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journal homepage: www.elsevier.com/locate/comnetIncentivizing the global wireless village[☆]Gergely Biczók^{a,*}, László Toka^b, András Gulyás^a, Tuan A. Trinh^a, Attila Vidács^a^a HSN Lab, Dept. of Telecommunications and Media Informatics, Budapest University of Technology and Economics, Hungary^b Networking and Security Dept., Eurecom, France

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ABSTRACT

The wireless community networking paradigm shows great promise in achieving a global status. However, in creating a “global wireless village”, both user participation and support from traditional Internet Service Providers (ISPs) are key ingredients; for this end a viable incentive system is essential. In this paper we investigate the economic interactions in global wireless community networks with regard to users, ISPs and community providers (called mediators) with both analysis and data-driven simulation. The main contribution of this paper is threefold. First, we develop a model of the global wireless community concept as a Stackelberg game of participation at two levels (the mediator as leader, and the users and ISPs as followers). Second, we analyze equilibrium properties of the game for users, ISPs and mediator. Our main finding is that the heterogeneity of user home location relevance is necessary for an economically feasible system. Third, we support our analytical claims with simulation results on the evolution of the user population. We show that the emergence of a truly global wireless community network is indeed possible.

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1. Introduction

User-provided networking has seen its stock rising recently. While some see this concept as an interesting but only moderately viable alternative to the traditional Internet Service Provider (ISP)-centric paradigm, others believe it has the potential to induce a complete shift in Internet communication patterns. The latter view can be justified by four important disruptive aspects of user-provided networking. First, since the end-user can share or sell her own resources (e.g., connectivity), the distinction between end-user device and network device disappears. Second, the nature of wireless media, human mobility and the rise of micro-operators create the need for protocols that inherently handle intermittent connectivity, opportunistic relaying and smooth roaming. Third, user-provided services require traditional trust relationships to be transformed: social networks of trust should be formed to

ensure the willingness to cooperate and to maintain network growth. And last, swift adoption of new technologies is possible as adopters are the end-users themselves. We strongly believe that these novel characteristics and features enable user-provided wireless networking to be the foundation of the future wireless Internet.

We refer to a wireless community network with worldwide coverage as the *global wireless village*. A working prototype of such a network already exists: the FON WiFi system [1]. Community members, referred to as Foneros, share their home Internet connections and gain access to free WiFi at other locations. The great interest of users in sharing their broadband connection over WiFi is reflected in the fact that there are more than 1 million FON access points all over the world. There are three different types of Foneros: Linus, Bill, and Alien. A Linus has a “La Fonera” WiFi router, shares his connection and gets free roaming at any FON Spot. A Bill, having the same rights as a Linus, gets further 50% of the revenues when a visitor purchases a FON pass at her FON Spot. An Alien does not have a “La Fonera” router and therefore does not share an Internet connection; she accesses FON Spots by purchasing FON passes. Since its inception in 2005, a number of prolific companies

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have partnered the FON movement: Google, Skype and British Telecom (BT) among others.

Since recent legislation efforts, e.g., in Germany [2], and ISP policy changes [3] require broadband users to secure their home wireless networks, the endorsement of ISPs for sharing home broadband is becoming essential. Furthermore, telecommunication law in certain countries, e.g., the United States, bans communities from providing their own community networks, therefore they have to go through a commercial ISP [4]. Unlike municipal wireless networks, which are mainly or partially operated by the local or national government, we focus on community networks, where users share their own (purchased from commercial ISPs) broadband connections over wireless. We argue that although FON and global wireless community networks (Whisher [5], WeFi [6] and local Meraki [7] among others) in general show great promise, their ultimate success depends on properly designed incentive mechanisms for both users and ISPs. The global wireless village should be as much user-provided as ISP-endorsed: a dual support is essential to achieve global wireless connectivity.

While the existing literature focuses either on local community networks or provider-based WiFi [11–14], our work aims at modeling a global wireless community, where ISPs also play an important role, and the presence of incentives and the proper dynamics of deployment are crucial. As evidenced by the growing number of ISPs partnering with FON (BT, Neuf, Time Warner, etc.), studying the economic interactions among users, ISPs and community providers is a key topic. Moreover, we argue that incorporating user mobility and home location relevance into our model brings us closer to the thorough understanding of the global wireless village. There has been a surge in research results in mobility measurements and models recently [10,21,22]. We use the findings of these papers and propose a mobility and location relevance framework.

In this paper we investigate the economic interactions in global wireless user-provided networks with regard to users, ISPs and community providers (called mediators) through game-theoretic analysis and simulation experiments driven by real-world data. Our main contribution is threefold. First, inspired by the FON concept, we build a game-theoretical model rooted in the Stackelberg game, incorporating user, ISP and mediator games as sub-games. We model the user participation decision both as single-shot and evolutionary games. Also, we consider homogeneous and heterogeneous payment distribution among members. At the ISP level, we propose a one-shot game of two players, which captures the essence of the ISPs' struggle whether to support the community access sharing. Also, the mediator acts as the leader, setting cost and revenue share parameters setting the pace. Second, we derive the Nash equilibrium points of the homogeneous and heterogeneous one-shot user games. Moreover, we show how the evolutionarily stable strategies of the homogeneous user game are among the equilibrium strategy profiles of the one-shot game. Our main finding is that the heterogeneity of user location relevance is necessary for an economically feasible system. Furthermore, we show how all ISPs either adopt or defect against the global wireless village in Nash equilibrium, depending on the parameter setting. Later,

we demonstrate two possible design goals for the mediator (maximizing profit or social welfare) and show numerically how she can achieve them. Third, we define a sophisticated mobility graph incorporating the social aspects of human mobility and location-dependent user home relevance. Using this graph as playing field, we simulate an evolutionary user game with heterogeneous payments. We find that high-relevance users drive the technology diffusion. Moreover, the current penetration and structure of community networks, such as FON, enable them to expand towards global coverage, possibly creating the global wireless village.

The structure of this paper is as follows. We give a short overview on related work in Section 2. We go through the basic notions of game theory in Section 3. We develop a model of the global wireless community concept as a Stackelberg game of participation and construct the respective payoff functions of the mediator, ISPs and users in Section 4. We derive equilibrium properties of the proposed one-shot and evolutionary games, and study the optimal parameter settings of the mediator in Section 5. Next, we define the mobility model and the evolutionary game with heterogeneous payments, and present simulation results in Section 6. We outline possible future research directions in Section 7, and finally conclude the paper in Section 8.

2. Related work

Here we give a brief overview on related research efforts. We focus on two main topics: wireless community networks (WCN), and mobility measurements and models.

Authors of [11] focus on the provision of free Internet access to mobile users through WCN-controlled wireless LAN access points. Their scheme is built on reciprocity: a person participates in the WCN and provides free Internet access to mobile users in order to enjoy the same benefit when roaming. The proposed reciprocity scheme is compatible with the distinctive structure of WCNs: it does not require registration with authorities, relying only on uncertified free identities (public-private key pairs). This work deals mostly with the basic concepts of WCN (sharing in order to gain access on the move) and authentication aspects. In our work we use the reciprocity for users as a starting point, but we also incorporate more complex incentives like monetary revenues and endorsement from ISPs. We do not deal with practical implementation issues like authentication: we simply assume an operating network.

It has not been clear if WCNs can serve as a replacement of existing centralized networks operating in licensed bands (such as cellular networks) or if they should be considered as a complimentary service only. Authors of [12] study the dynamics of wireless social community networks using a simplistic analytical model. In this model, users choose their service provider based on the subscription fee and the offered coverage. They show that the evolution of the respective network depends on its initial coverage, the subscription fee, and the user preferences for coverage. They find that by using an efficient static or dynamic pricing strategy, the wireless social community can obtain a high coverage. Efficient pricing strategies are also studied by authors of [13]. They study the problem comprised of modeling user subscription,

mobility behavior and coverage evolution with the objective of finding optimal subscription fees. They derive optimal prices with both static and semi-dynamic pricing. Coping with an incomplete knowledge about users, they calculate the best static price and prove that optimal fair pricing is the optimal semi-dynamic pricing. The mobility model used is graph-based, and the probabilities for visiting other locations are drawn uniformly. These latter two studies are very valuable theoretical studies focusing on possible users of WCNs. Our work is different in two main aspects. First, we use a model inspired by a practical, operating WCN – the FON network (one-time entry cost, possible monetary income for users). Second, we incorporate another level of entities, Internet Service Providers, who can have a critical impact on the evolution of WCNs.

Network design for a wireless service provider using a WAN technology with uniform spatial coverage and a set of LAN access points each with limited coverage is presented in [14]. The main assumption of the paper is that the system is designed so that users independently and greedily select among the two options based on maximizing a specified utility function which may be a function of the quality of the wireless link, distance to the access points, and/or congestion on system resources. Authors study system performance under such decision-making strategies. Authors deal with the importance of competition among different providers; while we examine a single WCN in this paper, we plan to extend our analysis to multiple provider settings in the future (see Section 7).

We believe that our initial work [8] constitutes the first attempt to incorporate the economic interactions of both users and ISPs into a global wireless community networking framework. It gives a first impression about the topic using strong assumptions and a limited analysis. We use the above paper as a stepping stone and motivation for this work.

Mobility measurements and models based on experimental data are highly relevant to our work: a valid mobility model is needed for experimental investigations. Authors of [21] report that human walks performed in outdoor settings of tens of kilometers resemble a truncated form of Levy walks commonly observed in animals such as monkeys, birds and jackals. The study is based on about 1000 h of GPS traces involving over 100 volunteers in various outdoor settings. They show that many statistical features of human walks follow a truncated power-law, showing evidence of scale-freedom and do not conform to the central limit theorem. These traits are similar to those of Levy walks. Furthermore, they construct a Truncated Levy Walk (TLW) mobility model which is versatile enough in emulating diverse statistical patterns of human walks observed during the measurements. The same authors also proposed another mobility model, correcting the main weakness of TLW: random behavior of mobile nodes. Further empirical data is collected and analyzed, and it is shown that people hardly move randomly and the waypoints where people make a stop show self-similarity. Based on these observations they define a context-based Levy Walk model, called SLAW (Self-similar Least Action Walk) [22]. While these works are technically sound and the models are tailor-made for simulation purposes, they lack a large-scale measurement proving their real-life validity.

A large-scale mobility measurement of 100,000 mobile phone users over 6 months is presented in [10]. They find that, in contrast to the random trajectories predicted by the prevailing Levy flight and random walk models, human trajectories show a high degree of temporal and spatial regularity, each individual being characterized by a time-independent characteristic travel distance and a significant probability to return to a few highly frequented locations. They show that humans follow simple reproducible patterns driven by truncated power-law distributions. Since this latter study is based on a sample three orders of magnitude larger than the previous ones, we consider it more accurate, hence we use its findings when building our own mobility model in Section 6.1.

3. Basic notions of game theory

Here we present elements of classical non-cooperative (players behave rationally and selfishly, i.e., maximize their individual profits) and evolutionary (in the biological sense) game theory we build upon throughout the paper. For a comprehensive material please refer to [25,26].

3.1. One-shot game

This type of game assumes that players act at the same time instant, therefore there is no causality. A game in strategic (normal) form can be described by three elements:

- the set of *players* $i \in \mathcal{I}$, which we take to be the finite set $\{1, 2, \dots, I\}$;
- the *pure-strategy space* $s_i \in S_i$ for each player i , where s_i is a possible action of player i ;
- and *payoff functions* π_i , which give player i 's utility $\pi_i(s)$ for each profile $s = (s_1, \dots, s_I)$ of strategies.

A *mixed strategy* σ_i is a probability distribution over pure strategies. Also, we denote the opponents of player i as a whole as $-i$.

A general solution concept for games of economic interest is the Nash equilibrium solution. A Nash equilibrium is a profile of strategies such that each player's strategy is an optimal response to the other players' strategies.

Definition 1 (*Nash equilibrium*). A mixed strategy profile σ^* is a Nash equilibrium if, for all players i

$$\pi_i(\sigma_i^*, \sigma_{-i}^*) \geq \pi_i(s_i, \sigma_{-i}^*) \forall s_i \in S_i. \quad (1)$$

A pure-strategy Nash equilibrium is a pure-strategy profile that satisfies the same conditions. Practically speaking, a strategy profile is a Nash equilibrium point, when no individual player can get a higher payoff by unilaterally deviating from it.

3.2. Stackelberg game

Also known as a leader–follower game, it introduces multiple stages. The leader commits itself first, chooses its strategy, then the followers respond sequentially.

The Stackelberg model can be solved to find the *sub-game perfect Nash equilibrium* or equilibria, i.e., the strategy

profile that serves each player best, given the strategies of the other players and that entails every player playing in a Nash equilibrium in every subgame.

Definition 2 (*Subgame-perfect equilibrium*). A strategy profile s is a subgame perfect equilibrium if it represents a Nash equilibrium of every subgame of the original game.

3.3. Evolutionary game

Maynard Smith's evolutionary theory originates from the application of the mathematical theory of games to biological contexts, arising from the realization that frequency dependent fitness introduces a strategic aspect to evolution. Recently, however, evolutionary game theory has become of increased interest to economists, sociologists, and computer scientists, since evolutionary games can be adapted to their domains. The evolutionary game-theoretic approach have been also used in computer networking [28,29].

The main differences in an original evolutionary game compared to a classical game are:

- There is a large population of players.
- Players are hard-wired to their strategies; they play, and either reproduce and die, or are partially replaced.
- Payoffs represent the reproductive *fitness* of strategies, i.e., the chance of survival.

Practically speaking, evolutionary game theory analyzes the competition of strategies, without emphasizing the individual players. Hence its solution concept, *evolutionarily stable strategy* (ESS).

Definition 3 (*Evolutionarily stable strategy*). A strategy s_i is evolutionarily stable, if it has the property that if almost every member of the population follows it, no mutant (i.e., an individual who adopts another strategy s_j , $j \neq i$) can successfully invade.

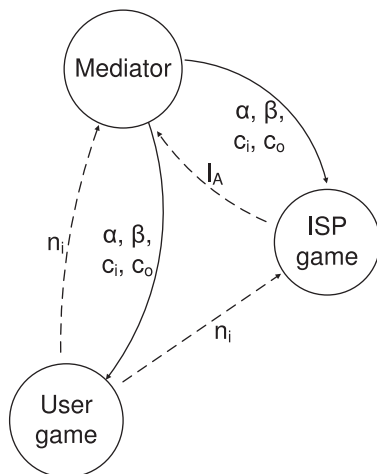


Fig. 1. Interactions among users, ISPs and mediator (structure of the Stackelberg model). Solid lines denote direct input (cost parameters and revenue shares), dashed lines denote implicit feedback (outcome of subgames, for subgame perfect design).

Along the lines of pure and mixed strategies, the concept of a *mixed ESS* emerges. However, in this case it has two different interpretations. Either it is an ESS in mixed strategies, i.e., every individual plays the same mixed strategy; or the population is partitioned to groups playing pure strategies, and relative group sizes represent the probability distribution assigned to pure strategies. The latter interpretation is used in this paper.

4. Model

In this section we present the framework that models the economic interactions of a global wireless community network as a Stackelberg game. The model captures the main decision processes of such an ecosystem: users figuring out if it is worth joining the community, ISPs contemplating whether to support access sharing, and the community provider trying to design the whole mechanism efficiently by setting prices and rewards.

We introduce the notion of a *mediator* (community provider), which operates as the leader of our leader–follower game. The stages of this extended game are the following:

1. The mediator decides on cost parameters and the distribution of income among the other participants (followers), i.e., ISPs and users, and announces these values.
2. At the second stage, Internet Service Providers (ISPs) play a one-shot game, and decide whether they support access sharing for their subscribers.
3. Then in the last stage Internet subscribers (users) play a game among themselves (we consider both a one-shot and an evolutionary model for the users' game in our analysis). Each user decides whether or not she wants to become a community member.

We propose investigating the subgame perfect equilibrium of this extended game setting. The leader, i.e., the mediator, opts for the strategy with which the second stage's (ISPs) reactions (in the view of the third stage's expected decision) and then the best response of the users

Table 1
Notation used in the model.

Symbol	Explanation
s	Strategy
π	Payoff
α	Insiders' share of roaming income
β	ISPs' share of the roaming income
S_j	Share of income of the j th insider
c_i	Insider entry cost
c_o	Outsider roaming cost
c_s	Internet subscription cost
c_r	ISP traffic cost due to roaming alien users
c_a	ISP adoption cost
c_m	Mediator operational cost
u	Roaming utility of a user
n	Number of users/subscribers
n_i	Number of insiders
n_o	Number of outsiders
G	User roaming activity
I_a	Indicator of adopter ISP
R	User home location relevance
T	Technology (community) penetration

leads to the highest possible payoff. This line of thoughts holds for each stage, as the definition of sub-game perfection states. The game-theoretic model is illustrated in Fig. 1 with the anticipated strategies of lower stages to be taken into account. Note, that in the second and third stages the strategy choices are not made by individual entities, but embedded games of multiple players on each. Please refer to Table 1 for notations.

Before the formal definition of strategies and payoff functions for each player, we make some ground assumptions for our model.

4.1. Assumptions

The following assumptions enable the game theoretic modeling of the global wireless Internet-access sharing community.

- We assume that the system consists of two ISPs and a static user set. First, we model the users' game as a one-shot, then as an evolutionary game; the ISPs' game is studied as a two-player one-shot game based on the expected outcome of the users' game. Note, that the ISP game can be easily extended to n players, but it yields similar results with respect to equilibrium properties, thus omitted in this paper.
- We assume games of complete information and observed actions in this paper, meaning that every player knows the exact payoff functions of other players at the same game level, while every player observes the outcome of the games at lower levels.
- When discussing the user and the ISP games, we suppose that the parameters set by the mediator are determined steadily so the followers (i.e., users and ISPs) could choose their strategies based upon those. Hence, our equilibrium analysis of the followers' game is provided in this perspective.
- Every user has broadband Internet access at home and electricity is relatively cheap, hence community members' shared WiFi networks are only used by roaming users, and their Internet access boxes and wireless routers are never turned off. On the other hand, with traffic prioritization in place, insiders do not experience any serious degradation in quality of service at their home routers.
- The mediator-tuned price provides the cheapest option among roaming WiFi networks, so users favor community wireless networks to other commercial WiFi hot-spots, if available.
- Community member users switch to the adopter ISP (if available) in case their original ISP chooses to defect. If both ISPs play the defecting strategy, users are supposed to stay at their original ISP, and continue sharing their access regardless.
- Roaming connectivity demand among users (later denoted by G) is homogeneous.
- We do not target ISPs' Internet subscription pricing, because we consider this issue to be orthogonal to our analysis on wireless Internet access sharing. Therefore, we simply assume that ISPs have the same number of subscribers (with the same rate of user strategies) and

that they lose existing subscribers only due to their policy on connection sharing, i.e., insiders leave a defector for an adopter ISP.

While our assumptions are numerous, we believe that our model captures the essence of the interactions in the global wireless community ecosystem.

4.2. The mediator

In the following we define the role of the mediator as the leader in our game through her payoff function. Exact values of parameters α , β , c_o and c_i (see below) are determined by this entity. Note, that the mediator also decides on the revenue distribution model for users: homogeneous (see Section 5.1) or differentiated (Section 5.3).

Definition 4 (*Mediator payoff*). The instantaneous payoff function π_m of the mediator is defined as

$$\pi_m = GTn_o^*(\alpha, \beta, c_o, c_i)c_o(1 - \alpha - \beta\mathcal{I}_a^*(\alpha, \beta, c_o, c_i)) - c_m + n_i^*(\alpha, \beta, c_o, c_i)c_i, \quad (2)$$

where

- $G \in [0..1]$ represents a user's roaming intensity and her demand to be online;
- $T \in [0..1]$ represents the technology penetration (see Section 4.5) in a general sense, which is an increasing function of the number of insiders. Intuitively, this gives the probability that a roamer requests Internet connection at a given geographic location and at least one community member provides WiFi access at that location;
- n_o^* is the global number of outsiders, i.e., users who do not share their broadband access, in the equilibrium of the user game;
- n_i^* is the global number of insiders, i.e., users who share their broadband access ($n_o + n_i = n$ is the number of all Internet subscribers), in equilibrium of the user game;
- $\alpha \in [0..1]$ is the ratio of the roaming fee that the insider receives from an outsider's payment (c_o) through the mediator when she uses the insider's connection;
- $\beta \in [0..1]$ is the ratio of the roaming fee that the ISP receives from an outsider's payment (c_o) through the mediator when she uses the shared Internet connection of one of the ISP's subscribers ($\alpha + \beta < 1$);
- \mathcal{I}_a^* is an indicator function of the ISPs' best response, and $\mathcal{I}_a = 1$ if at least one adopter ISP is present, otherwise $\mathcal{I}_a = 0$ (the mediator is not required to split its income with the defector ISPs).
- c_o is the time-unit price for roaming WiFi access, paid by the outsider;
- c_i represents the insider's entry cost (e.g., the price of a system-dedicated box, that is supposed to be upgraded periodically) paid to the mediator, discounted to time-unit;
- c_m represents the time-unit operational cost of the mediator.

4.3. The user

First, we define the strategy set for users.

Definition 5 (*User strategies*). The possible strategies of users are the following: one may join the Internet connection sharing community or stay out and pay when roaming. We denote the strategy set as shown here:

- s_i : user strategy to become a community member, i.e., insider;
- s_o : user strategy not to become a community member, i.e., outsider.

Next, we define the payoff function for the two possible user strategies, i.e., joining the global community (*insider*) or staying out (*outsider*).

Definition 6 (*User payoff*). The instantaneous payoff functions π_u of the outsider and of the insider strategies are defined as

$$\pi_u(s_o) = GT(u - c_o) - c_s; \quad (3)$$

$$\pi_u(s_i^j) = GT(u + S_j n_o c_o \alpha) - c_i - c_s. \quad (4)$$

where

- u is a user's time-unit utility for being online when roaming (assumed to be the same for every user);
- c_s is the time-unit Internet subscription fee paid by the user to her ISP;
- $S_j \in [0,1]$ denotes the share of income of the j th insider from the roaming outsiders' contributions.¹

4.4. The Internet Service Provider

First, we introduce the strategy set for ISPs.

Definition 7 (*ISP strategies*). The possible strategies for ISPs are to support or not to support the access sharing for their own end-users. We denote the strategies as

- s_a : ISP strategy to support its subscribers in becoming community members, i.e., adopter;
- s_d : ISP strategy not to support its subscribers in becoming community members, i.e., defector;

Next, we define the instantaneous payoff functions π_p of the defector and adopter ISP strategies.

Definition 8 (*ISP payoff*). Before we lay down the ISPs' payoffs for the possible outcomes, first we define the following cost parameters that characterize ISP-related expenses:

¹ We will analyze two settings of this parameter, a homogeneous case, where the income from the outsiders' roaming costs is shared evenly between the insiders, and a heterogeneous case, where the income is shared as a function of the geographic relevance of the insiders' home locations.

Table 2

ISP game payoffs.

	s_a^2	s_d^2
s_a^1	(5), (5)	(6), (7)
s_d^1	(7), (6)	(8), (8)

- c_t represents the time-unit cost associated with carrying extra traffic caused by the other ISP's roaming subscribers using the shared connections at the ISP's subscribers who are insiders;
- c_a is the ISP's time-unit cost of adoption of the community sharing concept, incorporating trust, security and infrastructural expenditures.

If both ISPs play the adopter strategy, then their payoffs are the same:

$$\pi_p = \frac{n}{2} c_s + GT \frac{n_o^*(\alpha, \beta, c_o, c_i)}{2} c_o \beta - GT \frac{n}{2} c_t - c_a. \quad (5)$$

In case only one ISP plays adopter and the other one defects the payoff of the prior is

$$\pi_p = \frac{n + n_i^*(\alpha, \beta, c_o, c_i)}{2} c_s + GT n_o^*(\alpha, \beta, c_o, c_i) c_o \beta - GT n c_t - c_a; \quad (6)$$

and that of the latter is

$$\pi_p = \frac{n_o^*(\alpha, \beta, c_o, c_i)}{2} c_s. \quad (7)$$

If both ISPs choose to defect, their payoffs are same again:

$$\pi_p = \frac{n}{2} c_s - GT \frac{n}{2} c_t. \quad (8)$$

As a representative summary, we show the payoff matrix in Table 2.

4.5. Technology penetration

The technology penetration in our model gives the probability that a roamer requests Internet connection at a given geographic location and at least one community member provides WiFi access at that location. Here, we show that the heterogeneity in the geographic relevance of user home locations highly affects the experienced technology penetration of the roamers. This happens because the roamers prefer to visit the more relevant places most of the time, which means, that providing WiFi access at relevant places makes roamers "feel" a larger penetration than the actual.

Here, we study several distributions of the geographic home relevance of users (R) since it is highly related to the *perceived* technology penetration (T).

First we assume homogeneous home relevance, as in [8], then the perceived penetration experienced by the roamers is given by:

$$T^{hom} = \frac{n_i}{n}. \quad (9)$$

This setting implements the case where the penetration scales linearly with the number of insider users. However,

the roamers may feel larger perceived penetration if we assume that the players at the most relevant places join to the community of insiders first, or their home locations become highly relevant exactly because they have become insiders.

Second, in the general case, the perceived penetration for a relevance distribution R , given by this latter's cdf $F(r)$ can be calculated for a given insider ratio $\frac{n_i}{n} = Z$ as follows. Now we calculate the appropriate relevance value above which the tail probability equals Z :

$$1 - F(r_z) = Z. \quad (10)$$

From this we can calculate the perceived penetration:

$$T = \frac{\int_{r_z}^{\infty} r f(r) dr}{E(R)}, \quad (11)$$

where $f(r)$ is the probability density function of R , and $E(R)$ is the expected value of R . Without loss of generality, we consider distributions for which $E(R) = 1$ hereafter.

We opt for the Pareto distribution to describe user home relevances because of their intuitive resemblance to empirical statistics of real-world popularity [23] which are usually approximated with Pareto distribution. For this particular distribution of which the cdf is $F(r) = 1 - \frac{r_m^\rho}{r^\rho}$, after some calculation we get:

$$T = Z^{\frac{\rho-1}{\rho}}, \quad (12)$$

with $\rho > 1$. This finally gives:

$$T_{n_i} = \left(\frac{n_i}{n}\right)^{\frac{\rho-1}{\rho}}, \quad (13)$$

for the perceived penetration. Eq. (13) has a convenient property, that it reveals the homogeneous penetration value if $\rho \rightarrow \infty$.

5. Analysis

In this section we investigate the characteristics of the games that arise on the previously defined payoffs and we present our findings about equilibrium states. Building upon the assumption that every participant behaves selfishly (in the game-theoretic sense), first we deduce the expected number of insiders in a one-shot game with homogeneous payment distribution among insiders, then, having the same assumption, we analyze the game from an evolutionary perspective. Second, we relax the uniform payment restriction, and we investigate the users' one-shot game with heterogeneous payments based on the insiders' home relevances. Third, we discuss the game between ISPs and calculate its equilibrium properties. Last, we demonstrate the optimization problem of the mediator.

For tractability reasons, in the following we consider only those Nash equilibria [25] which consist of all insiders having higher home relevance than a given threshold and the rest, below the relevance threshold, play the outsider strategy (see Section 6 for justification). Also, the one-shot game models are investigated in the asymptotic regime, implicitly assuming that the total number of users is large, i.e., $n \gg 1$.

5.1. One-shot user game with homogeneous payments

Here we analyze the scenario when all insiders receive the same share of income from the roaming outsiders' contributions (the mediator's decision). Therefore, their supposed S_j parameters are equal, and from the budget-balance condition (the sum of insider incomes is equal to the α portion of the sum of outsider payments, i.e., $\alpha n_o c_o$) we get:

$$S_j = \frac{1}{n_i}. \quad (14)$$

We model the decision situation of users with a one-shot game, and we derive the stable outcomes, i.e., the pure and mixed strategy Nash equilibria, based on the relation between the user payoffs and the number of insiders. Selfish users join the sharing community and become insiders if their payoffs are higher than the payoffs they get as outsiders, i.e.,

$$\pi_u(S_i) > \pi_u(S_o). \quad (15)$$

This constraint transforms to the following by substituting (3) and (4) into (15), considering (14):

$$G(T_{n_i} - T_{n_i - \Delta n_i})u + Gc_o \alpha \frac{T_{n_i}}{\Delta n_i} \left(\frac{n}{\Delta n_i} - \frac{n_i}{\Delta n_i} \right) + Gc_o T_{n_i - \Delta n_i} > c_i, \quad (16)$$

where T_{n_i} is the perceived penetration as defined in (13).

Considering the limit $\Delta n_i \rightarrow 0$ we get:

$$Gc_o T_{n_i} \left[\alpha \frac{n}{n_i} - \alpha + 1 \right] > c_i, \quad (17)$$

Solving (17) for n_i we infer several roots, which point out the number of insiders, where the two payoff functions are balanced. From these roots we can determine the Nash equilibria of the game.

Lemma 1

$$Gc_o T_{n_i} \left[\alpha \frac{n}{n_i} - \alpha + 1 \right] - c_i = 0 \quad (18)$$

has at most two roots presuming that $n_i > 1$.

Proof. Considering the first derivative of the function we get:

$$Gc_o \frac{T_{n_i}}{n_i} \left[-\frac{1}{\rho} \frac{\alpha n}{n_i} + (1 - \alpha) \frac{\rho - 1}{\rho} \right]. \quad (19)$$

This function in n_i has only one root:

$$n_i = \frac{1 - \frac{\rho-1}{\rho} \alpha n}{(1 - \alpha) \frac{\rho-1}{\rho}}. \quad (20)$$

This means, that (17) may have at most one local minimum or maximum, which completes the proof. \square

Corollary 1 (Equilibria of the user game with homogeneous payments). *The homogeneous user game can produce the following outcomes:*

1. Every user playing the insider strategy is a pure strategy Nash equilibrium, if (17) has at most one real root or it has two real roots $n_{i_1}^*, n_{i_2}^*$ in increasing order, but $n_{i_1}^* > n$.
2. Every user playing the mixed strategy $P(\text{insider}) = \frac{n_{i_1}^*}{n}$ and $P(\text{outsider}) = 1 - P(\text{insider})$ is a mixed strategy Nash equilibrium, if (17) has two real roots and $n_{i_1}^* \leq n$ ($n_{i_1}^* > 0$ always holds).
3. There are at least two Nash equilibria at $n_i = n_{i_1}^*$ (mixed, see previous) and $n_i = n$ (pure, every user is insider), if both real roots exist and $n_{i_2}^* < n$.

Proof. The following considerations support our claims in the respective order.

1. In this case an insider's payoff is not lower than an outsider's, irrespective of the actual number of insiders n_i .
2. In this case at the point $n_i = n_{i_1}^*$ no outsider would become insider to have a negative payoff growth and similarly no insider would leave the community since the outsider payoff gets worse as the number of insiders shrinks at this point (see (3)).
3. In this case the second root is not a Nash equilibrium, since a new insider has higher payoff than as outsider. Therefore the positive payoff difference implies that the second Nash equilibrium is at $n_i = n$. \square

Corollary 1 shows an important characteristic of the system: under certain circumstances an equilibrium state may stall the growing insider population. This phenomenon hinders technology diffusion and it can impact future payoffs in a negative way. Dynamically tuning parameters during the different life cycles of the system may alleviate the problem; this is an interesting topic for future work. The investigated user game has another interesting property regarding to the geographic location relevance of users' homes (see Fig. 2(a)).

Lemma 2. If $\rho < \frac{1}{1-\alpha}$, the first derivative of (17) is always negative.

Proof. The first derivative of (17) is negative, when $\frac{\rho-1}{\rho} < 1 - \frac{1-\alpha}{(1-\alpha)+\frac{2\rho}{n_i}}$. Taking the minimum value of the right-hand side expression at $n_i = n$ gives the stated condition for ρ . \square

Corollary 2 (Massive heavy-tail relevance distribution). User home relevances, described by a Pareto distribution with an exponent less than $\frac{1}{1-\alpha}$, ensure that if a Nash equilibrium, where users with the highest relevance constitute the team of insiders, exists, then full penetration cannot be achieved.

Proof. If the Pareto distribution of relevances has an exponent less than $\frac{1}{1-\alpha}$ then the difference of the insider and outsider payoff is a strictly decreasing function based on Lemma 2. If, based on the parameters and the penetration expression, the first root of (17) is lower than n , then by Corollary 1 it is a mixed Nash equilibrium. However, in this parameter setting the second presented Nash equilibrium at $n_i = n$ does not exist, since by Lemma 2 $\forall n_i > n_{i_1}^*, \pi_u(S_i) < \pi_u(S_0)$. \square

Corollary 2 has important consequences regarding economically feasible game scenarios. The equilibrium state, when all users join the community is clearly not favorable, since in this setting all income from roaming vanishes. In case of heavy tailed user relevance distributions the game naturally transforms into a scenario, where the all-insider equilibrium cannot be present in the system. The heavy tailed relevance distribution also ensures that there is a large number of insiders even when considering lower α values compared to the situation with homogeneous home relevance.

We explore the parameter space of the game numerically in Fig. 2. Parameter ρ affects the insider ratio significantly: the smaller ρ is, the heavier the tail of the user relevance distribution becomes, thus higher penetration can be achieved. This supports the consequence of Corollary 2. Now, if we fix all parameters but roaming cost c_o , we can observe that higher c_o results in higher penetration: it is worth joining if roaming is expensive. Note, that it is the relation of roaming and entry cost (c_i), and roaming intensity G (see Section 5.2) which determines the outcome. Furthermore, a higher user share α motivates users to join the wireless community, as can be seen in Fig. 2(c).

5.2. Evolutionarily stable strategy with homogeneous payments

Since we are interested in the temporal evolution of the system's state, particularly concerning the actual insider number, we derive the evolutionary game theoretic model

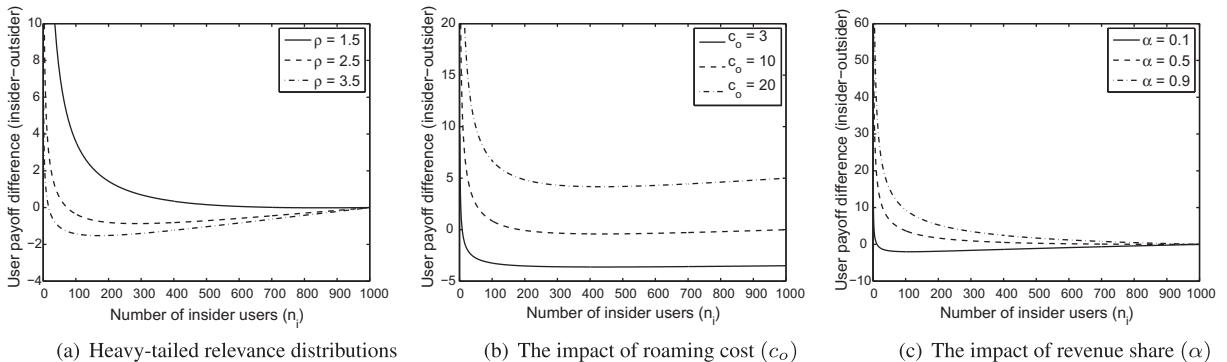


Fig. 2. Expected revenue for users under various parameters settings (default parameters: $c_o = 10$, $c_i = 5$, $\alpha = 0.3$, $\rho = 2$).

of the presented payoffs, and investigate equilibrium states that are also evolutionarily stable. We motivate the choice of applying evolutionary game theoretic tools by the fact that users intend to make practical decisions (e.g., about joining a community) based on their environment's position and experience about the question, and on personal comparison to others' strategies "performance".

Before the analysis of the user game, we introduce the related notions and draw Maynard Smith's requirements on evolutionarily stable strategies [27]. Let $\Pi(s')$ denote the total fitness of an individual following strategy s' ; furthermore, suppose that each individual in the population has an initial fitness of Π_0 . If s^* is an ESS and s' a mutant attempting to invade the population, then

$$\begin{aligned}\Pi(s^*) &= \Pi_0 + (1-p)\Delta\Pi(s^*, s^*) + p\Delta\Pi(s^*, s'); \\ \Pi(s') &= \Pi_0 + (1-p)\Delta\Pi(s', s^*) + p\Delta\Pi(s', s');\end{aligned}$$

where p is the proportion of the population following the mutant strategy s' , and $\Delta\Pi(s^*, s')$ denotes a player's fitness growth playing s^* against another player with strategy s' .

Since s^* is evolutionarily stable, the fitness of an individual following s^* must be greater than the fitness of an individual following s' (otherwise the mutant following s' would be able to invade), and so $\Pi(s^*) > \Pi(s')$. Now, as p is very close to 0, this requires either

$$\Delta\Pi(s^*, s^*) > \Delta\Pi(s', s^*), \quad (21)$$

or

$$\Delta\Pi(s^*, s^*) = \Delta\Pi(s', s^*) \quad \text{and} \quad \Delta\Pi(s^*, s') > \Delta\Pi(s', s'). \quad (22)$$

In other words, a strategy s^* is an ESS if one of two conditions holds: (21) s^* does better playing against s^* than any mutant does playing against s^* , or (22) some mutant does just as well playing against s^* as s^* , but s^* does better playing against the mutant than the mutant does.

Corollary 3 (Evolutionarily stable strategies). *The evolutionarily stable equilibria are included among the one-shot game Nash equilibria presented in Corollary 1 (for $\rho = 2$).*

Proof. We consider the extension of the ESS definition to mixed strategies [15]. In order to find the mixed ESS of the users' game with homogeneous payments, we write (21) and (22) for mixed-strategy user payoffs, defined in Section 4. We denote the mixed ESS by s^* , where a user plays s_i with probability γ ($0 \leq \gamma \leq 1$), and by s' the invader strategy, in which δ is the fraction of insider strategy ($\delta \neq \gamma$). By deriving the expected payoff growths for mixed strategies given the majority's strategy, e.g., $\Delta\Pi(s^*, s') = \gamma\pi_u^\delta(s_i) + (1-\gamma)\pi_u^\delta(s_o)$ where s^* is the evaluated strategy in a state where the majority follows s' and thus $\pi_u^\delta(s_i)$ denotes the insider payoff played with probability γ against a penetration described by δ , after some algebra we arrive at the following conditions:

$$(\gamma - \delta)\left(\gamma^{-\frac{1}{\rho}}(\alpha - \alpha\gamma + \gamma)c_oG - c_i\right) > 0, \quad (23)$$

or

$$\gamma^{-\frac{1}{\rho}}(\alpha - \alpha\gamma + \gamma) = \frac{c_i}{c_oG}, \quad (24)$$

and

$$(\gamma - \delta)\left(\delta^{-\frac{1}{\rho}}(\alpha - \alpha\delta + \delta)c_oG - c_i\right) > 0. \quad (25)$$

The only γ that satisfies condition (23) for any δ ($0 \leq \delta \leq 1$, $\delta \neq \gamma$) is $\gamma = 1$, given that $c_oG > c_i$ holds. On the other hand, the roots of (24) also provide ESSs if they satisfy (25), which transforms to the following with (24):

$$(\gamma - \delta)\left(\delta^{-\frac{1}{\rho}}(\alpha - \alpha\delta + \delta) - \gamma^{-\frac{1}{\rho}}(\alpha - \alpha\gamma + \gamma)\right) > 0. \quad (26)$$

Substituting (13) and $\gamma = \frac{n_i}{n}$ into (17), we get:

$$Gc_o\gamma^{-\frac{1}{\rho}}(\alpha - \alpha\gamma + \gamma) - c_i > 0, \quad (27)$$

which is identical to (24) with equality. It directly follows that $\gamma = 1$ is a pure strategy Nash equilibrium in the one-shot game provided that $c_oG > c_i$ holds. Based on Corollary 1, the lower root n_{i1}^* of (17) gives another Nash equilibrium in the one-shot game if $n_{i1}^* \leq n$. Hindered by the model's complexity, we show the coincidence of the ESS provided by (24) and (25) and this one-shot Nash equilibrium only for $\rho = 2$. In this special case (26) transforms to

$$\left(\frac{\alpha}{\sqrt{\gamma\delta}} + \alpha - 1\right)(\sqrt{\gamma} - \sqrt{\delta})^2(\sqrt{\gamma} + \sqrt{\delta}) > 0, \quad (28)$$

and (28) holds if

$$\frac{\alpha}{\sqrt{\gamma\delta}} + \alpha - 1 > 0, \quad (29)$$

δ may take any value, therefore by substituting $\delta = 1$ we set the left-hand side expression to its the minimal value, thus creating the tightest condition for γ :

$$\frac{\alpha}{1-\alpha} > \sqrt{\gamma}. \quad (30)$$

If $\rho = 2$, the roots of (24) (and those of (27)) are given by

$$\sqrt{\gamma}_{1,2} = \frac{\frac{c_i}{c_oG} \pm \sqrt{\frac{c_i^2}{c_o^2G^2} - 4(1-\alpha)\alpha}}{2(1-\alpha)}. \quad (31)$$

We show that if the lower root satisfies (30), it provides the shown mixed strategy Nash equilibrium of the one shot game, while the higher root provides nothing. After some algebra it follows that $\frac{\alpha}{1-\alpha} > \sqrt{\gamma}_1$ if $c_i > c_oG$, that is, if $c_i > c_oG$ holds, the first root always exists and fulfills the condition, thus gives equilibrium state n_{i1}^* . Furthermore, $\frac{\alpha}{1-\alpha} > \sqrt{\gamma}_2$ holds only if $c_i < c_oG(2\alpha - 1) < c_oG$ for which condition we have seen that $\gamma = 1$ is the ESS. Therefore the relation between c_oG and c_i always determines the ESS, which directly gives either the $\gamma = 1$ or the non-trivial ($\gamma \neq 1$) equilibrium of the one-shot game, proving our claim. Note, that even if both states are Nash equilibria in the one-shot game model, only one of them is evolutionarily stable. \square

5.3. One-shot user game with heterogeneous payments

In what follows, we investigate the scenario where the distribution of income among insiders is not uniform. Instead, an insider receives the fraction of the total income

proportional to her home's relevance. Therefore the term S_j in the insiders' payoff function, i.e., (4), is replaced by

$$S_j = \frac{R_j}{\sum_{j=1}^{n_i} R_j}, \quad (32)$$

for user j , where R_j is a relevance value according to an arbitrary distribution.

We define the condition that needs to hold for the insider and outsider payoffs in the asymptotic regime again:

$$GT_{n_i} \left(u + \frac{R_{n_i}}{T_{n_i}} c_o \left(\frac{n}{\Delta n_i} - \frac{n_i}{\Delta n_i} \right) \alpha \right) - GT_{n_i - \Delta n_i} (u - c_o) > c_i. \quad (33)$$

Lemma 3. (33) has at most two roots presuming that $n_i > 1$.

Proof. The asymptotic analysis transforms R_{n_i} to:

$$R_{n_i} = T_{n_i} - T_{n_i - \Delta n_i}. \quad (34)$$

Combining (34) with (33), after some simplifications:

$$G(T_{n_i} - T_{n_i - \Delta n_i})u + G \frac{T_{n_i} - T_{n_i - \Delta n_i}}{\Delta n_i} c_o (n - n_i) \alpha + GT_{n_i - \Delta n_i} c_o > c_i. \quad (35)$$

By taking the limit $\Delta n_i \rightarrow 0$ in (35) we get:

$$GT'_{n_i} c_o (n - n_i) \alpha + GT_{n_i} c_o > c_i. \quad (36)$$

After calculating the derivative of T assuming (13) for the relevance distribution, and some simplification we arrive at:

$$Gc_o T_{n_i} \left[\frac{(\rho - 1) \alpha n}{n_i} - \frac{\rho - 1}{\rho} \alpha + 1 \right] > c_i. \quad (37)$$

To determine the number of roots of (37) we calculate the roots of its first derivative:

$$Gc_o T_{n_i} \frac{\rho}{(\rho - 1) n_i} \left[\frac{(\rho - 1) \alpha n}{n_i} - \frac{\rho - 1}{\rho} \alpha + 1 - \frac{\alpha n}{n_i} \right] = 0. \quad (38)$$

Eq. (38) has only one root at:

$$n_i = \frac{\alpha n \left(\frac{\rho - 1}{\rho} - 1 \right)}{\alpha \frac{\rho - 1}{\rho} - 1}, \quad (39)$$

which means that (33) has at most two roots. \square

Based on Lemma 3, we can derive some Nash equilibria of the heterogeneous payment distribution model.

Corollary 4 (Equilibria of the user game). The heterogeneous user game can produce the following outcomes:

1. Every user playing the insider strategy is a pure strategy Nash equilibrium, if (33) has at most one real root or it has two real roots $n_{i_1}^*, n_{i_2}^*$ in increasing order, but $n_{i_1}^* > n$.
2. Every user playing the mixed strategy $P(\text{insider}) = \frac{n_{i_1}^*}{n}$ and $P(\text{outsider}) = 1 - P(\text{insider})$ is a mixed strategy Nash equilibrium, if (33) has two real roots and $n_{i_1}^* \leq n$ ($n_{i_1}^* > 0$ always holds).

3. There are at least two Nash equilibria at $n_i = n_{i_1}^*$ (mixed, see previous) and $n_i = n$ (pure, every user is insider), if both real roots exist and $n_{i_2}^* < n$.

Proof. The proof follows the same line of thoughts as in the proof of Corollary 1. \square

We illustrate the impact of the payment structure on the users' willingness to join the community in Fig. 3. While equal incomes motivate more users to join (even users at low-relevance locations), a heterogeneous revenue structure allows high-relevance users to generate higher profits. Here lies a management possibility for the mediator: whether she wants to share the wealth evenly in the community, or to provide incentives for highly relevant users to join and thus facilitate the quick diffusion of the technology. More on design choices can be found in Section 5.5.

5.4. The ISP game

ISPs can either allow and support their users' intent of joining a global wireless community or prohibit it. On one hand, allowing it results in extra profit coming from roaming users through the mediator; on the other hand, there are also associated costs, namely the adoption cost c_a and the cost of the extra traffic c_t stemming from roamers. Furthermore, users of an ISP which does not cooperate with the community can look for another, more flexible and user-friendly ISP. This way ISPs compete for potential broadband subscribers.

With these thoughts in mind, we model the interaction of ISPs with a one-shot game of two players which takes the number of insiders n_i as an input from the user game. The strategy space and respective payoffs are defined in Section 4.4. To facilitate understanding, we show the payoff matrix in Table 3. Since payoffs are symmetric, we omit the value for the second player in each bracket for simplicity.

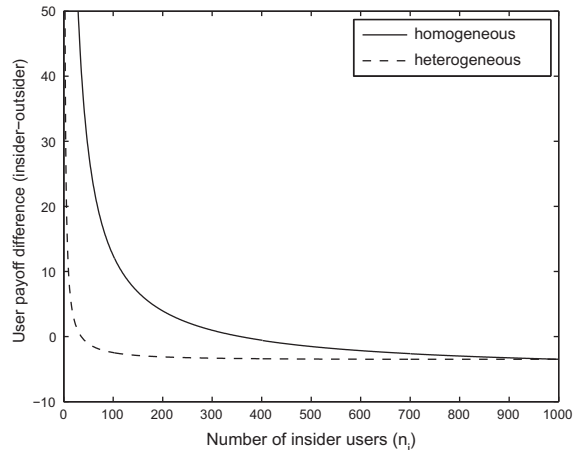


Fig. 3. Expected revenue for users: heterogeneous vs. homogeneous payment structure (all other parameters fixed).

Table 3

Simplified payoff matrix for the ISP game.

	s_a^2	s_d^2
s_a^1 (5)	$\frac{n_i}{2}c_s + GT\frac{n_o}{2}c_o\beta - GT\frac{n_i}{2}c_t - c_a$	(6) $\frac{n_i+n_o}{2}c_s + GTn_o c_o\beta - GTn_i c_t - c_a; \frac{n_o}{2}c_s$
s_d^1 (7)	$\frac{n_o}{2}c_s$	(8) $\frac{n_i}{2}c_s - GT\frac{n_i}{2}c_t$

Corollary 5 (Equilibria of the ISP game). Let $X = \frac{n_i}{2}c_s + \frac{n_o}{2}GTc_o\beta - \frac{n_i}{2}GTc_t - c_a$.

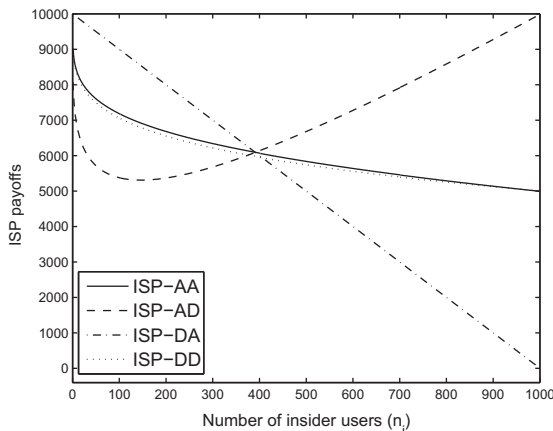
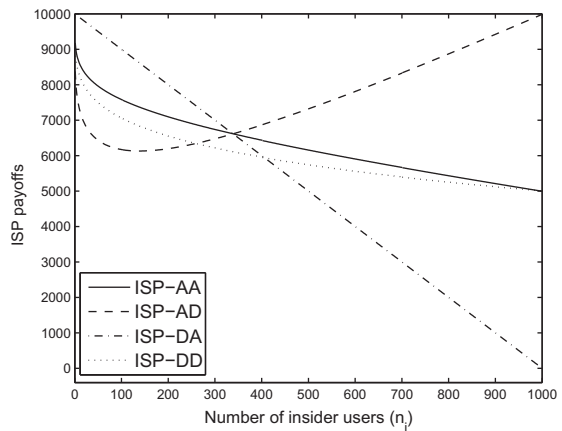
1. The strategy profile (A,A) is a pure-strategy Nash equilibrium, if $X > 0$.
2. The strategy profile (D,D) is a pure-strategy Nash equilibrium, if $X < 0$.

Proof. If the above condition holds, no payoff increase can be achieved by any player unilaterally deviating, which is the definition of the Nash equilibrium. For (A,D) and (D,A) to be Nash equilibrium points, there are two contradicting conditions, therefore these two strategy profiles are not stable. \square

From a different perspective, it is important to understand, if and how the above conditions could be achieved.

Corollary 6 (Influence of the mediator). The mediator determines the outcome of the ISP game and hence the Nash equilibrium by setting parameters for roaming cost c_o and ISPs' share β with respect to the number of insiders n_i . Specifically (let $K = \frac{n_iGTc_t + c_a - n_oGT}{n_oGT}$):

1. if $c_o\beta > K$, the strategy profile (A,A) is a pure-strategy Nash equilibrium.
2. if $c_o\beta < K$, The strategy profile (D,D) is a pure-strategy Nash equilibrium.

(a) ISP payoffs ($\beta = 0.2$)(b) ISP payoffs ($\beta = 0.8$)**Fig. 4.** Expected revenue for ISPs: low and high revenue shares (β).

We illustrate the effect of revenue share β on ISP payoffs in Fig. 4. The gap between (A,A) and (D,D) payoffs increases with β : a higher revenue share makes supporting the global wireless village concept more favorable for ISPs. On the other hand, there is an intersection of (A,A), (A,D) and (D,A) in both cases: it points out the number of insiders that satisfies $c_o\beta = K$. To the left of this point it is better to defect, on the contrary, to the right of this point it is better to adopt. This is further supported by the changing relation between $\pi_{(D,A)}$ and $\pi_{(A,D)}$. Note, how this point moves to the left with a higher β value.

5.5. Mediator: an optimization problem

The mediator acts as the leader of the Stackelberg game defined in Section 4. It is in her power to set roaming cost c_o , entry cost c_i , user revenue share α and ISP revenue share β . Additionally, the mediator can influence the growth of the insider population by choosing either a homogeneous or a heterogeneous payment structure (See Fig. 3). By consciously implementing a given parameter set, the mediator can achieve different goals, e.g., she can maximize her own profit, aim for the maximum utility of insiders, side with the ISPs or target social welfare. While we leave the investigation of most cases to future work, we do explore the greedy (maximizing mediator profit) and the social (maximizing social welfare) case in the followings.

In the greedy scenario the objective function to be maximized is:

$$\pi_m^g = GTn_o^*(\alpha, \beta, c_o, c_i)c_o(1 - \alpha - \beta T_a^*(\alpha, \beta, c_o, c_i)) - c_m + n_i^*(\alpha, \beta, c_o, c_i)c_i. \quad (40)$$

On the other hand, when the mediator wants to maximize social welfare, she has to consider all parties: insider and outsider users, adopter and defector ISPs, and herself. Formally:

$$\pi_m^s = \pi_m^g + \sum_{j=1}^{n_i^*} \pi_u^j(s_i) + n_o^* \pi_u(s_o) + n_a^* \pi_a + n_d^* \pi_d \quad (41)$$

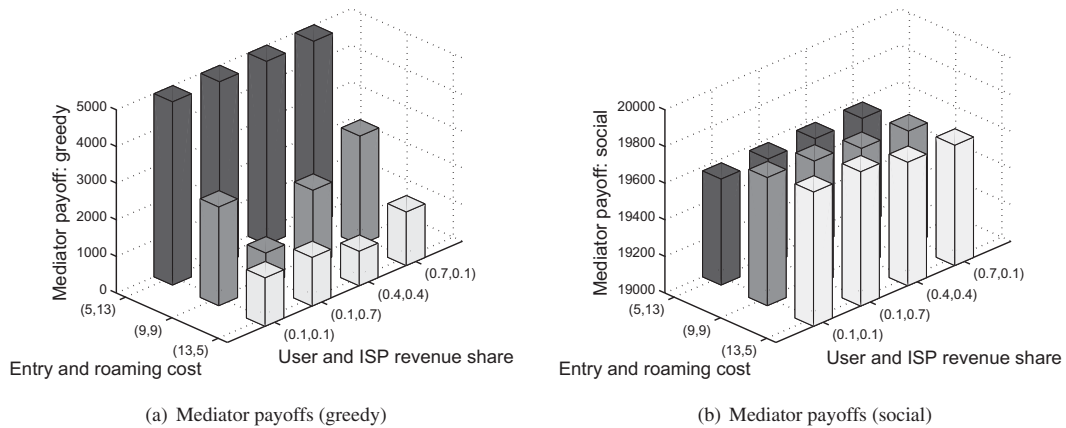


Fig. 5. Mediator payoffs: greedy vs. social welfare (x axis: revenue shares (α, β) , y axis: cost parameters (c_i, c_o)).

where n_a^* is the number of adopter and n_d^* is the number of defector ISPs in equilibrium. In either case, the optimization problem (which is essentially a one-player game, hence the notation s_m) becomes:

$$s_m = \operatorname{argmax}_{c_o, c_i, \alpha, \beta} \pi_m(c_o, c_i, \alpha, \beta). \quad (42)$$

The solution method for Stackelberg games is *backward induction*. The leader (mediator) considers what the best response of the followers (ISPs and users) are, i.e., how they will respond once they have observed the strategy of the leader. The leader then picks a quantity that maximizes its objective function, anticipating the predicted response of the followers. The followers actually observe this and in equilibrium pick the expected strategy as a response. This process is also visualized in Fig. 1. In our case, backward induction is very complex.

In order to give a little insight into the mediator's actions, we map her parameter space numerically, while adhering to the concept of backward induction. We use the evolutionary game with homogeneous payments at the user level. We show the results in Fig. 5. In the greedy case, the mediator achieves the highest profit when $c_i < c_o G$; then every user is an insider (please refer to Section 5.2). In the social case, community welfare is the highest when user income share α is low and roaming cost c_o is equal or greater than entry cost c_i . Note, how the all-insider scenario (where $(c_i, c_o) = (5, 13)$) is inefficient in this case.

6. Experimental evaluation

Here, we investigate the evolutionary behavior of the community in the heterogeneous payment case. For this end we develop a Matlab simulator based on real-world data from the FON map [16], and inspired by state-of-the-art mobility measurement campaigns and models [10,21,22]. Note, that we assume a single Internet Service Provider; by doing so we are able to focus on the intricate details of user behavior. Also, we assume that users and home routers are in a 1–1 relationship, i.e., we do not consider multiple users in a single household.

First, we define the mobility graph (the playing field). Second, we define the interaction among players. Last, we present the simulation results.

6.1. Modeling user mobility and relevance

Since we can handle greater complexity when building a simulation model compared to an analytically tractable one, we chose to incorporate exact geographic locations of users home, mobility (captured by parameter G in the analytic model) and relevance (R) for every individual user. We present a case study on the city of Berlin because of two reasons. First, the number of currently existing, shared FON hotspots is the highest there among cities. Second, we can greatly reduce simulation run time compared to the entire globe. Note nonetheless that our simulator is capable of simulating any geographic area and user population.

6.1.1. Geographic location

Locations of user homes are determined in two distinct ways. First, the coordinates of insiders are extracted from

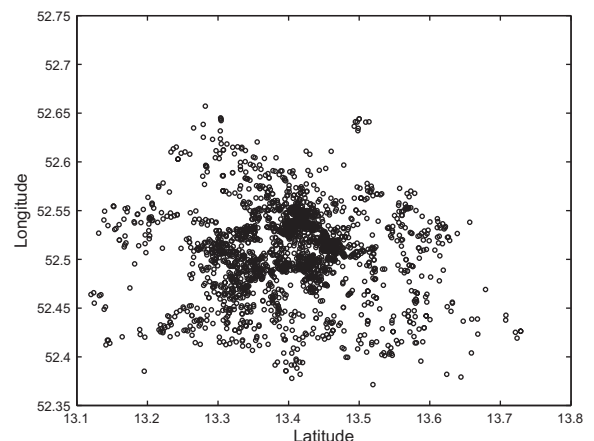


Fig. 6. Geographic distribution of FON users in Berlin.

the FON data set. The Berlin scenario is depicted in Fig. 6. Second, outsiders are deployed according to the population density of the respective area. For Berlin, we turn to the district-level population density map ([17,18]). Then, we simplify and aggregate the map into sections of concentric circles and generate outsiders' home locations accordingly. We denote a single geographic location by (x,y) .

6.1.2. Mobility

The individual mobility of users are calculated based on the observations in [10]. Every user has a *mobility radius* b , a random variable drawn from a truncated power-law distribution, such as

$$P(r) = (b + b^0)^{-\beta} e^{-b/c}, \quad (43)$$

where shape parameter $\beta = 1.65$, minimum value $b^0 = 1$ km and cutoff value $c = 15$ km (all parameters are drawn from large-scale experiments reported in [10], adjusted to Berlin). Practically speaking, the cutoff value ensures that only few users travel from an outer suburb to a suburb in the opposite side of the city.

In addition, each user spends a different portion of her day away from home, roaming. According to [24] overall mobility of individual users are characterized by a power law. So for every user the *willingness-to-roam* m is drawn from the distribution $P(m)$, analogous to (43) with cutoff parameter $c = 50$ to ensure greatly varying values.

Every user is characterized by the number of places she visits regularly when roaming. We refer to this value as the *number of neighbors* q . Authors of [10] found that most mobile phone users visit between 5 and 50 places repeatedly. We consider this as an acceptable estimation for roaming WiFi users. Additionally, human mobility is highly affected by one's social connections, i.e., the number of relationships one has [20]. It is well-known since [23] that social connections can be modeled as complex networks, where node degrees follow power-law characteristics. Thus for $P(q)$ we use (43) but with minimum value $q_0 = 5$ and cutoff $c = 50$.

Choosing q specific locations inside the circle determined by center (x,y) and mobility radius b is not trivial. On one hand, it is shown that nodes in a network characterized by social relationships have a certain *attraction factor* a , which follows a power-law distribution (thus producing the “rich getting richer” phenomenon). On the other hand, we have not seen specific empirical evidence for this behavior in the context of roaming wireless Internet users. Therefore, we treat the attraction type as a parameter, i.e., in a “diverse” scenario we draw a from a truncated power-law distribution ($\beta = 1.6$, $a_0 = 5$, $c = 50,000$), while in a “uniform” scenario a is a constant at all locations.

Note that for the actual generation of b, m, q, a values, we use the code from Clauset and Shalizi [19].

It is shown by extensive measurements [10] that for user i the *frequency* (w_{ij}) of visiting place j is inversely proportional to the rank (L_j) of the respective place (descending order), i.e., $w \propto \frac{1}{L}$. This value should be weighted by user i 's respective willingness-to-roam m_i , so we get

$$w_{ij} = m_i \frac{1}{L_j} \sum_{k=1}^{q_i} \frac{1}{k}, \quad (44)$$

where rank L_j comes from an ordering on the respective attraction factors (a_j). Now we can construct the vertices and edges of the mobility graph, using Algorithm 1.

Algorithm 1. Constructing the edges of the mobility graph

Input: Set of vertices V , each $v_i \in V$ characterized by a 6-tuple $(x_i, y_i, b_i, q_i, m_i, a_i)$

Output: Set of edges E , $e_{ij} = (v_i, v_j, w_{ij})$

foreach v_i **do**

$t = []$;

foreach $v_j \neq v_i$ **do**

if $(x_i, y_i) - (x_j, y_j) < b_i$ **then**

 append (t, v_j) ;

if $q_i > |t|$ **then**

$q_i = |t|$;

for $k \leftarrow 1$ **to** q_i **do**

 assign L_k to t_k based on a_{t_k} ;

 calculate $w(i, t_k)$;

$e(i, j) = (v_i, t_k, w_{i,t_k})$;

Definition 9 (*Mobility graph*). $\mathcal{G} = (V, E)$ constitutes the mobility graph. Note, that the outdegree of nodes are given by \underline{q} , and the sum of outgoing weights are contained by \underline{m} .

6.1.3. Relevance

In Section 4.5, we gave a formula for the perceived technology penetration T based on tail distributions. Here we can explicitly compute a user's relevance R_i from \mathcal{G} :

$$R_i = \sum_{e(v_j, v_i) \in E} w_{j,i}, \quad (45)$$

in other words, relevance is equal to the sum of weights for incoming edges.

6.2. The evolutionary game

Our simulations capture the evolution of the user population with respect to their chosen strategies (insider vs. outsider). Since users roam at their neighbors, they are affected by the strategies of their neighbors. The effect of a single neighbor j to user i is proportional to the weight w_{ij} ; the more she visits a neighbor the greater effect she will have on her behavior. In the real world, this would require complete information on the neighbors' payoff, which may be only partially available (e.g., the wireless router could broadcast the number of currently connected users, but the neighbor's utility of being online can only be estimated). Here, we assume complete information (in the context of neighbors) for tractability. Furthermore, to model early adopters and dissatisfied former users we make use of mutation: in each round, every user changes its strategy with the probability of μ , the mutation rate. The game is formally defined by Algorithm 2.

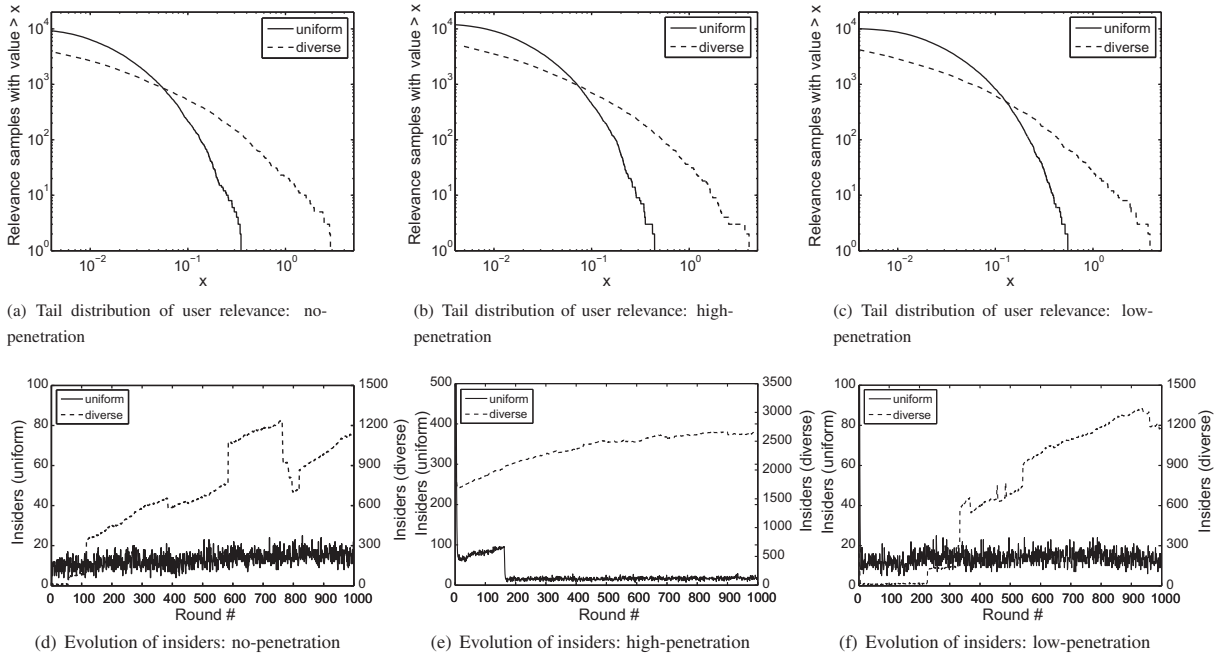


Fig. 7. Tail distribution of user relevance and evolution of the user population under different initial conditions ($u = 4, c_o = 3, c_i = 3, \alpha = 0.5, \mu = 0.001$).

Algorithm 2. The evolutionary game

Input: Set of nodes V , Set of edges E , $e_{ij} = (v_i, v_j, w_{i,j})$,
Vector of initial strategies S , $s(j) \in \{s_i, s_o\}$, $|S| = n$,
mutation rate μ , rounds

Output: Matrix of strategies A , $a_{ij} \in S_i, S_o$,

$|A| = \text{rounds} \times n$

$A(0, :) = S$;

$k = 0$;

while $k < \text{rounds}$ **do**

$k++$;

$z_i = 0$;

$z_o = 0$;

foreach v_j **do**

foreach $e_{j,i}$ **do**

if $A(k-1, i) = s_i$ **then**

$z_i = z_i + \pi_i w_{j,i}$;

else

$z_o = z_o + \pi_i w_{j,i}$;

if $z_i > \pi(A(k-1, j))$ **then**

$A(k, j) = s_i$;

else if $z_o > \pi(A(k-1, j))$ **then**

$A(k, j) = s_o$;

else

$A(k, j) = A(k-1, j)$;

if $\text{rand}() < \mu$ **then**

$A(k, j) = \overline{A(k, j)}$;

zero insiders ($n = 10,000$), and show how the system may or may not evolve to have a sizable insider population. Second, we add all the insiders ($n_i = 2410, n = 12,410$) found in the FON Berlin data set, and show the evolution of the system. Third, we scale down the initial insider population to reflect the state of the real-world Berlin (100,000 households with roaming users, hence $n_i = 241, n = 10,241$). In Figs. 7 and 8, these scenarios are shown in the first, second and third column, respectively.

We emphasize that it is the trends and characteristics of the results that are important, not the exact penetration ratio or the exact tail distribution of user relevance. We mainly focus on temporal characteristics, so results presented are from single simulation runs; however, only slight differences due to mutation can occur, otherwise the simulation is deterministic for a fixed mobility graph. Our Matlab simulation scripts can be downloaded from [9].

6.3.1. The impact of attraction factors

We characterize the relevance distribution of users in both “diverse” and “uniform” attraction cases in Fig. 7(a)–(c). User home relevance values are determined when building the mobility graph (see (45)), i.e., they do not change over the course of simulation. It can be observed that in the “diverse” case, relevance is very close to a power-law (linear in a log-log plot), meaning that power-law characteristics of the number of neighbors q and attraction factor a carries over to relevance, resulting in a heavy tail. On the other hand, a “uniform” neighbor selection process leads to a lighter tail for the relevance distribution, despite the other power-law characteristics of the graph. Note the significant difference between curves near $x = 0$; while “uniform” attraction ensures incoming roaming relations (edges) to virtually all users

6.3. Simulation results

All simulations are run with 10,000 outsider users ($n_o = 10,000$), mutation rate $\mu = 0.001$ and for 1000 rounds. We simulate three different scenarios. First, we start with

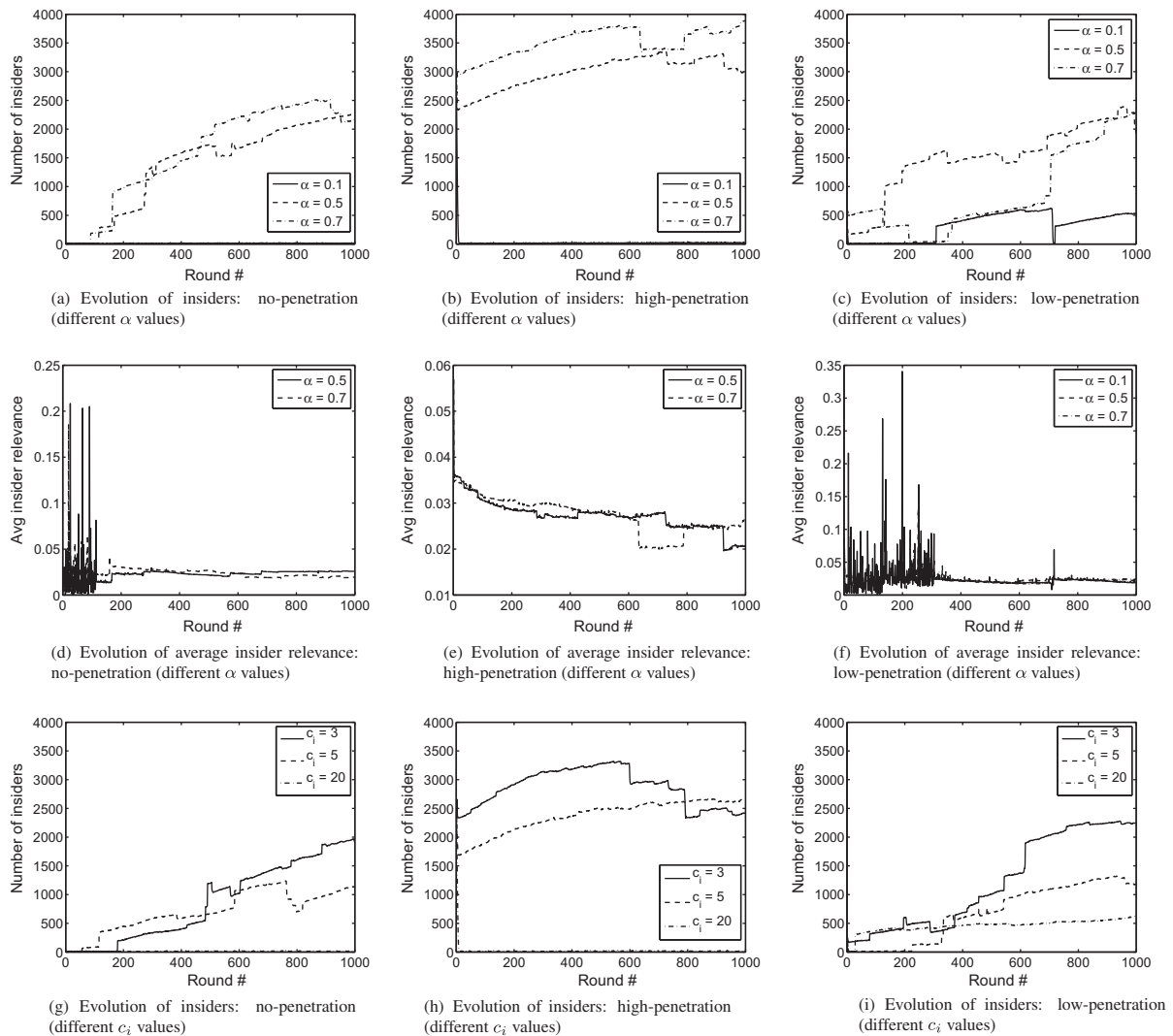


Fig. 8. Evolution of the user population and the average insider relevance under different initial conditions (default parameters: $u = 4$, $c_o = 3$, $c_i = 3$, $\alpha = 0.5$, $\mu = 0.001$).

(vertices), a significant number of users does not get incoming roamers when applying the “diverse” model, hence their zero relevance.

As can be seen in Fig. 7(d)–(f), there are very few insiders in the “uniform” attraction case. This stems from the fact that there are few to none high-relevance users, so bootstrapping other users is increasingly difficult. Even in the high initial penetration case, the insider ratio drops to a very low level after some early drop and increase. On the other hand, high initial penetration and “diverse” attraction results in progress. In addition, even in scenarios with lower initial penetrations there is a growing number of insiders. Our experiments showed similar results for various parameter settings with regard to attraction models, therefore we only demonstrate scenarios with “diverse” attraction from this point forward.

6.3.2. Impact of user revenue share (α)

Evolution of the user population and the corresponding average insider relevance with respect to different values of revenue share α is shown in Fig. 8(a)–(f). (Note, that $\alpha = 0.1$ is omitted in two relevance plots, because they are fluctuating wildly during the entire simulation run.) The first observation is that at $\alpha = 0.1$, virtually no users are joining the wireless community (although some high-relevance users help technology diffusion in Fig. 8(c)). Looking at the average insider relevance, high-relevance users seem to join first in Fig. 8(d) and (f). This justifies the assumptions made in Section 5: they are based on realistic user behavior. Note, how the valleys and peaks occur at the same time with the evolution of insiders in all plots. In the high initial penetration case, the large number of initial insiders smoothes out the fluctuation in average user relevance.

Note, that Fig. 8(c) and (f) represent the current state of the FON community. Specifically, $\alpha = 0.5$ is implemented in the FON system. These plots show steady-rate progress, forecasting potential further growth for existing global wireless communities.

6.3.3. Impact of roaming and entry cost (c_o, c_i)

In Fig. 8(g)–(i), we plot the evolution of the user population with different entry costs c_i . Since all other parameters are fixed (including roaming cost c_o), the impact of cost parameters can be clearly seen. An inadequately high entry cost keeps the users from joining, except for our down-scaled scenario (Fig. 8(i)), where a small but steady insider population is present. Naturally, a lower c_i generally results in more insiders (although, there are some fluctuations).

Again, if we look at the down-scaled plot, an interesting trend emerges: even at comparatively high entry costs, there is a steady-rate growth in technology penetration. This implies that wireless communities networks are in an expanding phase today. This shows the possibility of the emergence of a global wireless village.

7. Future work

This study presents our effort towards the understanding of economic interactions in a wireless, global, user-provided and ISP-supported networking framework. Here we give an outlook at further issues and possible research directions which can bring us closer to a complete picture.

7.1. Individual modeling of user characteristics

Throughout our analytic investigations we made several simplifying assumptions that could be relaxed. We presumed that all users have similar mobility characteristics, demand and utility of being online (implicitly assuming a uniform data traffic profile), which is clearly not the case in real life; however, taking these issues into consideration in an analytical framework seems to be highly non-trivial task. For example in the case of heterogeneous payment (Section 5.3), where the payoff functions of the users basically depend on their relevance value, determining the ESS is not straightforward. Finding such equilibrium may benefit from techniques used in [34] in the future; although, more sophisticated (individual) user models may only be suitable for simulation studies.

7.2. Mediator payoff and business models

While the business model we used in this paper is based on the FON concept, we cannot yet determine if it is optimal in any (social welfare, maximum mediator profit, etc.) aspect. Further research applying mechanism design techniques to the problem of creating and evaluating efficient business models for user-provided networking is highly relevant. Additionally, the dynamic tuning of parameters could be studied: it could help in overcoming the stagnating technology penetration in certain scenarios (see Section 5.1). Furthermore, more sophisticated user categories

can be introduced to refine revenue flow. On the other hand, the existence of multiple mediators induce competition among them, which can be also taken into consideration. Last, the impact of a global wireless community network on 3G/4G mobile operators should also be studied (see [14] for guidelines).

7.3. Traffic measurements

Modeling ISP costs proportional to traffic is far from trivial in this context. For an advanced cost model one should take into account traffic and usage characteristics as measured in the very network. This would include different traffic profiles for users (e.g., heavy hitters vs. e-mail and browsing). Also, the utility of users is greatly dependent on traffic prioritization techniques and quality of experience. While there are some existing surveys dealing with the FON system [30], we still lack a large-scale measurement study to build upon.

7.4. Mobility patterns

For our simulations we constructed mobility graphs to be as realistic as possible in terms of users locations, mobility patterns and roaming intensities. However we believe, that such a network evolves according to the current state of the user, ISP and mediator games, which eventuates a dynamic structure, with temporally changing characteristics. Although the understanding and modeling of the dynamic properties of networks and communities is still in heavy development [31–33], it would be interesting to see how the defined game rules interact with the underlying roaming structure.

7.5. Regulatory aspects

Accountability (tying the traffic to a given user) and the feasibility of lawful interception are challenging issues: these features may be needed if/when a truly global wireless village emerges. Both the legal framework and its enabling implementation are far from trivial: these are possible directions for a different branch of highly interesting future work.

As it can be clearly seen, there is room for improvement both in modeling and designing a global wireless community network. We believe that the main results of this paper are sound qualitatively; nevertheless, pursuing the above directions of research could yield insights and methods that are needed in order to devise a practical “handbook” for community providers and ISPs. In particular, a real-life measurement campaign would be of highest benefit.

8. Conclusion

In this paper, we have presented a multi-faceted study on the economic interactions in a global wireless community network, incorporating users, Internet Service Providers and the community provider (mediator). We have developed a model based on a leader–follower game,

where every stage (user, ISP and mediator) is a game in itself.

We have shown by game-theoretic analysis that, assuming homogeneous payment distribution, the user game could have different pure and mixed strategy Nash equilibria depending on the actual parameters. We have also found that heterogeneity in the user location relevance is a driving force behind an economically feasible system. Furthermore, the evolutionary extension of this game reveals that the ESSs are among the equilibrium profiles of the one-shot game. Specifically, if $c_i > c_o G$, a balanced regime arises, where technology penetration is not 100%, i.e., there are outsiders who generate profit for others.

Next, considering a heterogeneous payment structure, where a user's income is proportional to its respective home relevance, we have shown that high-relevance users can achieve high profit. In exchange, they expedite the technology diffusion process. Determining the payment structure is a design choice for the mediator.

ISPs tend to act as one: either all of them defect against the community or all of them adopt the technology. We have demonstrated how the mediator can motivate them by setting their revenue share β to a higher value, thus increasing the chance of adoption.

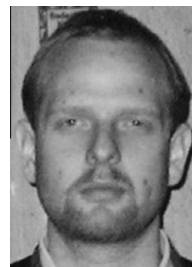
We have also illustrated the various possible goals of the mediator (maximizing different objective functions). As an example, we have shown how a 100% penetration can be efficient in a greedy scenario and totally inefficient when maximizing social welfare.

We have constructed a mobility model based on the social interactions of users, and real-world location and population density data. Using the mobility model and a heterogeneous payment structure we have demonstrated by simulation of user population evolution that high-relevance users drive the technology diffusion. Moreover, the current real-world penetration level and structure of global community networks could imply steady growth for the future.

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