UNBIASED MAXIMUM SINR PREFILTERING FOR REDUCED–STATE EQUALIZATION

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ABSTRACT

We consider prefiltering for a single–carrier transmission over frequency–selective channels, where reduced–state trellis–based equalization is employed at the receiver. While previously proposed prefiltering schemes are based on the optimum filters of decision–feedback equalization (DFE), the prefiltering scheme introduced in this paper is designed according to a signal–to–interference–plus–noise ratio (SINR), whose definition takes into account explicitly the subsequent trellis–based equalizer and its complexity. In addition to the prefilter, a finite–length target impulse response for trellis–based equalization and a feedback filter for state–dependent decision feedback in equalization, respectively, is determined. However, it is shown that a direct maximization of the SINR with respect to all filters results in a *biased* solution, and the relation of this solution to an optimum unbiased solution with improved error rate performance, which is also derived, is investigated.

1. INTRODUCTION

Single–carrier transmission is still of significant interest for various practical applications because in contrast to multi–carrier transmission, it does not suffer from a high peak–to–average power ratio (PAPR). In particular for devices in which transmitters of low cost should be employed, single–carrier transmission is advantageous because amplifiers with a reduced linearity range can be used due to the low PAPR of single–carrier signals. For example, single–carrier frequency–division multiple access (FDMA) transmission has been standardized for the uplink of the Long Term Evolution (LTE) mobile communications system, and conventional single–carrier transmission schemes are still frequently employed in military communications.

For a transmission with single–carrier modulation over frequency–selective channels producing intersymbol interference (ISI), optimum maximum–likelihood sequence estimation (MLSE) [1] using the Viterbi algorithm (VA) is often too complex for an implementation. In such a case, suboptimum trellis–based equalization with a reduced number of states might be adopted because it offers a very good trade-off between performance and complexity.

For a reduction of the number of states of the trellis diagram of MLSE, the principles of trellis–based equalization and decision–feedback equalization (DFE) might be combined [2]. However, such a scheme is prone to error propagation if the feedback of decisions is done outside of the VA, using tentative decisions. This problem can be circumvented if state–dependent feedback is performed within the VA, exploiting the symbols of the surviving paths of the VA

assigned to the trellis states as has been proposed by Duel–Hallen and Heegard [3]. The resulting *delayed decision–feedback sequence estimation (DDFSE)* algorithm is characterized by an excellent tradeoff between performance and complexity. A further refinement of DDFSE has been introduced by Eyuboğlu and Qureshi [4] which is referred to as *reduced–state sequence estimation (RSSE)*. Here, an additional reduction of the number of states of the VA is accomplished by defining the trellis states via the index numbers of the subsets of a set partitioning of the signal constellation.

However, DDFSE/RSSE offers an excellent tradeoff between performance and complexity only if it is combined with suitable prefiltering in front of equalization. Typically, the prefilter is designed to transform the channel impulse response (CIR) approximately into its minimum–phase version. Corresponding finite impulse response (FIR) prefilters have been proposed e.g. in [5,6].

However, the aforementioned prefilters seem to be not optimally adjusted to DDFSE/RSSE because they have been originally designed for DFE, neglecting the fact that not only the first tap of the prefiltered CIR is relevant for the detection performance but several consecutive taps whose number depends on the trellis definition of DDFSE/RSSE. In this paper, we propose a prefiltering scheme which offers an improved performance compared to the DFE–based prefilters and is better matched to the characteristics of DDFSE/RSSE.

This paper is organized as follows. In Section 2, the underlying system model for a single–carrier transmission over an ISI channel with DDFSE/RSSE in the receiver is described, and a filter design criterion which is adjusted to the characteristics of DDFSE/RSSE is introduced. In Section 3, it is demonstrated that an optimization of this criterion without any further constraints, as has been done in [7], results in a *biased* solution which causes a certain mismatch in subsequent trellis–based equalization. In particular, the assumed useful signal and the error signal of equalization for a certain time instant are no longer statistically independent for a biased solution, resulting in a degraded performance of trellis–based equalization. In Section 4, the optimum unbiased FIR prefilter is derived which guarantees an improved error rate performance of equalization. Both solutions are related to each other in Section 5. Numerical results for the proposed scheme are presented in Section 6 which demonstrate that it is able to outperform previously proposed schemes.

Notation: $\mathscr{E}\{\cdot\}$, $*$, $(\cdot)^T$ and $(\cdot)^H$ denote expectation, convolution, transposition and Hermitian transposition, respectively. Bold lower case letters and bold upper case letters stand for column vectors and matrices, respectively. $\mathbf{I}_X$ is the $X \times X$ identity matrix. $P(z)$ stands for the z–transform of a sequence $p[k]$. The correlation sequence of signals

$p[k]$ and $q[k]$ and its z–transform are denoted as $\varphi_{pq}[\kappa] = E\{p[k]\,q^*[k-\kappa]\}$ and $\Phi_{pq}(z)$, respectively.

2. SYSTEM MODEL

We consider a single–carrier transmission with linear modulation over a frequency–selective channel producing ISI. In discrete–time equivalent complex baseband representation, the received signal is given by

$$r[k] = \sum_{\kappa=0}^{q_h} h[\kappa]\,a[k-\kappa] + n[k], \qquad (1)$$

where $a[k]$ denote the independent, identically distributed (i.i.d.) symbols of the transmit sequence of variance σ_a^2 which are taken from a signal constellation $\mathscr{A}$, e.g. an M–ary phase–shift keying (PSK) or quadrature amplitude modulation (QAM) constellation. The discrete–time CIR $h[k]$ of order q_h comprises the effects of transmit filtering, channel, receiver input filtering, and symbol–spaced sampling. $n[k]$ stands for additive white Gaussian noise (AWGN) of variance σ_n^2. The received signal is prefiltered by an FIR filter with transfer function $F(z) = \sum_{k=0}^{q_f} f[k]\,z^{-k}$.

The prefiltered signal is processed by a DDFSE or RSSE algorithm. For simplicity, we discuss only DDFSE in more detail[1]. The states of the reduced–state trellis diagram of DDFSE are defined as

$$\tilde{\mathbf{s}}_r[k] = [\tilde{a}[k-1]\ \tilde{a}[k-2]\ \dots\ \tilde{a}[k-q_d]]^T, \qquad (2)$$

where $\tilde{a}[\cdot] \in \mathscr{A}$ are equalizer trial symbols. The number of states per time step is $Z = M^{q_d}$, $0 \le q_d \le q_h$. Here, the extreme cases of $q_d = 0$ and $q_d = q_h$ correspond to DFE and a full–state VA, respectively. The metric of the DDFSE trellis branch emerging from state $\tilde{\mathbf{s}}_r[k-k_d]$ with trial symbol $\tilde{a}[k-k_d]$ is given by [3]

$$\lambda\left(\tilde{a}[k-k_d], \tilde{\mathbf{s}}_r[k-k_d]\right) = \left| u[k] - \sum_{\kappa=0}^{q_d} d[\kappa]\,\tilde{a}[k-k_d-\kappa] - \right.$$
$$\left. \sum_{\kappa=0}^{q_b} b[\kappa]\,\hat{a}[k-k_d-(q_d+1)-\kappa, \tilde{\mathbf{s}}_r[k-k_d]] \right|^2. \qquad (3)$$

Here, $u[k]$ denotes the prefiltered received signal, $u[k] = f[k] * r[k]$, and $d[k]$ refers to the target impulse response of prefiltering of order q_d which is used for trellis definition. $b[k]$ is the causal impulse response of the feedback filter of order q_b employed in metric calculations by which postcursor taps with delays higher than q_d can be taken into account properly, exploiting the contents $\hat{a}[k-k_d-\kappa, \tilde{\mathbf{s}}_r[k-k_d]]$ of the registers of the survivor paths of states $\tilde{\mathbf{s}}_r[k-k_d]$ via state–dependent decision feedback. Here it is assumed that q_b is sufficiently high to cancel the postcursor ISI completely. The decision delay k_d is necessary because $f[k]$ is a causal FIR filter.

Commonly, $f[k]$ is chosen as the impulse response of the feedforward filter of a zero–forcing (ZF) or minimum mean–squared error (MMSE) DFE, and $d[k]$ and $b[k]$ are selected as the leading and backmost portion of the causal part of the prefiltered CIR, respectively. However, in general this seems to be not the optimum choice.

As a novel criterion for filter optimization, we consider the signal–to–interference–plus–noise ratio (SINR) seen by the DDFSE algorithm,

$$\text{SINR} = \frac{\sigma_a^2 \sum_{\kappa=0}^{q_d} |d[\kappa]|^2}{\mathscr{E}\{|e[k]|^2\}}, \qquad (4)$$

cf. also [7], where the error signal $e[k]$ is defined as

$$e[k] = \sum_{\kappa=0}^{q_f} f[\kappa]\,r[k-\kappa] - \sum_{\kappa=0}^{q_d} d[\kappa]\,a[k-k_d-\kappa] -$$
$$\sum_{\kappa=0}^{q_b} b[\kappa]\,a[k-k_d-(q_d+1)-\kappa], \qquad (5)$$

representing the difference of the signal after feedforward and feedback filtering from the desired signal $d[k] * a[k-k_d]$ of trellis–based equalization. For (5), perfect feedback in branch metric computations has been assumed corresponding to error–free symbols in the survivor path registers, as usual in the design of systems with decision feedback.

In the proposed criterion for filter optimization, the effect of a possible correlation of $e[k]$ on the performance of trellis–based equalization with Euclidean metric has been not taken into account. However, it can be assumed that the performance degradation due to correlation is only slight, at least for the most interesting case of low q_d, because an excessive noise correlation can be avoided by allowing a feedback filter in addition to the feedforward filter, similar to a DFE.

3. BIAS PROBLEM OF MMSE SOLUTION

In [7], the SINR according to (4) has been maximized, assuming a two–sided infinite–length feedforward filter and setting $k_d = 0$.[2] The optimum transfer function $F(z)$ is given by [7]

$$F(z) = F_{\text{LE}}(z)\,(D(z) + z^{-k_0}\,B(z)), \qquad (6)$$

where $D(z) = \sum_{k=0}^{q_d} d[k]z^{-k}$, $B(z) = \sum_{k=0}^{q_b} b[k]z^{-k}$, $k_0 = q_d + 1$, and $F_{\text{LE}}(z)$ is the transfer function of the optimum infinite–length MMSE linear equalizer [8],

$$F_{\text{LE}}(z) = \frac{H^*(1/z^*)}{H(z)\,H^*(1/z^*) + \zeta}, \qquad (7)$$

with $H(z) = \sum_{k=0}^{q_h} h[k]z^{-k}$ and $\zeta = \sigma_n^2/\sigma_a^2$.

The optimum feedback filter transfer function is obtained as [7]

$$B(z) = -\sum_{\kappa=0}^{k_0-1} d[\kappa]\,P_{k_0-\kappa}(z), \qquad (8)$$

where

$$P_\mu(z) = \frac{\Phi_{\mu,\min}(z)}{\Phi_{\min}(z)}, \quad \mu \in \{1, 2, \dots, k_0\} \qquad (9)$$

can be identified as the optimum causal infinite impulse response (IIR) transfer function of a μ–step forward predictor for the error signal of MMSE linear equalization $e_{\text{LE}}[k]$ [7]. Here, the z–transform $\Phi_{\min}(z)$ corresponds to a causal, stable

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[1]The resulting filters are also directly applicable to RSSE.

and minimum–phase sequence $\varphi_{\min}[k]$ and results from spectral factorization of the z–transform of the autocorrelation sequence of the error signal of MMSE linear equalization,

$$\Phi_{e_{\text{LE}}e_{\text{LE}}}(z) = \frac{\sigma_n^2}{H(z)H^*(1/z^*) + \zeta}, \tag{10}$$

i.e., $\Phi_{\min}(z)$ satisfies

$$\Phi_{\min}(z)\Phi_{\min}^*(1/z^*) = \Phi_{e_{\text{LE}}e_{\text{LE}}}(z). \tag{11}$$

Furthermore, the definition

$$\Phi_{\mu,\min}(z) = \sum_{\nu=\mu}^{+\infty} \varphi_{\min}[\nu]z^{-\nu}, \quad \mu \in \{1, 2, \ldots, k_0\} \tag{12}$$

has been used.

Finally, the desired response $d[k]$ can be obtained via an eigenvalue problem [7] which completes the solution. It turns out that the error signal $e[k]$ of the optimum solution is a moving average (MA) process of order q_d. Hence, it is white only for the case of $q_d = 0$ which is equivalent to MMSE–DFE.

In the following, we show that this solution suffers from a bias problem which is detrimental to reduced–state equalization. To this end, we consider the overall transfer function of prefilter and channel, $H_o(z) = F(z)H(z)$,

$$H_o(z) = \frac{H(z)H^*(1/z^*)}{H(z)H^*(1/z^*) + \zeta}(D(z) + z^{-k_0}B(z)). \tag{13}$$

Using the identity

$$\frac{H(z)H^*(1/z^*)}{H(z)H^*(1/z^*) + \zeta} = 1 - \frac{1}{\sigma_a^2}\Phi_{\min}(z)\Phi_{\min}^*(1/z^*), \tag{14}$$

it is clear that the overall transfer function can be expressed as

$$H_o(z) = (D(z) + z^{-k_0}B(z)) + H_e(z), \tag{15}$$

with

$$H_e(z) = -\frac{1}{\sigma_a^2}\Phi_{\min}(z)\Phi_{\min}^*(1/z^*)(D(z) + z^{-k_0}B(z)). \tag{16}$$

The first term of the right hand side of (15) represents the transfer function to which the metrics of reduced–state equalization including feedback filtering are adjusted; $H_e(z)$ represents the deviation of the overall transfer function from this desired overall transfer function which is investigated in the following in detail for the special case of $q_d = 1$. Here, we can rewrite $H_e(z)$ as

$$H_e(z) = -\frac{1}{\sigma_a^2}\Phi_{\min}(z)\Phi_{\min}^*(1/z^*)\times$$

$$(d[0](1 - z^{-2}P_2(z)) + d[1]z^{-1}(1 - z^{-1}P_1(z))). \tag{17}$$

Using the fact that the prediction–error transfer function of the 2–step ahead predictor $P_2(z)$ is given by [7]

$$(1 - z^{-2}P_2(z)) = \frac{\varphi_{\min}[0] + \varphi_{\min}[1]z^{-1}}{\Phi_{\min}(z)}, \tag{18}$$

and that the prediction–error transfer function of the 1–step ahead predictor $P_1(z)$ is the classical whitening filter

$$(1 - z^{-1}P_1(z)) = \frac{\varphi_{\min}[0]}{\Phi_{\min}(z)}, \tag{19}$$

we can further simplify $H_e(z)$ as

$$H_e(z) = -\frac{1}{\sigma_a^2}\Phi_{\min}^*(1/z^*)(d[0](\varphi_{\min}[0] + \varphi_{\min}[1]z^{-1})$$
$$+ d[1]z^{-1}\varphi_{\min}[0]). \tag{20}$$

Because $\Phi_{\min}^*(1/z^*)$ is an anticausal transfer function,

$$\Phi_{\min}^*(1/z^*) = \sum_{k=0}^{+\infty} \varphi_{\min}^*[k]z^k, \tag{21}$$

it is immediately clear that $h_e[k] = 0$ for $k > 1$, i.e., the feedback filter cancels all interference within its time span and there is no residual postcursor interference after feedback filtering. However, (20) shows that there is residual anticausal intersymbol interference typical for MMSE filtering and also residual intersymbol interference within the time span of filter $D(z)$ which is not taken into account in the metrics of trellis–based equalization, causing a bias. In particular,

$$h_e[0] = -\frac{1}{\sigma_a^2}(d[0](|\varphi_{\min}[0]|^2 + |\varphi_{\min}[1]|^2) +$$
$$d[1]\varphi_{\min}[0]\varphi_{\min}^*[1]), \tag{22}$$

$$h_e[1] = -\frac{1}{\sigma_a^2}(d[0]\varphi_{\min}^*[0]\varphi_{\min}[1] + d[1]|\varphi_{\min}[0]|^2). \tag{23}$$

Hence, while $d[k]$ is assumed as the overall impulse response for metrics calculation in trellis–based equalization, the true overall channel coefficients within the time span of $d[k]$ take on different values. As a consequence, the assumed useful signal of trellis–based equalization and the error signal for time k are no longer statistically independent, causing a performance degradation.

Similar considerations pointing out a degraded probability of error due to mismatch, in particular bias, in the assumed target response hold for any q_d. For instance for the case of an MMSE-DFE ($q_d = 0$),

$$h_e[0] = -\frac{1}{\sigma_a^2}d[0]|\varphi_{\min}[0]|^2, \quad h_e[k] = 0, \, k > 0. \tag{24}$$

Here it is well known that an unbiased solution can be obtained by a simple scaling of the filters, decreasing the SINR by one [9]. Because useful signal and error signal are statistically independent for the unbiased MMSE–DFE solution, the error rate performance improves compared to the biased solution although the SINR decreases, cf. [9]. Also for $q_d > 1$, a bias always leads to a performance degradation of equalization because the error signal for time k contains parts depending on the assumed useful signal for time k.

Inspecting (22) and (23) closer, a simple relationship between the SINRs of biased and unbiased solution cannot be deduced in a straightforward way from these equations for $q_d = 1$ and also from similar equations for $q_d > 1$. Thus, for

$q_d \neq 0$, the construction of an unbiased solution and the relationship between the SINRs of both solutions requires more detailed investigations.

As was to be expected, the bias will vanish for $\sigma_n^2 \to 0$ because $\Phi_{\min}(z) \to 0$ in this case.

In the following, we introduce an SINR criterion which is unbiased per definition and perform a corresponding filter optimization. In Section 5, both solutions are related to each other.

4. OPTIMUM UNBIASED SOLUTION

For calculation of an optimum unbiased FIR solution, we again consider the overall forward impulse response $h_o[k] = f[k] * h[k]$. In order to exclude a bias a priori, the desired impulse response is required to be equal to the overall forward impulse response within its time span, and the feedback filter is required to cancel the postcursor ISI completely,

$$d[k] = h_o[k], \quad k \in \{k_d, k_d+1, \dots, k_d+q_d\}, \quad (25)$$
$$b[k] = h_o[k], \quad k \in \{k_d+q_d+1, \dots, q_h+q_f\}. \quad (26)$$

Hence, for a given feedforward filter, the desired response and the feedback filter are completely specified in the unbiased solution. Thus, in the following only the determination of $f[k]$ is addressed. Considering (4), (5), (25), and (26), the SINR of the unbiased solution can be specified as

$$\text{SINR} = \frac{\sigma_a^2 \sum_{k=k_d}^{k_d+q_d} |h_o[k]|^2}{\sigma_a^2 \sum_{k=0}^{k_d-1} |h_o[k]|^2 + \sigma_n^2 \sum_{k=0}^{q_f} |f[k]|^2}. \quad (27)$$

Using $h_o[k] = \sum_{\kappa=0}^{q_f} f[\kappa] h[k-\kappa]$ or equivalently $h_o[k] = \mathbf{f}^H \mathbf{h}[k]$ with $\mathbf{f} = [f^*[0] f^*[1] \dots f^*[q_f]]^T$, $\mathbf{h}[k] = [h[k] h[k-1] \dots h[k-q_f]]^T$, (27) can be rewritten as

$$\text{SINR} = \frac{\mathbf{f}^H \mathbf{A} \mathbf{f}}{\mathbf{f}^H \mathbf{B} \mathbf{f}}, \quad (28)$$

with the matrices

$$\mathbf{A} = \sum_{k=k_d}^{k_d+q_d} \mathbf{h}[k] \mathbf{h}^H[k], \quad (29)$$

$$\mathbf{B} = \sum_{k=0}^{k_d-1} \mathbf{h}[k] \mathbf{h}^H[k] + \zeta \mathbf{I}_{q_f+1}. \quad (30)$$

Thus, maximizing the SINR is equivalent to solving the generalized eigenvalue problem

$$\mathbf{A} \mathbf{f} = \text{SINR}_{\max,\text{unbiased}} \mathbf{B} \mathbf{f}, \quad (31)$$

i.e., determining the maximum eigenvalue of $\mathbf{B}^{-1} \mathbf{A}$ and the corresponding eigenvector. Hence, the complexity of computation of the optimum unbiased FIR solution is mainly governed by a matrix inversion and an eigenvalue decomposition, both for a matrix of size $(q_f+1) \times (q_f+1)$.

It should be noted that the obtained solution is similar in spirit to a solution developed in [10] for joint reduced–state equalization of several users in a multiple–input single–output transmission in the GSM/EDGE system. Here we push the analysis and interpretation of this approach in a different context further.

5. RELATION OF BIASED AND UNBIASED SOLUTION

In the following, a relation between the optimum biased and unbiased solution is established for arbitrary q_d. First, let us assume that we have a certain IIR unbiased solution with

$$\text{SINR}_u = \frac{\sigma_a^2 \sum_{k=0}^{q_d} |h_o[k]|^2}{\sigma_{e,u}^2}. \quad (32)$$

Here, $\sigma_{e,u}^2$ contains noise and precursor ISI contributions, but no ISI terms corresponding to coefficients of the overall impulse response with lags $k \geq 0$. We now modify only the target response from $h_o[k]$ to $d[k]$, $0 \leq k \leq q_d$, and optimize $d[k]$ for the corresponding biased SINR,

$$\text{SINR}_b = \frac{\sigma_a^2 \sum_{k=0}^{q_d} |d[k]|^2}{\sigma_{e,u}^2 + \sigma_a^2 \sum_{k=0}^{q_d} |h_o[k] - d[k]|^2}. \quad (33)$$

Differentiating SINR_b with respect to $d[k]$, $0 \leq k \leq q_d$, and setting all derivatives to zero yields the solution

$$d[k] = \frac{\text{SINR}_b}{\text{SINR}_b - 1} h_o[k], \quad 0 \leq k \leq q_d, \quad (34)$$

where SINR_b is still unknown and can be determined by inserting $d[k]$ into (33), yielding the quadratic equation

$$\text{SINR}_b^2 - (\text{SINR}_u + 2) \text{SINR}_b + 1 + \text{SINR}_u = 0, \quad (35)$$

with the solution

$$\text{SINR}_b = \text{SINR}_u + 1. \quad (36)$$

Hence, the SINR of any unbiased solution can be improved by one, introducing a bias of the coefficients in an optimum way. This implies that, if we choose the feedforward filter of the optimum biased solution and construct an unbiased solution by selecting $h_o[k] = f[k] * h[k]$, $0 \leq k \leq q_d$, as the target response, we can return from this solution to the biased solution by introducing a bias as described above, which increases the SINR by one. Thus, for this unbiased solution $\text{SINR}_u = \text{SINR}_{\max} - 1$ holds, where $\text{SINR}_{\max}$ is the maximum SINR of the biased solution. On the other hand, there cannot be a better unbiased solution with higher SINR, because introducing again a bias would increase the SINR beyond that of the optimum biased solution, which is a contradiction.

Therefore, the optimum feedforward filter of the unbiased solution is identical to that of the optimum biased solution (neglecting an arbitrary scale factor). This also holds for the optimum feedback filters which totally remove the postcursor ISI in both cases. However, according to (34), there is a factor of $\text{SINR}_b/(\text{SINR}_b - 1)$ between the desired responses of both solutions, and the SINRs differ by one. Hence, the well–known results for MMSE–DFE ($q_d = 0$) have been generalized to arbitrary q_d. As a consequence,
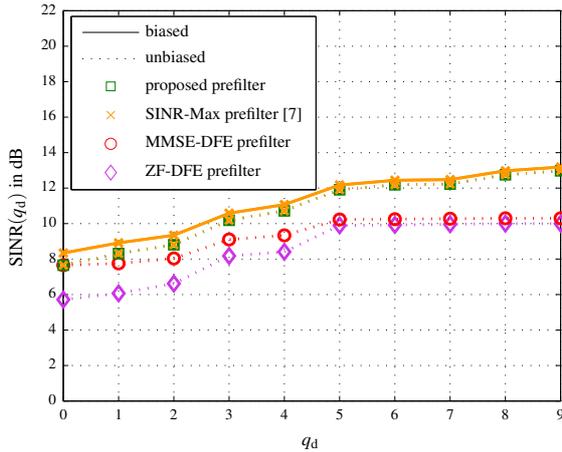
Figure 1: SINR vs. q_d for a snapshot of a random channel ($q_h = 9$) for various prefiltering approaches (SNR $= \sigma_a^2/\sigma_n^2 = 10$ dB).

Figure 2: SINR vs. SNR (SNR $= \sigma_a^2/\sigma_n^2$) for a snapshot of a random channel ($q_h = 9$) for various prefiltering approaches ($q_d = 2$).

the optimum unbiased FIR feedforward filter of (31) tends to the filter (6) for large filter orders.

It should be again emphasized that although the SINR is reduced for the unbiased solution, its error rate performance is improved compared to the biased solution, cf. Section 3.

6. NUMERICAL RESULTS

Fig. 1 shows the SINR according to (27) vs. q_d for different prefiltering approaches for a snapshot of a random channel with $q_h = 9$ and equal tap powers. SNR = 10 dB (SNR $= \sigma_a^2/\sigma_n^2$) is valid. All filter orders have been set sufficiently high to approach closely an IIR solution. Also shown is the SINR of the solution of [7] and the SINR which results when the feedforward filter of [7] is inserted in (27). The latter is equal to the SINR of the proposed unbiased scheme, which was to be expected. Also, it can be verified from Fig. (1), that $\text{SINR}_{\text{max,unbiased}} = \text{SINR}_{\text{max}} - 1$ on a linear scale not only for $q_d = 0$ (MMSE–DFE) but for general q_d.

Fig. 2 shows the SINR vs. SNR for the same scenario as for Fig. 1 for the different schemes and $q_d = 2$. Again, it can be confirmed that the prefilter of [7] is equivalent to that of the derived optimum unbiased scheme and that performance gains compared to the DFE–based prefilters can be obtained.

It should be noted that experiments with different snapshots have led to similar results.

7. CONCLUSIONS

We have introduced a novel prefiltering scheme for reduced–state trellis–based equalization which is better adjusted to the requirements of subsequent equalization than previously proposed schemes. In particular, all filters are optimized according to an appropriate SINR criterion. It is shown that an unconstrained SINR maximization leads to a biased solution. Introducing suitable constraints, the optimization problem can be modified in order to avoid a bias. The relation between both solutions is pointed out.

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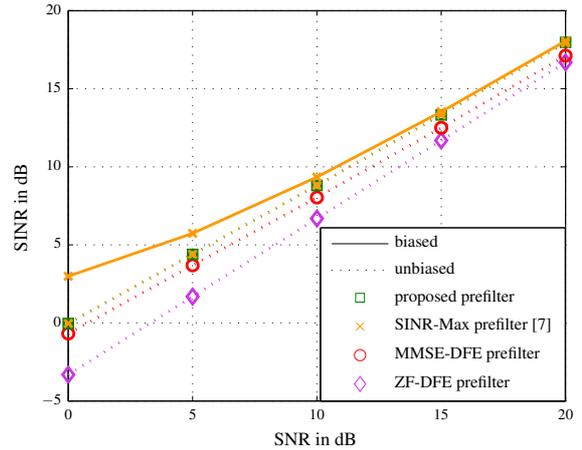[1] G.D. Forney, Jr. Maximum–Likelihood Sequence Estimation of Digital Sequences in the Presence of Inter-symbol Interference. *IEEE Transactions on Information Theory*, 18:363–378, May 1972.

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