{P-1}\sum_{q=1}^{Q} \left(\frac{r_{k,p,q}}{\alpha_k^R \gamma_p^R \lambda_q^R}\right)\left(\frac{t_{k,m,p,q}\,\alpha_k^R}{\beta_m^T \gamma_p^T \lambda_q^T}\right) \left(\frac{a_{f,p,q}\,\gamma_p^R \gamma_p^T}{\delta_f^A \lambda_q^A}\right)(s_{f,n,m,q}\,\delta_f^A \beta_m^T \lambda_q^R \lambda_q^T \lambda_q^A)$$

All those indeterminacies can be fixed by imposing constraints **C1**, **C2**, **C3**, and **C4**, presented in section 2.

4.2. Generic sufficient conditions

In accordance with **C1**-**C4**, and considering a stationary channel during one OFDM symbol, we obtain:

$$z_{k,f,n} = \sum_{p=0}^{P-1}\sum_{q=1}^{Q} h_{k,p,q} e^{jf\theta_p} s_{f,n,q} \quad (6)$$

Another way to assess uniqueness is to check whether the number of free parameters in the model for a given value of the pair (P,Q) is smaller than the dimensionality of the set generated by the parametric model (6). Yet the latter can be obtained as the dimension of the tangent space, that is, by the rank of the Jacobian [15]. For instance, for (F,K,N)=(4,5,4), the dimensionality of the set and the number of free parameters are given as a function of (P,Q) by table 1. This shows that for (P,Q)=(2,2), there are infinitely many solutions, whereas for (P,Q)=(3,1), there exists an essentially unique LS solution.

5. COMPUTER RESULTS

We run simulations under the conditions stated before, with the sources $s_{f,m,n,q}$ being QPSK. When initial values for

	$Q = 1$	$Q = 2$	$Q = 3$	$Q = 4$
$P = 1$	0	2	6	12
$P = 2$	-1	1	7	29
$P = 3$	0	4	20	46
$P = 4$	3	13	33	63

Table 1: Dimentionality of the set of solution as a function of (P, Q).

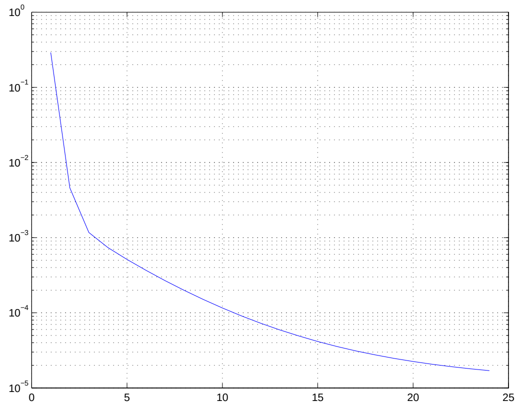

Fig. 2: The error Υ as a function of the number of iterations, with initial matrices close to the original ones.

matrices $\boldsymbol{R}$, $\boldsymbol{T}$, $\boldsymbol{A}$, and $\boldsymbol{S}$ are taken close to the original ones (figure 2), the ALS algorithm provides good estimated matrices as shown by table 2. At the opposite, when we initialize randomly, the ALS algorithm still converges (figure 3) but the estimated matrices are wrong.

Initialization vs. Gap	$\boldsymbol{R}$	$\boldsymbol{T}$	$\boldsymbol{A}$	$\boldsymbol{S}$
Neighborhood of original matrices	0.013	0.028	0.009	0.012
Random	1.279	2.049	0.976	1.484

Table 2: The gap between estimated matrices and original ones for different initializations.

This means that the necessary conditions stated in section 4 are not sufficient for uniqueness of the DEBRE. This was expected by results of section 4.2.

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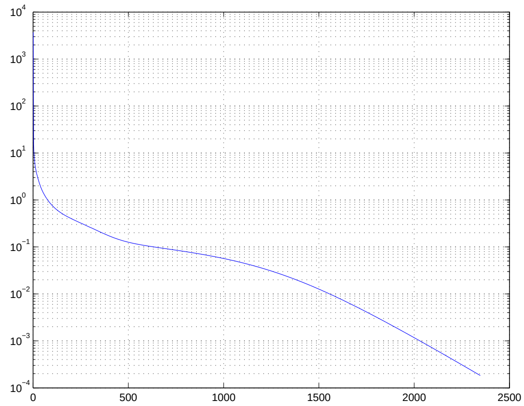

Fig. 3: The error Υ as a function of the number of iterations, with random initial matrices.

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