Performance Analysis of Preconditioned Iterative Inter-Carrier Interference Cancellation for OFDM

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***Abstract*—The performance of the classical low complexity OFDM detection is known to rapidly degrade with the raising of inter-carrier interference in the presence of fast time-varying channels. Advanced equalization techniques to mitigate the impact of the inter-carrier interference under those circumstances, generally involve significantly higher complexity. In this paper we address a class of reduced-complexity fast-converging iterative equalization algorithms yielding nearly-optimal performance with respect to other well-known methods. The complexity is optimized by suitable pre-conditioning in order to drastically reduce the number of required iterations necessary to achieve reliable signal detection. A detailed analysis supported by numerical results under realistic scenarios shows the nearly-optimal performance achievable by the proposed techniques with a very limited complexity, when comparing with classical zero-forcing and minimum-mean square error linear equalization.**

***Index Terms*—OFDM, Inter-Carrier Interference, Iterative Interference Cancelling, Preconditioning**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM), adopted by numerous existing wireless telecommunication standards, allows for flexible bandwidth allocation and low-complexity transmitter and receiver architectures. However the performance of classical OFDM low complexity receivers is severely impacted by fast time-varying propagation channels causing the rising of inter-carrier interference (ICI).

Those circumstances occur in the presence of high Doppler spread relative to the OFDM symbol rate due to the mobile receiver velocity. In practice, the increased ICI prevents classical OFDM receiver schemes from reliably detecting the desired signal. Hence, more advanced receiver equalization techniques are required to mitigate the effect of the ICI.

Optimal linear ICI equalization techniques generally involve complex full channel matrix inversion. In existing OFDM telecommunication systems, the typical size of the required discrete Fourier transform renders such a full channel matrix inversion operation prohibitively complex for practical implementation. Hence, several approaches have been adressed to reduce the complexity while maintaining acceptable performance. For instance, the use of time-domain windowing of the OFDM received signal has been shown to limit the significant span of the ICI, generating *banded* channel transfer matrices. In addition, iterative equalization and detection techniques have been proposed to further reduce the complexity of the receiver operating in the frequency-domain, as in e.g. [1], [2] and references therein, or in the time-domain as in [3], [4].

We introduce here an alternative and original framework for iterative ICI cancellation. Our analysis of the detection performance, the convergence speed, and the complexity provides guidelines to derive novel fast-converging iterative ICI cancellation algorithms. We show that proper *preconditioning* exploiting the inherent structure of the OFDM signal and the ICI yields to nearly optimal detection performance with very fast-converging and affordable complexity iterative algorithms.

II. Signal and System Model

We consider the transmission over a time-varying, frequency-selective fading channel with continuous-time impulse response $h(t,\tau) = \sum_m \alpha_m(t)\psi(\tau - \tau_m)$ assumed to obey the wide sense stationary uncorrelated scattering (WSS-US) model [5], where $\psi(\tau)$ represents the equivalent transmit-receiver front-end low-pass filter, τ_m represents the p-th path delay, $\alpha_m(t)$ is the time-varying complex channel coefficient associated with the m-th path of the propagation channel respectively. We shall refer to $h[k,l]$ as the corresponding low-pass sampled discrete-time impulse response, and assume $h[k,l]$ to be well-approximated by a finite-impulse response model with a maximum delay spread of L samples. Then we assume a classical OFDM system with cyclic-prefix (CP) of duration $N_{\rm cp} \geq L$ to avoid inter-symbol-interference. By letting N denote the number of sub-carriers the OFDM symbol duration is given by $N_{\rm block} = N + N_{\rm cp}$. The frequency-domain k-th OFDM transmit symbol $\boldsymbol{s}[k] = [s[kN] \; \dots \; s[kN - N + 1]]^{\mathsf{T}}$, where $(\cdot)^{\mathsf{T}}$ denotes transpose, comprising the encoded symbols $s[i]$ at the output of channel encoding, interleaving and mapping onto a finite-symbol constellation $\mathcal{S}$ assumed i.i.d. with unit energy, is modulated by an $N \times N$ discrete Fourier transform unitary matrix $\boldsymbol{F}$ so as to obtain

$$\boldsymbol{x}[k] = \boldsymbol{F}^{\mathsf{H}}\boldsymbol{s}[k] \tag{1}$$

where $(\cdot)^{\mathsf{H}}$ denotes Hermitian operator. Without accounting for the CP, the k-th received symbol can be written as

$$\boldsymbol{r}[k] = \boldsymbol{F}\boldsymbol{\mathcal{H}}[k]\boldsymbol{x}[k] + \boldsymbol{z}[k] \tag{2}$$

where $\boldsymbol{r}[k] = [r[kN] \ \ldots \ r[kN - N + 1]]^{\mathsf{T}}$, $\boldsymbol{\mathcal{H}}[k]$ represents the $N \times N$ time domain channel convolution matrix, and $\boldsymbol{z}[k] = [z[kN] \ \ldots \ z[kN - N + 1]]^{\mathsf{T}}$ represents a circularly symmetric complex additive white Gaussian noise such that $\boldsymbol{z}[k] \sim \mathcal{N}_{\mathcal{C}}(\boldsymbol{0}, \sigma_z^2\mathbf{I})$.

For the sake of the notational simplicity and without loss of generality, we shall drop the time index k in the sequel. Thus equation (2) can be rewritten as follows

$$\boldsymbol{r} = \boldsymbol{F}\boldsymbol{\mathcal{H}}\boldsymbol{F}^{\mathsf{H}}\boldsymbol{s} + \boldsymbol{z} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{z} \tag{3}$$

Since in general $L \ll N$ the channel matrix $\boldsymbol{\mathcal{H}}$ will tend to be *sparse* and *banded*. When the channel is time invariant within an OFDM symbol, $\boldsymbol{\mathcal{H}}$ is circulant and therefore the frequency-domain channel matrix, $\boldsymbol{H} = \boldsymbol{F}\boldsymbol{\mathcal{H}}\boldsymbol{F}^{\mathsf{H}}$, is diagonal. This characteristic is widely exploited to perform one-tap frequency-domain equalization.

In case of time-varying channel though, $\boldsymbol{\mathcal{H}}$ is no longer circulant and results in a full frequency-domain channel matrix. Thus the classical OFDM equalization approach is highly suboptimal and more complex equalization is required (see [1], [2] and references therein).

III. Preconditioned Iterative Reception

For the sake of the convenience for our analysis we shall start from classical linear Zero-Forcing (ZF) equalization problem, where an estimate of the transmitted signal $\boldsymbol{s}$ is computed from the received signal $\boldsymbol{r}$ of (3) as

$$\widehat{\boldsymbol{s}}_{\mathrm{ZF}} = \left(\boldsymbol{H}^{\mathsf{H}}\boldsymbol{H}\right)^{-1}\boldsymbol{H}^{\mathsf{H}}\boldsymbol{r} \tag{4}$$

Typically, Equation (4) is solved by first performing the Matched Filter (MF) operation on the received signal $\boldsymbol{r}$ and then by applying classical iterative techniques to approximate the inversion of $\boldsymbol{H}^{\mathsf{H}}\boldsymbol{H}$ based on its *Hermitian* structure. This approach has been widely adopted in a large number of works, especially for Code Division Multiple Access (CDMA) multiuser Minimum Mean Square Error (MMSE) linear equalization (for example in [6] – [7] and references therein).

When $\boldsymbol{H}$ is full-rank as in OFDM, Equation (4) reduces to

$$\widehat{\boldsymbol{s}}_{\mathrm{ZF}} = \boldsymbol{H}^{-1}\boldsymbol{r} \tag{5}$$

Following this formulation, it is straightforward to show that the problem of approximating the inverse of $\boldsymbol{H}$ is inherently better conditioned than the one of inverting $\boldsymbol{H}^{\mathsf{H}}\boldsymbol{H}$ by observing the relation of their respective condition numbers (CN): $\kappa(\boldsymbol{H}) \leq \kappa(\boldsymbol{H}^{\mathsf{H}}\boldsymbol{H})$ where $\kappa(\boldsymbol{X}) = \|\boldsymbol{X}\|\|\boldsymbol{X}^{-1}\|$. As it is well-known from the literature the smaller the CN, the faster an iterative algorithm will converge.

We therefore shall concentrate on iterative solution of the linear system of Equation (5). The most common iterative procedure, used throughout this discussion, can be derived by the *Taylor* expansion of matrix $\boldsymbol{H}^{-1}$ to give an *iterative* ZF (IT-ZF) receiver expressed by

$$\widehat{\boldsymbol{s}}_{\mathrm{IT-ZF}} = \left(\sum_{k=0}^{K-1}\left(\mathbf{I} - \boldsymbol{H}\right)^{k}\right)\boldsymbol{r} \tag{6}$$

Obviously, $\widehat{\boldsymbol{s}}_{\mathrm{IT-ZF}} = \widehat{\boldsymbol{s}}_{\mathrm{ZF}}$ for $K \to \infty$ if and only if $\rho\left(\mathbf{I} - \boldsymbol{H}\right) < 1$, where $\rho\left(\boldsymbol{X}\right)$ denotes the *spectral radius* of matrix $\boldsymbol{X}$.

For finite (and low) number of iterations, the IT-ZF receiver of equation (6) can be interpreted as a *polynomial expansion* receiver since it can be formulated as $(K-1)$-th order polynomial in $\boldsymbol{H}$ as $\sum_{k=0}^{K-1}\left(\mathbf{I} - \boldsymbol{H}\right)^{k} = \sum_{k=0}^{K-1} w_k\boldsymbol{H}^{k}$ with $w_k = (-1)^k\binom{K}{k+1}$. Polynomial expansion receivers have been extensively studied [8] [9], in particular for the CDMA case, as an effective mean of approximating ZF or MMSE linear equalizers. Relying in particular on Cayley-Hamilton theorem [10], these techniques aim at optimizing combining coefficients w_k for a reduced number of iterations (i.e. polynomial order) than N, the dimension of the linear system to be solved.

Alternatively to polynomial expansion receivers and the problem of finding optimized combining coefficients w_k, we instead focus on *preconditioning* as an advantageous mean to improve performance of iterative interference cancellation and signal detection by reducing the CN of the linear system to be solved and allow for faster convergence. For this purpose, the linear system

$$\widehat{\boldsymbol{s}}_{\mathrm{ZF}} = \left(\boldsymbol{P}\boldsymbol{H}\right)^{-1}\boldsymbol{P}\boldsymbol{r} \tag{7}$$

easily proves being exactly equivalent to (5). Using the same derivation of (6) from (5), we can therefore approximate (5) by a *preconditioned* iterative ZF (P-IT-ZF) receiver such as

$$\widehat{\boldsymbol{s}}_{\mathrm{P-IT-ZF}} = \left(\sum_{k=0}^{K-1}\left(\mathbf{I} - \boldsymbol{P}\boldsymbol{H}\right)^{k}\right)\boldsymbol{P}\boldsymbol{r} \tag{8}$$

if and only if $\rho\left(\mathbf{I} - \boldsymbol{P}\boldsymbol{H}\right) < 1$ and where $\boldsymbol{P}$ is a suitable nonsingular preconditioning matrix such that $\kappa\left(\boldsymbol{H}\right) \geq \kappa\left(\boldsymbol{P}\boldsymbol{H}\right) \geq 1$. The equality holds in the trivial case where $\boldsymbol{P}^{-1} = \boldsymbol{H}$ and the linear system in (7) is solved in one iteration.

It is worth noting that the ZF signal detection problem as formulated in (4) is a particular case of the preconditioned system (8) with $\boldsymbol{P} = \boldsymbol{H}^{\mathsf{H}}$, but where the CN is increased instead.

The asymptotic convergence in the receiver order K of the P-IT-ZF receiver of (8) to the ZF solution is independent of the choice of $\boldsymbol{P}$. For lower iteration orders instead, its convergence and performance behavior strongly depend on the chosen preconditioning.

In the absence of noise, i.e. $\sigma_z^2 \to 0$, the error of the $(K-1)$-th order P-IT-ZF receiver $\widetilde{\boldsymbol{s}}_{\mathrm{P-IT-ZF}} = \boldsymbol{s} - \widehat{\boldsymbol{s}}_{\mathrm{P-IT-ZF}}$ is

$$\widetilde{\boldsymbol{s}}_{\mathrm{P-IT-ZF}} = \left(\sum_{k=0}^{K}(-1)^k\binom{K}{k}\left(\boldsymbol{P}\boldsymbol{H}\right)^{k}\right)\boldsymbol{s} = \left(\mathbf{I} - \boldsymbol{P}\boldsymbol{H}\right)^{K}\boldsymbol{s}$$

For the P-IT-ZF receiver, by defining

$$\widetilde{\boldsymbol{G}} = \left(\mathbf{I} - \boldsymbol{P}\boldsymbol{H}\right)^{K}, \tag{9}$$

the error norm is $\|\widetilde{\boldsymbol{s}}\| = \|\widetilde{\boldsymbol{G}}\boldsymbol{s}\| \leq \|\widetilde{\boldsymbol{G}}\|\|\boldsymbol{s}\|$. $\|\widetilde{\boldsymbol{G}}\|$ and $\rho\left(\mathbf{I}-\boldsymbol{P}\boldsymbol{H}\right) = \lim_{K\rightarrow\infty}\|\widetilde{\boldsymbol{G}}\|^{\frac{1}{K}}$ are the $(K-1)$-th order and *asymptotic* convergence-factors of the iterative receiver, respectively.

IV. Iteration-Dependent Preconditioned Iterative Reception

The P-IT-ZF receiver of Equation (8) is based on a constant preconditioning over iterations. By construction, this receiver is intrinsically sub-optimal because, as $K \rightarrow \infty$, it degenerates to the ZF solution independently of the choice of the preconditioner.

The approach can be generalized and improved to give an *Iteration-Dependent* Preconditioned ITerative (ID-P-IT) receiver as follows

$$\widehat{\boldsymbol{s}}_{\mathrm{ID-P-IT}} = \left(\left(\sum_{k=1}^{K-1}\prod_{j=1}^{k}\left(\mathbf{I}-\boldsymbol{P}_j\boldsymbol{H}\right)\right)+\mathbf{I}\right)\boldsymbol{P}_0\boldsymbol{r} \tag{10}$$

where the preconditioning matrix $\boldsymbol{P}$ is optimized at each iteration forming the set $\mathcal{P}_K = \{\boldsymbol{P}_0, \boldsymbol{P}_1, \cdots, \boldsymbol{P}_{K-1}\}$.

This generalization introduces new degrees of freedom to our receiver that can be exploited to achieve better performances than those achievable with $\boldsymbol{P}$ constant.

The block diagram of the ID-P-IT receiver is depicted in Figure 1, the P-IT-ZF receiver of Equation (8) can be obtained imposing the same preconditioning matrix $\boldsymbol{P}$ for all stages.

Similarly as for the P-IT-ZF in (9), in case of the ID-P-IT receiver we can define

$$\widetilde{\boldsymbol{G}} = \prod_{k=0}^{K}\widetilde{\boldsymbol{G}}_k = \prod_{k=0}^{K}\left(\mathbf{I}-\boldsymbol{P}_k\boldsymbol{H}\right) \tag{11}$$

Then, by denoting $\widetilde{\boldsymbol{T}}_{K-1} = \prod_{k=0}^{K-1}\widetilde{\boldsymbol{G}}_k$, the error of the ID-P-IT receiver of the $(K-1)$-th order can be written as

$$\begin{aligned}\widetilde{\boldsymbol{s}} &= \widetilde{\boldsymbol{G}}\left(\boldsymbol{s}+\boldsymbol{H}^{-1}\boldsymbol{z}\right)-\boldsymbol{H}^{-1}\boldsymbol{z} = \\ &= \widetilde{\boldsymbol{G}}_K\widetilde{\boldsymbol{T}}_{K-1}\boldsymbol{s}+\left(\widetilde{\boldsymbol{G}}_K\widetilde{\boldsymbol{T}}_{K-1}-\mathbf{I}\right)\boldsymbol{H}^{-1}\boldsymbol{z}\end{aligned} \tag{12}$$

and its *mean square error* (MSE) as

$$\begin{aligned}\mathsf{E}\|\widetilde{\boldsymbol{s}}\|^2 =& \mathrm{tr}\left\{\sigma_s^2\widetilde{\boldsymbol{G}}_K\widetilde{\boldsymbol{T}}_{K-1}\widetilde{\boldsymbol{T}}_{K-1}^{\mathsf{H}}\widetilde{\boldsymbol{G}}_K^{\mathsf{H}}\right\}+ \\ &+\mathrm{tr}\left\{\sigma_z^2\left(\widetilde{\boldsymbol{G}}_K\widetilde{\boldsymbol{T}}_{K-1}-\mathbf{I}\right)\boldsymbol{R}^{-1}\left(\widetilde{\boldsymbol{G}}_K\widetilde{\boldsymbol{T}}_{K-1}-\mathbf{I}\right)^{\mathsf{H}}\right\}\end{aligned} \tag{13}$$

where $\mathsf{E}\{\cdot\}$ denotes the expectation operator and $\boldsymbol{R} = \boldsymbol{H}^{\mathsf{H}}\boldsymbol{H}$.

V. Sparse Preconditioning Derivation

The optimal preconditioning matrix $\boldsymbol{P}_K$ for K-th iteration can be found minimizing the MSE of Equation (13), that is

$$\begin{aligned}\boldsymbol{P}_K &= \arg\min_{\boldsymbol{P}_K\in\mathcal{P}_K}\mathsf{E}\|\widetilde{\boldsymbol{s}}\|^2 = \left(\sigma_s^2\widetilde{\boldsymbol{T}}_{K-1}+\sigma_z^2(\widetilde{\boldsymbol{T}}_{K-1}-\mathbf{I})\boldsymbol{R}^{-1}\right)\cdot \\ &\cdot\widetilde{\boldsymbol{T}}_{K-1}^{\mathsf{H}}\boldsymbol{H}^{\mathsf{H}}\left(\boldsymbol{H}\widetilde{\boldsymbol{T}}_{K-1}\left(\sigma_s^2\mathbf{I}+\sigma_z^2\boldsymbol{R}^{-1}\right)\widetilde{\boldsymbol{T}}_{K-1}^{\mathsf{H}}\boldsymbol{H}^{\mathsf{H}}\right)^{-1}\end{aligned} \tag{14}$$

Nevertheless, as we can see for the first iteration, i.e. $\boldsymbol{T}_{K-1} = \mathbf{I}$, Equation (14) gives

$$\boldsymbol{P}_0 = \sigma_s^2\boldsymbol{H}^{\mathsf{H}}\left(\sigma_s^2\boldsymbol{H}\boldsymbol{H}^{\mathsf{H}}+\sigma_z^2\mathbf{I}\right)^{-1} \tag{15}$$

which is the trivial solution corresponding to MMSE receiver. In this case, such preconditioning coefficients would allow any preconditioned receiver to converge to the optimal solution in the MMSE sense in one iteration but at the cost of unaffordable complexity entailing the inversion of a $N \times N$ matrix. In general, this operation requires complexity orders of $\mathcal{O}\left(N^3\right)$ or $\mathcal{O}\left(N^2\right)$ order when classical techniques are used, such as *Gauss-Jordan elimination* or *Cholesky decomposition* (exploiting the Hermitian nature of $\boldsymbol{\mathcal{H}}^{\mathsf{H}}\boldsymbol{\mathcal{H}}$) respectively [10].

In order to avoid the trivial and extremely complex solution, we therefore need to add a complexity-limitation constraint to the minimization problem in (14).

The optimal *constrained* MMSE preconditioning matrix can be obtained by estimating the transmitted symbol $s(n)$ at sub-carrier n by adopting *sparse* MMSE Finite-Impulse-Response (FIR) preconditioning filter $\bar{\boldsymbol{p}}_n$ across L_{FIR} (neighboring) tones to limit the complexity of a full *per-tone* equalization across all N sub-carriers.

Considering for example the first iteration case, this corresponds to selecting a subset of the elements of vector $\boldsymbol{r}$ as $\bar{\boldsymbol{r}}_n = \boldsymbol{S}_n\boldsymbol{r}$ with $\boldsymbol{S}_n$ being a $L_{\mathrm{FIR}} \times N$ *selection* matrix obtained by extracting L_{FIR} rows of the identity matrix $\mathbf{I}_{\mathrm{N}}$ for a given filter-length L_{FIR} and sub-carrier n to have

$$\hat{s}(n) = \bar{\boldsymbol{p}}_n^{\mathsf{H}}\bar{\boldsymbol{r}}_n = \bar{\boldsymbol{p}}_n^{\mathsf{H}}\boldsymbol{S}_n\boldsymbol{r} = \boldsymbol{p}_n^{\mathsf{H}}\boldsymbol{r} \tag{16}$$

with $\boldsymbol{p}_n^{\mathsf{H}} = [\boldsymbol{P}_0]_{i=n;j=1,\ldots,N}$.

Therefore, the *sparse* MMSE filtering coefficients are computed such that $\bar{\boldsymbol{p}}_n^{\mathsf{H}} = \mathsf{E}\{s(n)\bar{\boldsymbol{r}}_n^{\mathsf{H}}\}\left(\mathsf{E}\{\bar{\boldsymbol{r}}_n\bar{\boldsymbol{r}}_n^{\mathsf{H}}\}\right)^{-1}$ to give

$$\bar{\boldsymbol{p}}_n^{\mathsf{H}} = \mathbf{1}_n\boldsymbol{H}^{\mathsf{H}}\boldsymbol{S}_n^{\mathsf{H}}\left[\boldsymbol{S}_n\left(\boldsymbol{H}\boldsymbol{H}^{\mathsf{H}}+\gamma^{-1}\mathbf{I}\right)\boldsymbol{S}_n^{\mathsf{H}}\right]^{-1} \tag{17}$$

with $\mathbf{1}_n$ being the $1 \times N$ vector containing 1 at n-th position and 0 elsewhere. It is noteworthy mentioning that the above expression stems from the multiplication of a $1 \times L_{\mathrm{FIR}}$ vector $\mathsf{E}\left\{s(n)\bar{\boldsymbol{r}}_n^{\mathsf{H}}\right\}$ and $L_{\mathrm{FIR}} \times L_{\mathrm{FIR}}$ inverse matrix of $\mathsf{E}\{\bar{\boldsymbol{r}}_n\bar{\boldsymbol{r}}_n^{\mathsf{H}}\}$ which varies across sub-carriers.

By limiting the complexity and determining preconditioning coefficients subject to this constraint, we are instead able to reduce the computational requirements to $\mathcal{O}\left(L_{\mathrm{FIR}}^3\right)$ at the expense of an increased number of iterations depending on the target performance.

The trade-off between the complexity for computing the preconditioning matrix elements and the number of iterations shall be considered in light of the fact that, even in time-varying channel OFDM reception, each iterative stage can be efficiently implemented with $\mathcal{O}\left(N\log_2 N\right)$ complexity, as shown in [3], using the using channel Polynomial Basis Expansion Modeling (Poly-BEM) approximation [11].

Similarly to (17), we can therefore derive the *constrained* MMSE preconditioning matrix elements at n-th sub-carrier for the $(K+1)$-th iteration stage, provided the set $\mathcal{P}_{K-1} =$

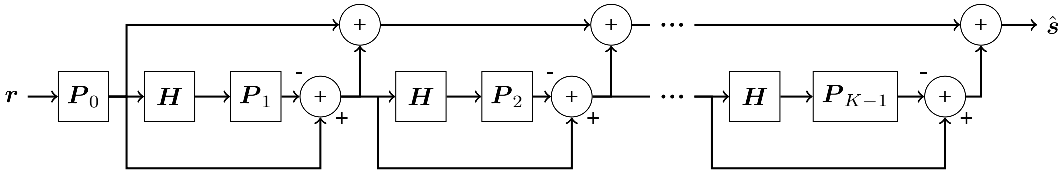

Fig. 1. Iteration-Dependent Iterative ICI cancellation receiver

$\{\boldsymbol{P}_0, \boldsymbol{P}_1, \cdots, \boldsymbol{P}_{K-1}\}$ and subject to the limited-complexity constraint $\boldsymbol{S}_n$, as

$$\begin{aligned} &[\boldsymbol{P}_K]_{i=n;j=1,\ldots,N} = \\ &= \mathbf{1}_n \left(\sigma_s^2 \widetilde{\boldsymbol{T}}_{K-1} + \sigma_z^2 (\widetilde{\boldsymbol{T}}_{K-1} - \mathbf{I})\boldsymbol{R}^{-1}\right) \widetilde{\boldsymbol{T}}_{K-1}^{\mathsf{H}} \boldsymbol{H}^{\mathsf{H}} \boldsymbol{S}_n^{\mathsf{H}} \cdot \\ &\cdot \left[\boldsymbol{S}_n \left(\boldsymbol{H} \widetilde{\boldsymbol{T}}_{K-1} \left(\sigma_s^2 \mathbf{I} + \sigma_z^2 \boldsymbol{R}^{-1}\right) \widetilde{\boldsymbol{T}}_{K-1}^{\mathsf{H}} \boldsymbol{H}^{\mathsf{H}}\right) \boldsymbol{S}_n^{\mathsf{H}}\right]^{-1} \boldsymbol{S}_n \end{aligned} \tag{18}$$

The ID-P-IT receiver of Equation (10) making use of preconditioning matrices computed according to Equation (18) is optimal by construction subject to the aforementioned limited-complexity constraint.

As a result of this analysis, it is evident how the optimality allowing the method to achieve the best possible MSE performance along iterations relates to the fastest possible convergence property in the Euclidean norm sense for the given complexity constraint. For this reason, as $K \to \infty$, and contrarily to the P-IT-ZF receiver of Equation (8), does not show degeneration to ZF despite the initial derivation.

Nevertheless, the ID-P-IT receiver appears to be considerably more complex than the P-IT-ZF making use of constant MMSE preconditioning as of Equation (17) because the preconditioning coefficients are optimized at each iteration.

Quickly time-varying and frequency-selective channels, also known as doubly-selective channels, show different ICI level depending on the sub-carrier index. This suggests that the limited-complexity constraint can be made varying depending on the sub-carrier and its corresponding ICI level. Larger L_{FIR} filters can be used on those sub-carriers severely impacted by ICI, while minimum preconditioning filtering effort ($L_{\mathrm{FIR}} = 1$) can be dedicated to the others.

Strong complexity reduction for all above mentioned schemes can then be easily achieved by first estimating the ICI level then selecting only a subset of sub-carriers for which the ICI level is above a given acceptance threshold or, using a *fixed-complexity* (FC) principle, deciding for a fixed amount of indexes corresponding to those sub-carriers mostly impacted by ICI. The ICI level on each sub-carrier can be estimated effortlessly by considering that, in the OFDM case under examination, the full channel matrix can be decomposed into *time-invariant* and *time-varying* terms such as $\boldsymbol{H} = \boldsymbol{H}_{\mathrm{TI}} + \boldsymbol{H}_{\mathrm{TV}}$ where $\boldsymbol{H}_{\mathrm{TI}}$ is diagonal. By taking $\widetilde{\boldsymbol{G}}_{\mathrm{J}} = \boldsymbol{H}_{\mathrm{TI}}^{-1} \boldsymbol{H}_{\mathrm{TV}}$, the Signal-to-Interference Ratio (SIR) can be estimated by $\mathrm{SIR}(n) = 1/\|\widetilde{\boldsymbol{g}}_{\mathrm{J},n}\|^2$ with $\widetilde{\boldsymbol{g}}_{\mathrm{J},n}^{\mathsf{H}} = \left[\widetilde{\boldsymbol{G}}_{\mathrm{J}}\right]_{i=n;j=1,\ldots,N}$.

VI. Performance Evaluation

In the previous sections, we derived general expressions for the MSE. Here we will provide other significant performance metrics such SINR and mutual information. For both P-IT-ZF and ID-P-IT receivers of Equations (8) and (10), the transmitted sequence estimate can be written as

$$\widehat{\boldsymbol{s}}_{\mathrm{P-IT-ZF}} = \boldsymbol{G}\boldsymbol{s} + \boldsymbol{G}\boldsymbol{H}^{-1}\boldsymbol{z} \tag{19}$$

where $\boldsymbol{G} = \mathbf{I} - \widetilde{\boldsymbol{G}}$ denotes the cascade of the channel and of the iterative receiver computed by plugging respective expressions for $\widetilde{\boldsymbol{G}}$ from (9) and (11). Additionally, we consider the MMSE receiver as of Equation (15) as reference for performance evaluation.

The error term can be expressed as $\widetilde{\boldsymbol{s}} = \boldsymbol{s} - \boldsymbol{G}\boldsymbol{s} - \boldsymbol{W}\boldsymbol{z}$ with $\boldsymbol{W} = \boldsymbol{G}\boldsymbol{H}^{-1}$ being the receiver transfer-function.

Letting $\boldsymbol{g}_n^{\mathsf{H}} = [\boldsymbol{G}]_{i=n;j=1,\ldots,N}$ and $\boldsymbol{w}_n^{\mathsf{H}} = [\boldsymbol{W}]_{i=n;j=1,\ldots,N}$ be the n-th row of $\boldsymbol{G}$ and $\boldsymbol{W}$ respectively, the expected value of the n-th sub-carrier symbol estimate power is

$$\mathsf{E}|\widehat{s}(n)|^2 = \sigma_s^2 (\underbrace{|g_n(n)|^2}_{\text{useful signal}} + \underbrace{\|\boldsymbol{g}_n\|^2 - |g_n(n)|^2}_{\text{ICI}}) + \underbrace{\sigma_z^2 \|\boldsymbol{w}_n\|^2}_{\text{noise}}$$

Thus, we find the general error expression of the SINR at n-th sub-carrier to be

$$\mathrm{SINR}(n) = \frac{|g_n(n)|^2}{\|\boldsymbol{g}_n\|^2 - |g_n(n)|^2 + \gamma^{-1}\|\boldsymbol{w}\|^2} \tag{20}$$

with $\gamma = \sigma_s^2/\sigma_z^2$.

Finally, as a convenient performance metric for our discussion, we evaluate the average *mutual information*, using

$$\mathcal{I}(\mathrm{SINR}) = \frac{1}{N}\sum_{n=1}^{N} \log_2(1 + \mathrm{SINR}(n)) \tag{21}$$

assuming independent per sub-carrier symbol detection of transmitted sequence.

VII. Simulation Results

We compare the methods proposed in this paper by evaluation of the analytical expressions as described in Section VI. Monte Carlo simulations were conducted on an equivalent OFDM setup with $N = 128$ sub-carriers for a sufficient number of realizations of uniform power-delay-profile multi-path channel of length $L = 4$. The channel is time-varying within an OFDM symbol according to Jake's Doppler spectrum [5] with normalized Doppler frequency of 0.256 with respect to the sub-carriers spacing. The performances are measured in terms of average mutual information (bits/sub-carrier) assuming perfect channel and noise statistics knowledge. The receiver uses $P = 2$ orthonormal Poly-BEM channel. In figures 1 – 3, the methods presented are evaluated for MMSE preconditioning filtering lengths of $L_{\mathrm{FIR}} = 3$.

As a general behavior, all methods show to achieve the MMSE capacity for decoding full-rate QPSK (2 bits/sub-carrier) with 1 iteration, 16QAM (4 bits/sub-carrier) with 2 iterations and 64QAM (6 bits/sub-carrier) with 3 iterations. As expected by optimal analytical construction, the ID-P-IT receiver shows the best approximation to the full-blown MMSE solution compared to any other method for the same number of iterations. Interestingly, the P-IT-ZF using MMSE preconditioning approaches MMSE performances very well for a very limited number of iterations and short length of MMSE preconditioning filter. Moreover, the complexity reduction techniques proposed provide negligible performance loss with respect to P-IT-ZF receiver with full complexity MMSE preconditioning for QPSK and 16 QAM and 1 dB loss for 64 QAM compared to MMSE. In particular, the FC reduction method realizes a complexity reduction of a factor 10 but still provides similar performance to the full complexity P-IT-ZF for QPSK and 16QAM. Hence, in practical applications, the P-IT-ZF with MMSE preconditioning and FC reduction appears to be very attractive.

VIII. Conclusions

In this paper, we propose an original approach to the problem of linear equalization of doubly-selective channels in OFDM reception by means of fast-converging iterative techniques based on preconditioning. We derived a novel optimal receiver structure in the MMSE sense subject to limited-complexity constraint. We showed that for realistic propagation scenarios, the optimal MMSE detection capabilities are achievable with an extremely low number of iterations. Moreover, we propose complexity reduction guidelines at negligible performance penalty to show the attractiveness of the proposed methods in practical applications.

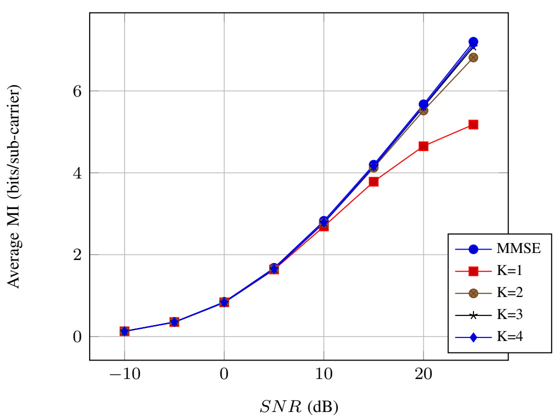

Fig. 2. Performances of ID-P-IT receiver with $L_{\text{FIR}} = 3$

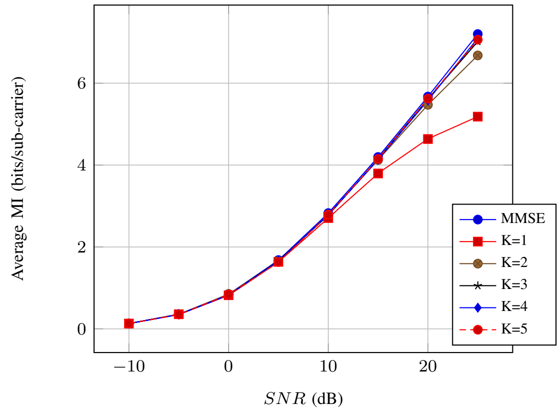

Fig. 3. Performances of P-IT-ZF receiver with $L_{\text{FIR}} = 3$

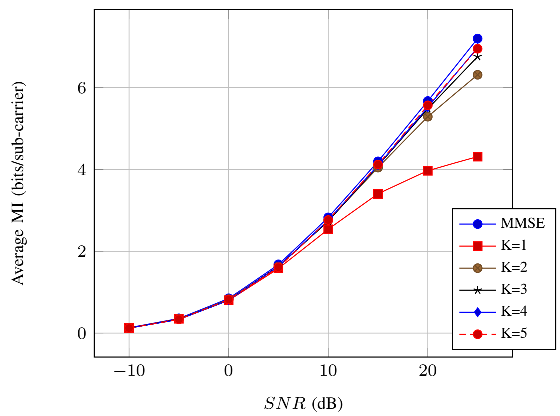

Fig. 4. Performances of P-IT-ZF receiver with $L_{\text{FIR}} = 3$ and FC mechanism

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