

LOW-RATE FEEDBACK FOR REAL MEASURED TEMPORALLY CORRELATED MIMO CHANNELS

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ABSTRACT

This work presents a summary of a proposal for a feedback scheme in a multi-input-multi-output (MIMO) communication system based on a differential quantization strategy applied to the channel response. The performance of this scheme is evaluated using real data and channel measurements obtained with the Eurecom's MIMO OpenAir Sounder (EMOS). More concretely, the impact of having a delay in the feedback link is also studied in terms of a loss of performance in the communication through several simulation results.

Topics: Precoding and limited feedback, Multi-antenna channel measurements, MIMO systems.

1. INTRODUCTION

Multi-input-multi-output (MIMO) communication systems are shown to provide improved performance when compared to single-antenna configurations, specially when both the transmitter and the receiver have some kind of channel state information (CSI). A possibility to obtain CSI at the transmitter consists in the exploitation of a low rate feedback channel from the receiver to the transmitter.

In the literature, several feedback schemes have been proposed in order to provide CSI to the transmitter side. For time-varying channels, where the coherence time is higher than the time difference between consecutive feedback instants, a good approach consists in quantizing the channel response in a differential way. This lowers the required feedback load or improves the quality of the quantization for a fixed capacity of the feedback link. Taking this philosophy, there are several techniques, such as the direct scalar quantization of the entries of the channel variation matrix, or more sophisticated approaches, such as those based on geodesic curves over Grassmannian manifolds or correlation-type matrices [1–3].

Another important aspect is the feedback delay. Such a delay causes a mismatch between the true channel and the available CSI, and consequently between the actual design of the transmitter and the optimum one, which results in a degradation of the performance.

The main objective of this paper is to evaluate experimentally in a real environment the impact of different val-

ues of the feedback delay on the system performance. This will be done taking as example the technique presented in [3] for feedback and channel quantization (this technique will be summarized in the subsequent sections) exploiting real channel measurements obtained with the Eurecom's MIMO OpenAir Sounder (EMOS) [4, 5].

In general terms, differential quantization is based on a quantization of the difference between the CSI at consecutive feedback intervals, instead of quantizing the complete CSI every time [6]. Depending on the design criterion and the allowed computational complexity, different strategies arise.

Some techniques can be based on the quantization of the variations of the MIMO channel matrix $\mathbf{H}(n)$ itself or even, on the differential quantization of the strongest right eigenspaces spanned by such matrices [1]. The technique that we will use in this paper to evaluate experimentally the performance of the communication setup corresponds to reference [3]. It relies on the fact that in general, all the joint transmitter-receiver designs for MIMO channels and different quality criteria (SNR, mean square error, mutual information, etc.) depend on the channel response matrix $\mathbf{H}(n)$ only through the channel correlation matrix defined as $\mathbf{R}_H(n) = \mathbf{H}^H(n)\mathbf{H}(n)$ [7]. Taking this into account, a possible strategy consists in applying a differential quantization exploiting the intrinsic geometry of the set of positive definite Hermitian matrices by means of the use of geodesic curves, as suggested in [2].

2. SYSTEM AND SIGNAL MODELS

This section and the next one summarize some of the ideas presented in [3] concerning the technique that will be used in this paper to evaluate a realistic system performance according to real channel measurements.

We consider the transmission through a MIMO channel with n_T and n_R transmit and receive antennas represented at time instant n by matrix $\mathbf{H}(n) \in \mathbb{C}^{n_R \times n_T}$. The n_R received signals at the same time instant, assuming a linear transmitter, can be expressed as

$$\mathbf{y}(n) = \mathbf{H}(n)\mathbf{B}(\hat{\mathbf{R}}_H(n))\mathbf{x}(n) + \mathbf{w}(n) \in \mathbb{C}^{n_R}, \quad (1)$$

where $\mathbf{x}(n) \in \mathbb{C}^{n_S}$ represents the n_S streams of signals to be transmitted with $\mathbb{E}[\mathbf{x}(n)\mathbf{x}^H(n)] = \mathbf{I}$, and $\mathbf{B} \in \mathbb{C}^{n_T \times n_S}$ is the linear transmitter matrix. Note that we explicitly indicate that the transmitter depends on the available estimate of the channel correlation matrix $\hat{\mathbf{R}}_H(n)$, where the exact matrix is $\mathbf{R}_H(n) = \mathbf{H}^H(n)\mathbf{H}(n)$. The AWGN at the receiver is represented by $\mathbf{w}(n) \in \mathbb{C}^{n_R}$ with $\mathbb{E}[\mathbf{w}(n)\mathbf{w}^H(n)] = \sigma_w^2\mathbf{I}$.

This work was supported by the Catalan Government under grants 2005SGR-00690 and 2005SGR-00996; by the Spanish Government under projects TEC2005-08122-C03 (ULTRA-PROCESS) and 2A103 (MI-MOWA) from MEDEA+ program (AVANZA I+D TSI-020400-2008-150); by the European Commission under projects NEWCOM++ (216715) and CHORIST; and by Eurecom.

In the system setup, it will be considered that the receiver knows perfectly the current channel matrix $\mathbf{H}(n)$ and that the transmitter designs \mathbf{B} assuming that the available CSI at its side represented by $\widehat{\mathbf{R}}_H(n)$ is also perfect, despite not being true in a general situation. The transmitter design can be done according to different criteria, such as the maximization of the mutual information or SNR, or the minimization of the MSE or the BER, among others. In all the cases, the optimum transmitter has been shown to depend only on the channel correlation matrix $\mathbf{R}_H(n)$ [7]. For each of them a *cost function* $d(\widehat{\mathbf{R}}_H(n), \mathbf{H}(n))$ can be defined, where the design objective is its minimization. A couple of examples of cost functions are given below, although any criterion can be applied (we drop the dependency with respect to the time index n for the sake of clarity in the notation):

- *Maximization of the SNR with single beamforming* ($n_S = 1$):

$$d(\widehat{\mathbf{R}}_H(n), \mathbf{H}(n)) = -\frac{1}{\sigma_w^2} \|\mathbf{H}\mathbf{B}\|_F^2, \quad (2)$$

where the transmission matrix $\mathbf{B} \in \mathbb{C}^{n_T \times 1}$ is defined as

$$\mathbf{B}(\widehat{\mathbf{R}}_H(n)) = \sqrt{P_T} \mathbf{u}_{\max}(\widehat{\mathbf{R}}_H(n)), \quad (3)$$

and $\mathbf{u}_{\max}(\cdot)$ stands for the unit-norm eigenvector of maximum associated eigenvalue. P_T represents the maximum transmission power, i.e., $\|\mathbf{B}\|_F^2 \leq P_T$, where subindex F stands for the Frobenius norm.

- *Maximization of the mutual information:*

$$d(\widehat{\mathbf{R}}_H(n), \mathbf{H}(n)) = -\log_2 \left| \mathbf{I} + \frac{1}{\sigma_w^2} \mathbf{B}\mathbf{B}^H \mathbf{H}^H \mathbf{H} \right|, \quad (4)$$

where the transmission matrix $\mathbf{B} \in \mathbb{C}^{n_T \times n_S}$ is defined as

$$\mathbf{B}(\widehat{\mathbf{R}}_H(n)) = \widetilde{\mathbf{U}}(n) \mathbf{P}^{1/2}(n), \quad \mathbf{P}(n) = \text{diag}(p_1, \dots, p_{n_S}), \quad (5)$$

and $\widetilde{\mathbf{U}}(n)$ consists of n_T columns that are the n_S eigenvectors of $\widehat{\mathbf{R}}_H(n)$ associated to its n_S maximum eigenvalues $\{\lambda_i\}_{i=1}^{n_S}$. The power $\mathbf{P}(n)$ is allocated according to the waterfilling solution ($p_i = \max\{0, \mu - 1/\lambda_i\}$ where μ is a constant such that $\sum_{i=1}^{n_S} p_i = P_T$) [7].

The next section is devoted to summarize algorithm [3] for quantizing the actual correlation matrix \mathbf{R}_H (instead of \mathbf{H}) from the receiver to the transmitter in a differential way. Since \mathbf{R}_H belongs to the set of Hermitian positive definite matrices,¹ exploiting its inherent geometry will improve the performance of the quantization.

3. ALGORITHM DESCRIPTION FOR QUANTIZATION IN FEEDBACK LINK

In this section first we will give some comments on the concept of geodesic curves on the set of positive definite Hermitian matrices and then we will summarize the basic ideas concerning the algorithm presented in [3] for differential quantization.

¹In the following, it will be assumed that the channel correlation matrix is strictly positive definite. If this cannot be guaranteed because, for example, $n_R < n_T$, it is possible to work with extended correlation matrices defined as $\mathbf{R}_H = \mathbf{H}^H \mathbf{H} + \varepsilon \mathbf{I}$, $\varepsilon > 0$, which are positive definite by construction.

3.1 Geodesic Curves

As shown in [2] the set of Hermitian positive definite matrices $\mathcal{S} = \{\mathbf{R} \in \mathbb{C}^{n_T \times n_T} : \mathbf{R}^H = \mathbf{R}, \mathbf{R} \succ \mathbf{0}\}$ is a convex cone², i.e., $\forall \mathbf{R}_1, \mathbf{R}_2 \in \mathcal{S}, \forall s \geq 0, \mathbf{R}_1 + s\mathbf{R}_2 \in \mathcal{S}$ [8]. The characterization of this set is described properly by means of differential geometry, which states a set of definitions for the distance, scalar products and routes within this set:

- *Scalar product and norm:* At any point in this set \mathcal{S} given by \mathbf{R} (also named as *base point*), the scalar product between two Hermitian matrices \mathbf{A} and \mathbf{B} is defined as $\langle \mathbf{A}, \mathbf{B} \rangle_{\mathbf{R}} = \text{Tr}(\mathbf{R}^{-1} \mathbf{A} \mathbf{R}^{-1} \mathbf{B})$. This definition implies that the norm is defined as $\|\mathbf{A}\|_{\mathbf{R}} = \sqrt{\text{Tr}(\mathbf{R}^{-1} \mathbf{A} \mathbf{R}^{-1} \mathbf{A})}$.
- *Geodesic curve:* Let us take two points \mathbf{R}_1 and \mathbf{R}_2 in the set \mathcal{S} . Then, the geodesic curve, which is the curve connecting these points with minimum distance and with all its points belonging to \mathcal{S} , is given by

$$\Gamma(t) = \mathbf{R}_1^{1/2} \exp(t\mathbf{C}) \mathbf{R}_1^{1/2}, \quad (6)$$

where $\mathbf{C} = \log(\mathbf{R}_1^{-1/2} \mathbf{R}_2 \mathbf{R}_1^{-1/2})$, $\Gamma(0) = \mathbf{R}_1$, and $\Gamma(1) = \mathbf{R}_2$. The derivative of the geodesic curve at $t = 0$, which is in fact the *direction* of such curve at $t = 0$, is given by the Hermitian matrix $\Gamma'(0) = \mathbf{R}_1^{1/2} \mathbf{C} \mathbf{R}_1^{1/2}$.

- *Geodesic distance:* The geodesic distance between any two points in \mathcal{S} is given by the length of the geodesic curve that connects them. According to the previous notation, it can be shown that this distance is given by

$$\text{dist}_g(\Gamma(0), \Gamma(t)) = |t| \|\mathbf{C}\|_F, \Rightarrow \text{dist}_g(\mathbf{R}_1, \mathbf{R}_2) = \|\mathbf{C}\|_F. \quad (7)$$

or, using an equivalent expression,

$$\text{dist}_g(\mathbf{R}_1, \mathbf{R}_2) = \left(\sum_i |\log \lambda_i|^2 \right)^{1/2}, \quad (8)$$

where $\{\lambda_i\}$ are the eigenvalues of matrix $\mathbf{R}_1^{-1/2} \mathbf{R}_2 \mathbf{R}_1^{-1/2}$.

3.2 Differential Quantization

The fundamentals of the algorithm proposed in [3] are based on a differential quantization of the channel correlation matrix $\mathbf{R}_H(n)$. The objective is to minimize the cost function as presented in section 2, which can be related to the quality measure of the system and, therefore, the receiver has to know which kind design that the transmitter will perform. If a more general setup is to be applied so that the feedback can be used for any transmitter design, another cost function could be added which is simply the geodesic distance between the actual channel correlation matrix and its feedback estimate, i.e., $d(\widehat{\mathbf{R}}_H(n), \mathbf{H}(n)) = \text{dist}_g(\widehat{\mathbf{R}}_H(n), \mathbf{H}^H(n) \mathbf{H}(n))$.

The differential quantization algorithm for the feedback of the channel correlation matrix is an iterative procedure. At each iteration n the initial situation is described as follows: the receiver has a perfect knowledge of the current channel matrix $\mathbf{H}(n)$ and both the transmitter and the receiver know which is the last estimate of the channel correlation matrix sent through the feedback channel $\widehat{\mathbf{R}}_H(n-1)$. A possible

²Actually, reference [2] is devoted to the case of real matrices, although the results and conclusions can be extended directly to the complex case.

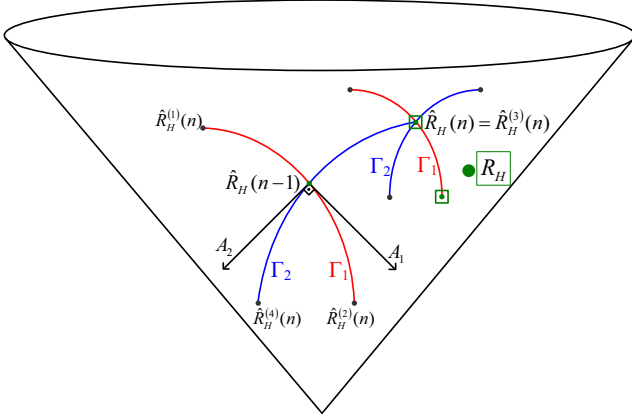


Figure 1: 2-bit differential quantization in the space of channel correlation matrices.

initialization of the algorithm would correspond to starting the run of the algorithm from the cone vertex before the first iteration: $\hat{\mathbf{R}}_H(0) = \mathbf{I}$.

At each iteration n , the following steps are followed (all these steps are represented conceptually in Fig. 1):

- **STEP 1:** Both the receiver and the transmitter generate a common set of Q random Hermitian matrices using the same pseudo-random generator and the same seed. Then, these matrices are orthonormalized using the Gram-Schmidt procedure [9] according to the definition of scalar product presented in section 3, producing the set $\{\mathbf{A}_i\}_{i=1}^Q$. Finally, each matrix \mathbf{A}_i is re-scaled individually so that $\mathbf{C}_i = \mathbf{R}^{-1/2} \mathbf{A}_i \mathbf{R}^{-1/2}$ has a norm equal to Δ ($\|\mathbf{C}_i\|_F = \Delta$) which is, in fact, the quantization step.
- **STEP 2:** Both the receiver and the transmitter use the previous matrices to generate a set of Q geodesic curves $\{\Gamma_i(t)\}_{i=1}^Q$ having all of them the same initial point $\mathbf{R} = \hat{\mathbf{R}}_H(n-1)$ and with orthogonal directions $\Gamma_i(t) = \hat{\mathbf{R}}_H^{1/2}(n-1) \exp(t \mathbf{C}_i) \hat{\mathbf{R}}_H^{1/2}(n-1)$.
- **STEP 3:** Each of these geodesic curves is used to generate two candidates for the feedback in the next iteration $\hat{\mathbf{R}}_H(n)$ corresponding to $\Gamma_i(-1)$ and $\Gamma_i(1)$.
- **STEP 4:** The receiver evaluates the cost function for each of the candidates (there are $2Q$ candidates), and sends the selected index i_{FB} through the feedback channel to the transmitter. This index is the one for which the corresponding candidate minimizes the cost function. According to this, the number of feedback bits per iteration has to be higher than or equal to $\log_2(2Q)$. The matrix corresponding to the selected candidate will be used for the transmitter design and as the starting point in the next iteration.

All the previous steps are represented graphically in Fig. 1 for the case of a feedback using 2 bits and taking as optimization criterion the minimization of the geodesic distance to the actual channel correlation matrix $\mathbf{R}_H(n)$. Starting from $\hat{\mathbf{R}}_H(n-1)$, the algorithm generates 2 orthogonal geodesic routes $\Gamma_1(t)$ and $\Gamma_2(t)$ with velocity matrices \mathbf{A}_1 and \mathbf{A}_2 , producing four quantization candidates, all of them at distance Δ from the initial point. At the receiver, each candidate is compared to the actual \mathbf{R}_H and the one with smallest distance (in this example candidate 3) is chosen. That is,

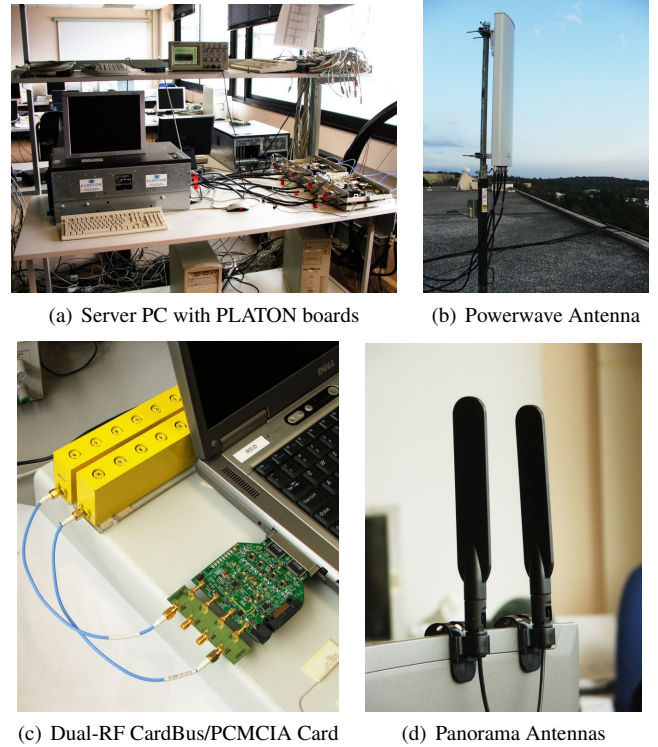


Figure 2: EMOS base-station and user equipment [10]

its index $i_{FB} = 3$ is sent to the transmitter through the feedback channel and $\hat{\mathbf{R}}_H(n) = \hat{\mathbf{R}}_H^{(3)}(n)$. The next iteration starts from this point, generates 2 orthogonal routes and 4 quantization candidates, selects the closest candidate to \mathbf{R}_H , and so on.

4. REAL CHANNEL MEASUREMENTS

Realistic MIMO channel measurements have been obtained using Eurecom's MIMO Openair Sounder (EMOS) [4, 5]. In this section we first describe the hardware of the EMOS platform and the channel sounding procedure and then the measurement campaign that was carried out for this paper. The obtained measurements are used in the next section to evaluate the previous feedback quantization technique from a realistic point of view.

4.1 Platform Description

The EMOS is based on the OpenAirInterface hardware/software development platform at Eurecom.³ It operates at 1.900-1.920 GHz with 5 MHz channels and can perform real-time channel measurements between a base station and multiple users synchronously. For the BS, a workstation with four PLATON data acquisition cards (see Fig. 2(a)) is employed along with a Powerwave 3G broadband antenna (part no. 7760.00) composed of four elements which are arranged in two cross-polarized pairs (see Fig. 2(b)). The UEs consist of a laptop computer with Eurecom's dual-RF CardBus/PCMCIA data acquisition card (see Fig. 2(c)) and two clip-on 3G Panorama Antennas (part no. TCLIP-DE3G, see Fig. 2(d)). The platform is designed for a full software-radio

³<http://www.openairinterface.org>

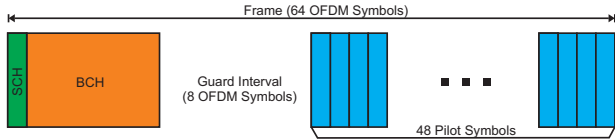


Figure 3: Frame structure of the OFDM Sounding Sequence. The frame consists of a synchronization channel, (SCH), a broadcast channel (BCH), and several pilot symbols used for channel estimation.

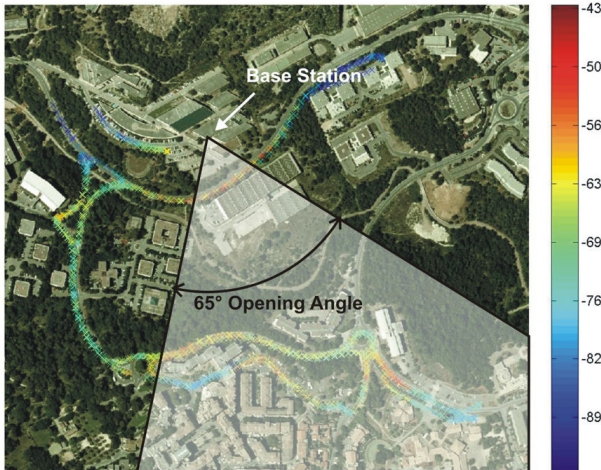


Figure 4: Map of the measurement scenario. The position and the opening angle of the BS antenna are also indicated. The users were driving in cars along the indicated routes (the colors show the received signal strength in dBm along the routes).

implementation, in the sense that all protocol layers run on the host PCs under the control of a Linux real time operation system.

The EMOS is using an OFDM modulated sounding sequence with 256 subcarriers (out of which 160 are non-zero). The sounding sequence contains a synchronization, data and pilot symbols (see Figure 3). The pilot symbols are taken from a pseudo-random QPSK sequence defined in the frequency domain. The subcarriers of the pilot symbols are multiplexed over the four transmit antennas to ensure orthogonality in the spatial domain. We can therefore obtain one full MIMO channel estimate for one group of M subcarriers. The estimated channels are stored to disk for offline analysis. For a more detailed description of the synchronization and channel estimation procedure see [10, 11].

4.2 Measurements

The measurements were conducted outdoors in the vicinity of Eurecom in Sophia Antipolis, France⁴. The scenario is characterized by a semi-urban hilly terrain, composed by short buildings and vegetation with a predominantly present LOS. Fig. 4 shows a map of the environment. The BS is located at the roof of Eurecom's southmost building. The antenna is directed towards Garbejaire, a small nearby village. The UEs were placed inside standard passenger cars which

⁴Eurecom has a frequency allocation for experimentation around its premises.

Parameter	Value
Center Frequency	1917.6 MHz
Bandwidth	4.8 MHz
BS Transmit Power	30 dBm
Number of Antennas at BS	4 (2 cross polarized)
Number of UE	1
Number of Antennas at UE	2
Number of Subcarriers	1

Table 1: EMOS Parameters

were being driven along the routes shown in Fig. 4 (The colors indicate the received signal strength along the measurement routes). The measurement parameters are summarized in Table 1.

5. RESULTS

In the simulations, we consider a particular real channel measured as commented in section 4 with 4 transmit and 2 receive antennas. Note that for the evaluations in this paper we have selected only one subcarrier to mimic a narrow-band system. We show results for three cases: perfect CSI at the transmitter, non-differential Grassmannian packaging [6], and differential quantization of the channel correlation matrices $\mathbf{R}_H(n)$ using geodesic curves [3]. In all the cases, the performance measure corresponds to the SNR obtained in the communication using at the transmitter the strongest eigenmode of the available channel response.

As shown in Fig. 5, the differential strategy exploits the time-correlation of the channel and converges to perfect CSIT case, while the performance using the non-differential quantization is always lower, even using more feedback bits.

Fig. 6 analyzes the impact of the feedback delay in the performance of the system. The plot shows the averaged SNR in the window containing frames from 500 to 520 versus the delay measured in frames (e.g., a delay equal to 10 means that the delay is equal to 10 frames). Three situations are compared: perfect CSI at the transmitter, differential feedback with no delay, and differential feedback with different values for the delay in the feedback link. The main conclusion is that the performance rapidly decreases when the delay exceeds a threshold. More work is still to be done in order to avoid such negative effect of the delay by means, for example, of channel prediction techniques.

6. CONCLUSIONS

This paper has presented a work related to the evaluation of differential and non-differential feedback strategies for MIMO systems. The main objective is the study of the impact of such techniques using real channel measurements and under different situations of delay in the feedback link.

The feedback strategy proposed for MIMO communications, which is based on a differential quantization of the channel correlation matrix using geodesic routes, has several advantages over other existing feedback strategies. This technique exploits the intrinsic geometry of correlation matrices (positive definite Hermitian) versus channel response matrices in order to improve quantization performance.

Simulations using real measurement data show this algorithm achieves better performance than other techniques based on the direct quantization of the channel response

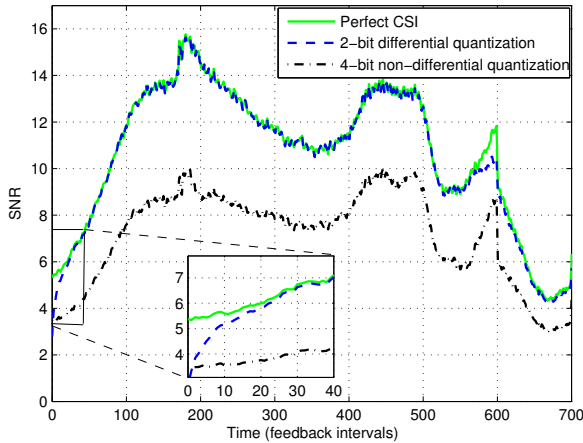


Figure 5: SNR achieved using different feedback techniques in a time-correlated channel (users with low mobility).

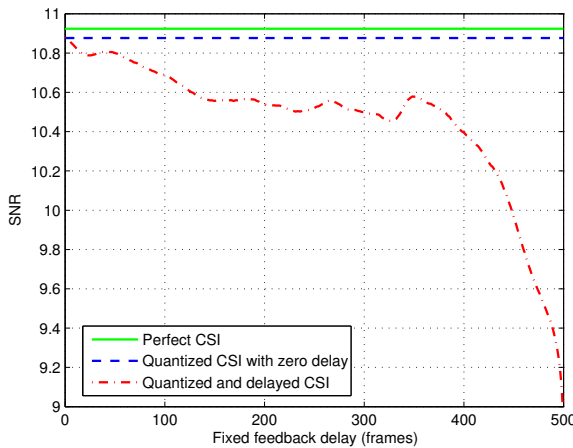


Figure 6: Effects of CSI quantization and feedback delay on the SNR performance.

matrix or the quantization of the subspace spanned by the strongest eigenmodes of the MIMO channel, as well as non-differential strategies like Grassmannian packaging.

REFERENCES

- [1] J. Yang and D.B. Williams, "Transmission Subspace Tracking for MIMO Systems With Low-Rate Feedback," *IEEE Trans. on Communications*, vol. 55, no. 8, pp. 1629–1639, Aug. 2007.
- [2] M. Talih, "Geodesic Markov Chains on Covariance Matrices," Tech. Rep., Statistical and Applied Mathematical Sciences Institute, March 2007.
- [3] D. Sacristan and A. Pascual-Iserte, "Differential Feedback of MIMO Correlation Matrices based on Geodesic Curves," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP 2009)*, Taipei, Taiwan, April 2009, accepted.
- [4] F. Kaltenberger, M. Kountouris, D. Gesbert, and R. Knopp, "Correlation and Capacity of Measured Multi-user MIMO

Channels," in *Proc. IEEE Intl. Symp. on Personal, Indoor and Mobile Radio Comm. (PIMRC)*, Cannes, France, Sept. 2008.

- [5] F. Kaltenberger, M. Kountouris, D. Gesbert, and R. Knopp, "Performance of Multi-user MIMO Precoding with Limited Feedback over Measured Channels," in *Proc. IEEE Global Communications Conference (IEEE GLOBECOM 2008)*, New Orleans, USA, Nov.–Dec. 2008.
- [6] D.J. Love, R.W. Heath, and T. Strohmer, "Grassmannian Beamforming for Multiple-Input Multiple-Output Wireless Systems," *IEEE Trans. on Information Theory*, vol. 49, no. 10, pp. 2735–2747, Oct. 2003.
- [7] D.P. Palomar, J.M. Cioffi, and M.A. Lagunas Hernandez, "Joint Tx-Rx Beamforming Design for Multicarrier MIMO Channels: a Unified Framework for Convex Optimization," *IEEE Trans. on Signal Processing*, vol. 51, no. 9, pp. 2381–2401, Sept. 2003.
- [8] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.
- [9] G.H. Golub and C.F. Van Loan, *Matrix Computations*, The Johns Hopkins University Press, Oct. 1996.
- [10] R. de Lacerda, L. S. Cardoso, R. Knopp, M. Debbah, and D. Gesbert, "EMOS platform: real-time capacity estimation of MIMO channels in the UMTS-TDD band," in *Proc. International Symposium on Wireless Communication Systems (IWCS)*, Trondheim, Norway, Oct. 2007.
- [11] F. Kaltenberger, L. Bernardo, and T. Zemen, "Characterization of measured multi-user mimo channels using the spectral divergence measure," Tech. Rep. TD(08) 640, COST 2100, Lille, France, Nov. 2008.