Decontaminating pilots in cognitive massive MIMO networks

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***Abstract*—Cognitive radio has been lately suggested as a promising technology in order to improve spectrum utilization. This paper addresses the problem of channel estimation in an underlay interference-prone cognitive radio setup. We consider a primary and a secondary base station, both with multiple antenna capability and serving multiple users. Although previous studies propose the use of beamforming to handle secondary-caused interference, this cannot be done in practice unless channels are correctly estimated in the first place. However channel estimation itself is plagued by interference (pilot contamination effects). Therefore we propose a method to address channel estimation at the primary system while removing contamination caused by the secondary transmitter. The approach is twofold: (i) We develop a robust channel estimator which makes use of covariance information. We show analytically that the performance of this estimator is identical to the interference free scenario under certain when the number of antennas becomes large under a condition on the distribution of the multipath model. (ii) We build a pilot assignment algorithm which seeks to fulfill this condition. Significant gains are reported.**

***Index Terms*—cognitive radio, massive MIMO, channel estimation, scheduling, covariance information.**

I. INTRODUCTION

Spectrum sharing is principally examined in the context of opportunistic spectrum usage and cognitive radio. In overlay cognitive radios, a primary radio system and a secondary one share the spectrum, in a way that the secondary devices can maximize the use of the spectrum under the condition of generating a small acceptable amount of interference to the primary ones [1], [2], [3], [4].

The use of multiple antennas at the secondary (and possibly primary) devices has proved to be very useful in order to allow interference rejection or avoidance mechanisms, and hence fulfilling the condition of limiting the impact caused by the secondary-generated interference to the primary system [5], [6], [7]. It should be noted, however, that MIMO-based approaches (often based on zero-forcing principles or its regularized variants) are efficient in dealing with interference, *provided* the channel vectors corresponding to intended users as well as interfering users can be estimated accurately enough. Interestingly, it was recently shown that the condition of knowing the interfering channel can be alleviated by sticking to simple spatial matched filter based receivers or transmitters. Although highly suboptimal in many interference scenarios, matched filters' (maximum ratio combining MRC) gains approach optimality when the base station antenna numbers are allowed to grow large, giving rise to the so-called *Massive MIMO* concept [8]. This phenomenon simply exploits the fact that as the number of antenna elements increases to infinity, the independent desired and interference channel vectors grow more orthogonal to each other, allowing the MRC to maximize the SNR at the desired user while rejecting (or avoiding on the downlink) interference to others "for free". Large numbers of antennas do not have to be installed on the same base tower. Instead, they can be spread around on a larger area, such as a large building's face or roof area, making Massive MIMO more realistic than initially thought.

Although the concept of large scale antenna systems was initially investigated for cellular networks, there is a clear potential in the area of cognitive radio networks as well where interference is even more problematic. However, this has not been addressed to the best of our knowledge. In particular, we put emphasis on the fact that even MRC filters assume that the channel vectors can be accurately estimated in Massive MIMO systems, as even a slight mismatch in the channel estimation will reduce system performance substantially when one is in the large antenna number regime. As primary and secondary systems cannot be fully coordinated, it is likely that the pilot sequences used in both systems do not satisfy orthogonality. This gives rise to the well known effect of *pilot contamination* (PC) [9], [10], [11], [12], [13]. PC causes the fast saturation of the interference rejection performance, as the number of antennas, M increases.

Recently a novel approach has been presented for decontaminating pilots in the context of Massive MIMO cellular networks [14]. The key idea lies in the exploitation of second order statistics (covariance matrix) for both the desired channel to be estimated as well as the interference channels. We assume such information can be collected and exchanged between base stations beforehand since this is slow-varying data. A robust Bayesian channel estimator is then developed. The surprising result in [14] is that when the number of antennas grows large, such an estimator will exhibit performance identical to that obtained in a zero-interference setting, given a condition on the distribution of multipath for both desired and interfering users.

In this paper, we take on this idea and adapt it to the context of underlay cognitive radios. The main difference between the cellular and underlay cognitive networks is the notion of priority for the cognitive scenario. To deal with this problem we propose a new pilot assignment scheme which is implemented in the secondary operator. This scheme aims at maximizing the channel estimation quality for secondary users, while minimizing the impact created by secondary users onto the primary channel estimation performance. It is shown analytically and by simulation that as the number of antennas grows large, one can have interference free channel estimation at the primary operator while letting the secondary users communicate.

II. Signal and Channel Models

Our model consists of a network of two service providers (SPs), a primary one, SP_P and a secondary one, SP_S, with full spectrum reuse (Fig.1). Estimation of (block-fading) channels in the uplink is considered, and the two base stations, BS_P and BS_S are equipped with M antennas each. We also assume that, both the primary (PU) and secondary (SU) users are equipped with a single antenna each and that K users belong to each SP's area. Moreover, the pilots, of length τ, used by users belonging to the same operator are mutually orthogonal. As a result, we assume that intra-operator interference is negligible. However, non-orthogonal (possibly aligned) pilots are reused among the operators, resulting in pilot contamination from the other SP's users. The pilot sequence used within the k-th operator's area ($k \in \{P, S\}$) is denoted by

$$\mathbf{s}_k = [\begin{array}{cccc} s_{k1} & s_{k2} & \cdots & s_{k\tau} \end{array}]^T \tag{1}$$

. The pilot symbols are normalized such that $|s_{k1}|^2 = \cdots = |s_{k\tau}|^2 = 1$ for every operator index $k = P, S$, where P denotes the primary service provider and S denotes the secondary (or cognitive) service provider.

We denote the receive covariance matrix $\mathbf{R}_i^{(k\to l)} \in \mathbb{C}^{M\times M}$ as $\mathbf{R}_i^{(k\to l)} = \mathbb{E}\left\{\mathbf{h}_i^{(k\to l)}\mathbf{h}_i^{(k\to l)H}\right\}$, where user i is assigned to SP_k and $\mathbf{h}_i^{(k\to l)}$ is the channel vector between this user and the l-th base station, $k, l \in \{P, S\}$. In this paper we assume that only one user per service provider is active at each resource allocation block, thus we momentarily omit the user index subscript, which will be useful later for the pilot assignment algorithm description. Channel vectors are assumed to be $M\times 1$ complex Gaussian, undergoing correlation due to the finite multipath angle spread at the base station side as [15]:

$$\mathbf{h}^{(k\to l)} = \mathbf{R}^{(k\to l)1/2}\mathbf{h}_{\mathrm{W}}{}^{(k\to l)}k, l \in \{P, S\} \tag{2}$$

where $\mathbf{h}_{\mathrm{W}}{}^{(k\to l)}$ is the spatially white $M \times 1$ SIMO channel with $\mathbf{h}_{\mathrm{W}}{}^{(k\to l)} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$, the user is assigned to SP_k, and $\mathbf{I}_M$ is the $M\times M$ identity matrix.

During the pilot phase, the $M \times \tau$ signal received at each base station from the user assigned to SP_k is

$$\mathbf{Y}^{(k)} = \mathbf{h}^{(P\to k)}\mathbf{s}_P^T + \mathbf{h}^{(S\to k)}\mathbf{s}_S^T + \mathbf{N}^{(k)} \tag{3}$$

where $\mathbf{N}^{(k)} \in \mathbb{C}^{M\times\tau}$ is the spatially and temporally white additive Gaussian noise (AWGN) with element-wise variance σ_n^2 [16].

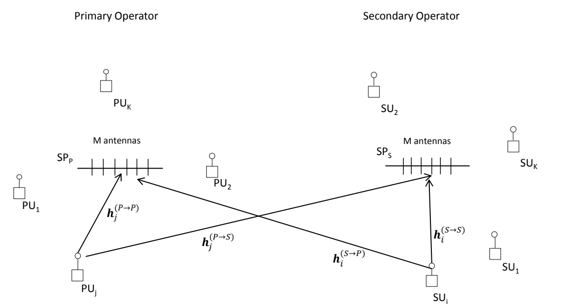

Fig. 1. Topology of a cognitive network comprising of two service providers

III. Covariance-Aided Channel Estimation

In this section we recall some key results obtained in [14] for the conventional cellular scenario. Then, in section IV the results are generalized to adapt to the cognitive radio scenario.

A. Covariance-based Bayesian estimator

By taking advantage of the known long-term covariance information of the channels, a Bayesian estimator, which is equivalent to a MMSE estimator, is given by [14], [17]:

$$\widehat{\mathbf{h}}^{(k\to k)} = \mathbf{R}^{(k\to k)}\bar{\mathbf{S}}^H(\bar{\mathbf{S}}(\mathbf{R}^{(k\to k)} + \mathbf{R}^{(l\to k)})\bar{\mathbf{S}}^H + \sigma_n^2\mathbf{I}_{\tau M})^{-1}\mathbf{y}^{(k)} \tag{4}$$

where $l \neq k$, $\bar{\mathbf{S}}$ is a matrix containing the pilot vectors, the user is assigned to SP_k, and $\mathbf{y}^{(k)}$ is the vectorized received training signal, that is: $\mathbf{y}^{(k)} = \mathrm{vec}(\mathbf{Y}^{(k)})$ [14]. It is noted that the expression (4) is similar to the traditional Bayesian estimator as shown in [17] [18]. The difference is that here identical pilot sequences are sent by users, and covariance information is assumed to be known at the BS side.

Applying the matrix inversion identity $\mathbf{A}(\mathbf{I}+\mathbf{B}\mathbf{A})^{-1} = (\mathbf{I}+\mathbf{A}\mathbf{B})^{-1}\mathbf{A}$, we can obtain a more convenient form:

$$\widehat{\mathbf{h}}^{(k\to k)} = \mathbf{R}^{(k\to k)}\left(\sigma_n^2\mathbf{I}_M + \tau\left(\mathbf{R}^{(k\to k)} + \mathbf{R}^{(l\to k)}\right)\right)^{-1}\bar{\mathbf{S}}^H\mathbf{y}^{(k)}. \tag{5}$$

B. MSE Performance Analysis

We are interested in the mean squared error (MSE) of the proposed Bayesian estimator developed in [14], which in the cognitive network case can be defined as:

$$\mathcal{M}_k \triangleq \mathbb{E}\left\{\left\|\widehat{\mathbf{h}}^{(k\to k)} - \mathbf{h}^{(k\to k)}\right\|_F^2\right\} \tag{6}$$

where the user is assigned to SP_k and $k \in \{P, S\}$. The MSE of the Bayesian estimator (5) with completely aligned pilots is recalled from [14] for convenience.

Proposition 1. The estimation MSE of (5) is given by

$$\mathcal{M}_k = \mathrm{tr}\left\{\mathbf{R}^{(k\to k)} - \mathbf{R}^{(k\to k)2}\left(\frac{\sigma_n^2}{\tau}\mathbf{I}_M + \mathbf{R}^{(k\to k)} + \mathbf{R}^{(l\to k)}\right)^{-1}\right\} \tag{7}$$

where $k \neq l$.

Proof: The proof can be found in [14]. ∎

We can easily derive the MSE of (5) obtained in an interference free scenario, by setting interference covariance matrices to zero in (5):

$$\mathcal{M}_k^{\text{no int}} = \text{tr}\left\{\mathbf{R}^{(k\to k)}(\mathbf{I}_M + \frac{\tau}{\sigma_n^2}\mathbf{R}^{(k\to k)})^{-1}\right\} \quad (8)$$

where superscript *no int* refers to the "no interference case". The corresponding channel estimate in this case is

$$\widehat{\mathbf{h}}^{(k\to k)\text{no int}} = \mathbf{R}^{(k\to k)}(\sigma_n^2\mathbf{I}_M + \tau\mathbf{R}^{(k\to k)})^{-1}\bar{\mathbf{S}}^H(\bar{\mathbf{S}}\mathbf{h}^{(k\to k)} + \mathbf{n}). \quad (9)$$

C. Large antenna number regime

Our objective is to analyze the performance of the above estimators in the large antenna number regime M. For tractability, our analysis is based on the assumption of a uniform linear array with supercritical antenna spacing (i.e. less than or equal to half wavelength).

In this paper we make use of the following multipath model

$$\mathbf{h}^{(k\to k)} = \frac{1}{\sqrt{P}}\sum_{p=1}^{P}\mathbf{a}(\theta_p^{(k\to k)})\alpha_p^{(k\to k)} \quad (10)$$

where P is the arbitrary number of independent spatially separated paths, $\alpha_p^{(k)} \sim \mathcal{CN}(0, \delta^{(k)2})$ is independent over the path index p, where $\delta^{(k)}$ is the user channel's average attenuation. $\mathbf{a}(\theta)$ is the steering vector,as shown in [19]

$$\mathbf{a}(\theta) \triangleq \begin{bmatrix} 1 \\ e^{-j2\pi\frac{D}{\lambda}\cos(\theta)} \\ \vdots \\ e^{-j2\pi\frac{(M-1)D}{\lambda}\cos(\theta)} \end{bmatrix} \quad (11)$$

where D is the antenna spacing at the BS and λ is the signal wavelength, such that $D \leq \lambda/2$. $\theta_p \in [0, \pi]$ is a random AOA. Note that we can limit angles to $[0, \pi]$ because any $\theta \in$ [-$\pi, 0]$ can be replaced by -θ giving the same steering vector.

A principal result of [14] is stated below, where the notations are adapted to our cognitive network model:

Theorem 1. *Assume the multipath angle of arrival θ yielding channel $\mathbf{h}^{(k\to k)}$ given in (10) for a user assigned to SP_k, is distributed according to an arbitrary density $p^{(k\to k)}(\theta)$ with bounded support, i.e. $p^{(k\to k)}(\theta) = 0$ for $\theta \notin [\theta^{\min,(k\to k)}, \theta^{\max,(k\to k)}]$ for some fixed $\theta^{\min,(k\to k)} \leqslant \theta^{\max,(k\to k)} \in [0, \pi]$. If the interfering user's AOA interval $[\theta^{\min,(l\to k)}, \theta^{\max,(l\to k)}]$, $k \neq l \in \{P, S\}$ is strictly non-overlapping with the desired channel's AOA interval $[\theta^{\min,(k\to k)}, \theta^{\max,(k\to k)}]$, we have*

$$\lim_{M\to\infty} \widehat{\mathbf{h}}^{(k\to k)} = \widehat{\mathbf{h}}^{no\ int,(k\to k)} \quad (12)$$

Proof: The proof can be found in [14]. ∎

From the above analysis and especially from Theorem 1, lemmas (1)-(3) and the rest of results given in [14] we conclude to the fact that the MSE performance of the covariance-aided channel estimation, in case this regards a PU, strongly depends on the degree with which the signal subspaces of the covariance matrices $\mathbf{R}^{(P\to P)}$ and $\mathbf{R}^{(S\to P)}$ overlap with each other. As a result, this will lead to an interference avoidance criterion for a primary user, PU, because in case the signal subspace of one of the two aforementioned covariance matrices will be achieved to be the orthogonal complement of the signal subspace of the other covariance matrix - or at least close to that one, then the pilot contamination effect will tend to vanish in the large antenna number, M, regime.

IV. Coordinated Cognitive Radio Pilot Assignment Algorithm

A. Introduction

In this section, we design a suitable coordination protocol for scheduling secondary users (SUs) to the above described two-operator cognitive radio network. We assume that the primary base station (BS_P) unconstrainedly selects a user $i_\star$, where $i_\star \in \{1\cdots K\}$ is assigned to SP_P and the secondary base station (BS_S) initially visits in an exhaustive manner all the secondary users, in order to acquire a subset, $\mathcal{L} \subset \{1\cdots K\}$, the elements of which will be the indices of the SUs which are ϵ-orthogonal to user $i_\star$. We suppose that a SU within subset $\mathcal{L}$ is indexed by $j \in \mathcal{L}$. Then, the secondary BS, BS_S will schedule user terminal $j_\star \in \mathcal{L}$ which will have the best channel estimation MSE performance.

B. Secondary user scheduling algorithm

The scheduling (optimization) problem with respect to the secondary network is the following

$$j_\star = \arg\min_{j\in\mathcal{L}} \frac{\mathcal{M}_S}{\text{tr}\left\{\mathbf{R}_j^{(S\to S)}\right\}} \quad (13)$$

subject to

$$\frac{\left\|\mathbf{R}_j^{(S\to P)}\mathbf{R}_{i_\star}^{(P\to P)}\right\|_2}{\text{tr}\left\{\mathbf{R}_j^{(S\to P)}\right\}\text{tr}\left\{\mathbf{R}_{i_\star}^{(P\to P)}\right\}} < \epsilon \quad (14)$$

where $j \in \mathcal{L} \subset \{1\cdots K\}$ belongs to SP_S and ϵ is a positive system design parameter which illustrates the degree of orthogonality between the signal spaces spanned by the covariance matrices $\mathbf{R}_j^{(S\to P)}$ and $\mathbf{R}_{i_\star}^{(P\to P)}$. In case ϵ is such that BS_S cannot find such a subset $\mathcal{L}$ of users ($|\mathcal{L}| = 0$), it selects no user for the present resource block; in other words, the secondary system becomes silent.
In summary, the above explained coordinated user scheduling algorithm can be described in terms of each scheduling interval by the following steps:

Step1 : BS_P selects a user $i_\star$ for transmission, $i_\star \in \{1\cdots K\}$

Step2 : Having this information, BS_S finds a user subset, $\mathcal{L}$ with $|\mathcal{L}| < K$ such that $j \in \mathcal{L}$ when

$$\frac{\left\|\mathbf{R}_j^{(S\to P)}\mathbf{R}_{i_\star}^{(P\to P)}\right\|_2}{\text{tr}\left\{\mathbf{R}_j^{(S\to P)}\right\}\text{tr}\left\{\mathbf{R}_{i_\star}^{(P\to P)}\right\}} < \epsilon. \quad (15)$$

In case no SU fulfills the above orthogonality criterion, BS_S schedules no user (the PU functions under interference-free conditions), otherwise move to step 3.

Step3 : BS_S schedules user $j_\star \in \mathcal{L}$ where

$$j_\star = \arg\min_{j\in\mathcal{L}} \frac{\mathcal{M}_S(\mathbf{R}_j^{(S\to S)}, \mathbf{R}_{i_\star}^{(P\to S)})}{\mathrm{tr}\left\{\mathbf{R}_j^{(S\to S)}\right\}}. \tag{16}$$

V. Numerical Results

In order to evaluate the performance of the proposed scheme, simulations of a two-SP system have been performed. Some basic simulation parameters are given in Table I. These parameters are being kept in the following simulations unless otherwise stated.

TABLE I
Basic simulation parameters

SP area radius	1 km
SP area edge SNR	20 dB
Number of users per SP	10
Distance from a user terminal to its BS	800 m
Path loss exponent	3
Carrier frequency	2 GHz
Antenna spacing	$\lambda/2$
Number of paths	50
Pilot length	10

In the following simulations the angles of arrival (AOAs) follow an unbounded (Gaussian) distribution. Moreover, all the user terminals have the same distance from their serving BS (800m) and the angles of their positions are uniformly distributed along this circle. The performance metric used to evaluate the proposed coordinated scheduling scheme is the normalized channel estimation mean squared error (MSE). Numerical results which depict the performance of the scheduling scheme proposed with an averaging over 100 independent system topologies (user positions) are shown both for a primary user (PU) and for a secondary user (SU), for an angle spread of 10 degrees and for two different values of the threshold ϵ, 0.03 and 0.08 respectively. In the figures, "CBC" stands for the proposed Covariance-Based (Bayesian) Coordinated estimation algorithm, "CBU" denotes the Covariance-Based Uncoordinated estimation case, "CBIF" denotes the Covariance-Based Interference-Free case , while "LSU" stands for the Least Squares (conventional) Uncoordinated case and finally, "LSIF" corresponds to the Least Squares Interference-Free case.

From Fig.2 and Fig. 3 we observe that the covariance-based (Bayesian) channel estimation methods -both CBU and CBC- outperform the conventional (LS) estimation method for the whole range of M under examination, both for the PU and for the SU. Also importantly, the proposed CBC scheduling algorithm shows a better performance behavior compared to the uncoordinated CBU algorithm. More specifically, as Fig.2 shows, the performance gain of the CBC over the CBU algorithm can reach or even overcome (for smaller values of

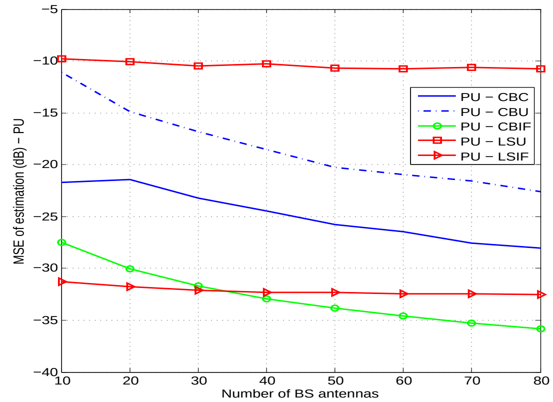

Fig. 2. PU Estimation MSE vs. BS antenna number, 2 SP network, Gaussian distributed AOAs, σ_θ=10 degrees, $\epsilon = 0.03$, τ = 10 symbols/pilot sequence

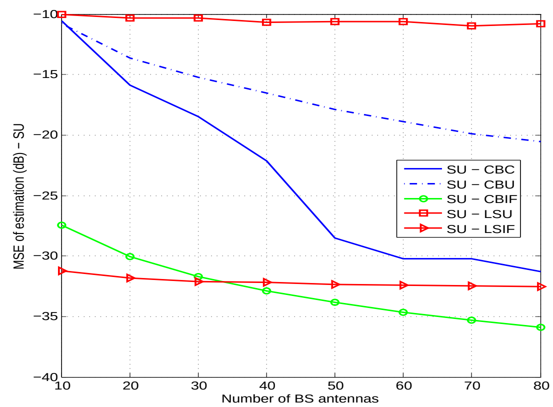

Fig. 3. SU Estimation MSE vs. BS antenna number, 2 SP network, Gaussian distributed AOAs, σ_θ=10 degrees, $\epsilon = 0.03$, τ = 10 symbols/pilot sequence

M) the value of 6dB plus the fact that the MSE performance of the PU slowly approaches the interference-free case for large values of M, which verifies Theorem 1. The above conclusions also hold for the SU case, with the difference being in the slope of the CBC curve. Here, the SU-CBC curve approaches the interference-free performance in a really fast way. This can be explained by the fact that the orthogonality threshold value ϵ remains unchanged for the whole M range, so consequently, as M grows really large and ϵ remains the same, the SU will tend to suffer from very few or even zero outages, thus causing interference to the PU and this is the reason why the slope of the PU-CBC curve is much smaller. Due to this way of implementation, looking at the smaller M regime, the PU behaves satisfactorily and the gap between the PU-CBC and the SU-CBC is significant for small values of M (MSE of about -22dB for the PU and -10dB for the SU when $M = 10$).

In Fig. 4 and Fig. 5 the MSE performance curves of the same algorithms are derived for a larger threshold parameter, ϵ. Here, we can make similar conclusions as the above with a big difference: the PU-CBC performance has now deteriorated (-13dB for $M = 10$) and it is just marginally better than the PU-CBU one. On the contrary, the SU-CBC curve is fastly approaching the interference-free regime and it behaves almost likewise over a value of M ($M = 60$). The explanation of this

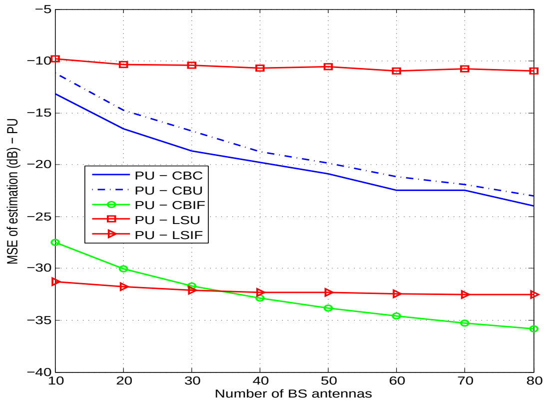

Fig. 4. PU Estimation MSE vs. BS antenna number, 2 SP network, Gaussian distributed AOAs, σ_θ=10 degrees, $\epsilon = 0.08$, τ = 10 symbols/pilot sequence

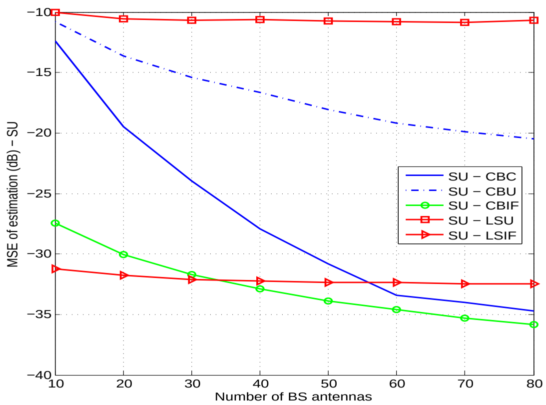

Fig. 5. SU Estimation MSE vs. BS antenna number, 2 SP network, Gaussian distributed AOAs, σ_θ=10 degrees, $\epsilon = 0.08$, τ = 10 symbols/pilot sequence

phenomenon is the same as the one described above: as the threshold value is decreased for an unchanged value of M and the angle spread, the coordinated CB scheduling method works more efficiently for the PU. Hence, an important part of the pilot assignment algorithm's design is deciding on an appropriate orthogonality threshold, which will compromise between guaranteeing the interference avoidance for the PU and simultaneously limiting the outage rate of the secondary system.

VI. Conclusions

This paper proposes a covariance-based pilot assignment algorithm within the channel estimation process itself regarding a cognitive radio network which consists of two service providers (SPs) and their corresponding users. The pilot assignment method is based upon the long-term knowledge of the second order statistics of the user channels within both service operators. In the large BS antenna number, M, regime, it is shown that the channel estimation performance approaches an interference-free scenario and significant performance gains are presented.

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References

[1] R. Etkin, A. Parekh, and D. Tse, "Spectrum sharing for unlicensed bands," *Selected Areas in Communications, IEEE Journal on*, vol. 25, no. 3, pp. 517 –528, april 2007.
[2] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *Selected Areas in Communications, IEEE Journal on*, vol. 23, no. 2, pp. 201 – 220, feb. 2005.
[3] I. Akyildiz, W.-Y. Lee, M. Vuran, and S. Mohanty, "A survey on spectrum management in cognitive radio networks," *Communications Magazine, IEEE*, vol. 46, no. 4, pp. 40 –48, april 2008.
[4] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proceedings of the IEEE*, vol. 97, no. 5, pp. 894 –914, may 2009.
[5] R. Zhang and Y.-C. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *Selected Topics in Signal Processing, IEEE Journal of*, vol. 2, no. 1, pp. 88 –102, feb. 2008.
[6] G. Scutari and D. Palomar, "Mimo cognitive radio: A game theoretical approach," *Signal Processing, IEEE Transactions on*, vol. 58, no. 2, pp. 761 –780, feb. 2010.
[7] K. Hamdi, W. Zhang, and K. Ben Letaief, "Joint beamforming and scheduling in cognitive radio networks," in *Global Telecommunications Conference, 2007. GLOBECOM '07. IEEE*, nov. 2007, pp. 2977 –2981.
[8] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun*, vol. 9, no. 11, pp. 3590–3600, Nov. 2010.
[9] J. Jose, A. Ashikhmin, T. L. Marzetta, and S. Vishwanath, "Pilot contamination problem in multi-cell TDD systems," in *Proc. IEEE International Symposium on Information Theory (ISITo09)*, Seoul, Korea, Jun. 2009, pp. 2184–2188.
[10] J. Hoydis, S. ten Brink, and M. Debbah, "Massive MIMO: How many antennas do we need?" in *Proc. 2011 49th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, Sep. 2011, pp. 545 –550.
[11] J. Jose, A. Ashikhmin, T. L. Marzetta, and S. Vishwanath, "Pilot contamination and precoding in multi-cell tdd systems," *IEEE Trans. Wireless Commun.*, vol. 10, no. 8, pp. 2640–2651, Aug. 2011.
[12] H. Q. Ngo, T. L. Marzetta, and E. G. Larsson, "Analysis of the pilot contamination effect in very large multicell multiuser mimo systems for physical channel models," in *Acoustics, Speech and Signal Processing (ICASSP), 2011 IEEE International Conference on*, may 2011, pp. 3464 –3467.
[13] B. Gopalakrishnan and N. Jindal, "An analysis of pilot contamination on multi-user mimo cellular systems with many antennas," in *Signal Processing Advances in Wireless Communications (SPAWC), 2011 IEEE 12th International Workshop on*, june 2011, pp. 381 –385.
[14] H. Yin, D. Gesbert, M. Filippou, and Y. Liu, "A coordinated approach to channel estimation in large-scale multiple-antenna systems," *IEEE J. Sel. Areas Commun.*, Feb. 2012, submitted. [Online]. Available: http://arxiv.org/pdf/1203.5924
[15] A. J. Paulraj, R. Nabar, and D. Gore, *Introduction to space-time wireless communications*. Cambridge University Press, 2003.
[16] A. Scherb and K. Kammeyer, "Bayesian channel estimation for doubly correlated MIMO systems," in *Proc. IEEE Workshop Smart Antennas*, 2007.
[17] E. Bjornson and B. Ottersten, "A framework for training-based estimation in arbitrarily correlated Rician MIMO channels with Rician disturbance," *IEEE Trans Signal Process.*, vol. 58, no. 3, pp. 1807–1820, 2010.
[18] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Englewood Cliffs, NJ: Prentice Hall, 1993.
[19] J.-A. Tsai, R. Buehrer, and B. Woerner, "The impact of aoa energy distribution on the spatial fading correlation of linear antenna array," in *Vehicular Technology Conference, 2002. VTC Spring 2002. IEEE 55th*, vol. 2, 2002, pp. 933 – 937 vol.2.