

On the Capacity of the Two-user Gaussian Causal Cognitive Interference Channel

Martina Cardone, Daniela Tuninetti, Raymond Knopp and Umer Salim

Abstract—This paper considers the two-user Gaussian Causal Cognitive Interference Channel (GCCIC), which consists of two source-destination pairs that share the same channel and where one full-duplex cognitive source can causally learn the message of the primary source through a noisy link. The GCCIC is an interference channel with unilateral source cooperation that better models practical cognitive radio networks than the commonly used model which assumes that one source has perfect non-causal knowledge of the other source’s message. First the sum-capacity of the symmetric GCCIC is determined to within a constant gap. Then, the insights gained from the study of the symmetric GCCIC are extended to more general cases. In particular, the whole capacity region of the Gaussian Z-channel, i.e., when there is no interference from the primary user, and of the Gaussian S-channel, i.e., when there is no interference from the secondary user, are both characterized to within 2 bits. The fully connected general, i.e., no-symmetric, GCCIC is also considered and its capacity region is characterized to within 2 bits when, roughly speaking, the interference is not weak at both receivers. The parameter regimes where the GCCIC is equivalent, in terms of generalized degrees-of-freedom, to the noncooperative interference channel (i.e., unilateral causal cooperation is not useful), to the non-causal cognitive interference channel (i.e., causal cooperation attains the ultimate limit of cognitive radio technology), and to bilateral source cooperation are identified. These comparisons shed lights into the parameter regimes and network topologies that in practice might provide an unbounded throughput gain compared to currently available (non cognitive) technologies.

Index Terms—Binning, causal cooperation, cognitive radio, constant gap, cooperative communication, dirty paper coding, generalized degrees of freedom, interference channel, superposition coding, Z-channel.

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I. INTRODUCTION

This work considers the cognitive radio overlay paradigm [3] that consists of two source-destination pairs sharing the same channel in which the pair with cognitive abilities attains its communication goals while helping the other (non cognitive) pair. The sources are indicated as PTx and CTx, and the destinations as PRx and CRx. PTx and PRx are referred to as the *primary* pair, while CTx and CRx as the *cognitive* pair. The prime features of overlay cognitive radio are to firstly allow the cognitive nodes to communicate without hindering the communication of the primary nodes, and secondly to enhance the communication reliability of the primary nodes. To this end, the CTx is assumed to operate in a full-duplex mode on the same channel as the PTx. Due to the broadcast property of the wireless media, the CTx overhears the PTx through a lossy communication link. Contrary to the commonly studied cognitive radio model that assumes perfect non-causal primary message knowledge available at the CTx [4], in this work we treat the causal case, that is, the CTx has access only to primary information it receives over the air. We refer to this system as the Causal Cognitive Interference Channel (CCIC).

From an application standpoint, the CCIC fits future 4G networks with heterogeneous deployments [5] where the CTx corresponds to the so-called *small-cell* base-station, or eNB. In this scenario, the CTx would listen to the PTx transmission but not make use of a dedicated point-to-point backhaul link (i.e., on either another channel or through a wired link). We consider deployment scenarios where the CTx→CRx link is on the same carrier frequency as PTx→PRx link and the CTx operates in a full-duplex mode. This implies that the CTx can listen to the PTx’s transmission while transmitting. Full-duplex communication is possible thanks to sophisticated self-interference cancellation techniques at the CTx [6]. Moreover, we assume that the PRx and CRx can implement sophisticated interference-mitigation techniques which exploit knowledge of the codebooks used at both PTx and CTx. These codebooks are conceived for the interference scenario (e.g. superposition-coding [7] or Dirty Paper Coding (DPC) [8]). It should be noted that, since 4G air-interfaces already specify up to 8-level superposition coding for point-to-point MIMO or point-to-multipoint MIMO transmission [5], it is feasible to assume that extensions for distributed superposition coding could also be envisaged.

Different interference scenarios are considered and can correspond to the choice of appropriate deployment configurations in cognitive radio networks. The first class is the fully

connected CCIC where both destinations suffer from interference, i.e., in this case both destinations are in the coverage area of both sources. The second class is the interference-asymmetric Gaussian CCIC where either the link $PT_x \rightarrow CR_x$ is non-existent (referred to as the Z-channel) or the link $CT_x \rightarrow PR_x$ is non-existent (referred to as the S-channel). In the noncooperative IC these two asymmetric scenarios are the same, up to a relabeling of the nodes. In the CCIC case, due to the asymmetry in the cooperation, the two scenarios are different and must be treated separately. The Z-channel models a situation such as an indoor $CT_x \rightarrow CR_x$ with another receiver (PR_x) connected to an outdoor base station (PT_x) in the vicinity of CT_x . The S-channel models the case where PR_x is out-of-range of CT_x and the base station (PT_x) schedules traffic to both PR_x and CT_x/CR_x concurrently. Both scenarios are relevant for practical cognitive radio deployments and their ultimate performance is investigated in this work.

A. Related past work

The presence of a lossy communication link between PT_x and CT_x enables CT_x to cooperate with PT_x . CT_x , in fact, through this noisy channel overhears the signal sent by the PT_x and gathers information about PT_x 's message, which serves as the basis for unilateral cooperation between the two sources. Unilateral source cooperation is a special case of the *IC with generalized feedback*, or bilateral source cooperation [9], [10], [11], [12], [13].

1) *IC with bilateral source cooperation*: Bilateral source cooperation has been actively investigated recently. Host-Madsen [9] first studied outer and inner bounds for the Gaussian IC with either source or destination bilateral cooperation. For outer bounds, the author in [9] evaluated the different cut-set upper bounds and then tightened the sum-rate upper bound by extending the sum-rate outer bounds originally developed by Kramer [14] for the Gaussian noncooperative IC in weak and strong interference to the cooperative case. Tuninetti [12] derived a general outer bound for the IC with bilateral source cooperation by extending Kramer's Gaussian noise sum-rate upper bounds in [14, Theorem 1] to any memoryless IC with source cooperation, and more recently to any form of source and destination cooperation [15]. Prabhakaran and Viswanath [11] extended the idea of [16, Theorem 1] to derive a sum-rate outer bound for a class of injective semi-deterministic IC with bilateral source cooperation in the spirit of the work by Telatar and Tse [17], and evaluated it for the Gaussian channel with independent noises (this assumption is not without loss of generality when cooperation and feedback are involved). Tandon and Ulukus [13] developed an outer bound for the IC with bilateral source cooperation based on the dependence-balance idea of Hekstra and Willems [18] and proposed a novel method to evaluate it for the Gaussian channel with independent noises.

The largest known achievable region for general bilateral source cooperation, to the best of our knowledge, is the one presented in [10, Section V]. In [10, Section V] each source splits its message into two parts, i.e., a *common* and a *private* message, as in the Han-Kobayashi's scheme for the

noncooperative IC [7]; these two messages are further subdivided into a *noncooperative* and a *cooperative* part. The noncooperative messages are transmitted as in the noncooperative IC [7], while the cooperative messages are delivered to the destinations by exploiting the cooperation among the two sources. In [10, Section V] each source, e.g. source 1, after learning the cooperative messages of source 2, sends the common cooperative message of source 2 and uses Gelfand-Pinsker's binning [19], or Dirty Paper Coding (DPC) [8] in the Gaussian noise case, against the private cooperative message of source 2 in an attempt to rid its own receiver of this interference. The achievable scheme in [10, Section V] only uses partial-decode-and-forward for cooperation. A possibly larger achievable region could be obtained by also including compress-and-forward as cooperation mechanism in the spirit of [20] for the relay channel.

For the two-user Gaussian noise IC with bilateral source cooperation, *under the assumption that the cooperation links have same strength*, the scheme of [10, Section V] was sufficient to match the sum-capacity upper bounds of [12], [11] to within a constant gap [11], [21]. [11] characterized the sum-capacity to within $20/2$ bits (in this work we consider the gap per user) of the IC with bilateral source cooperation under the condition that the cooperation links have the same strength, but otherwise arbitrary direct and interfering links. The gap was reduced to 2 bits in the 'strong cooperation regime' in [21] with symmetric direct links, symmetric interfering links and symmetric cooperation links. In this work we seek extensions of these results to the case where the cooperation links have different strengths. In particular, motivated by the cognitive radio technology, we focus on the case of unilateral source cooperation where one of the cooperation links is absent. Moreover, we seek to determine the whole capacity region to within a constant gap, not simply the sum-capacity. To the best of our knowledge, the whole capacity region with source cooperation has never been characterized to within a constant gap in the literature. Moreover, the case of asymmetric cooperation links, of which unilateral cooperation is a special case, to the best of our knowledge, has not been considered in the literature. The major contribution of this work is to approximately determine the capacity for the Gaussian Z- and S-channels and for the fully connected GCCIC in some parameters regimes which, roughly speaking, exclude the case of weak interference at both receivers.

2) *IC with unilateral source cooperation*: Unilateral source cooperation is clearly a special case of the general bilateral cooperation case where the cooperation capabilities of the two sources are not restricted to be the same. This case has been specifically considered in [22] where the cooperating transmitter works either in full-duplex or in half-duplex mode. For full-duplex unilateral cooperation, the authors of [22] evaluated the performance of two achievable schemes: one that exploits partial-decode-and-forward and binning and a second one that extends the first by adding rate splitting. It was observed, through numerical evaluations, that the proposed inner bounds are not too far from the outer bound of [13] for certain Gaussian noise channels. In this work we formally prove that the outer bound region obtained from [9], [11], [12]

is achievable to within a constant gap, for the two interference-asymmetric network scenarios considered, i.e., the Gaussian Z- and S-channels, and for the fully-connected GCCIC in some parameters regimes. Moreover, we use as unifying framework the achievable scheme of [10, Section V], of which the schemes of [22] are special cases.

An extension of the IC with unilateral source cooperation was studied in [23], where it was assumed that at any given time instant the cognitive source has a non-causal access to $L \geq 0$ future channel outputs. The case $L = 0$ corresponds to the strictly causal case considered in this paper, while the case $L \rightarrow \infty$ to the limiting non-causal cognitive IC [4]. The authors of [23] derived potentially tighter outer bounds for the CCIC channel (i.e., case $L = 0$) than those of [11], [12] specialized to unilateral source cooperation; unfortunately it is not clear how to evaluate these bounds in Gaussian noise because they are expressed as a function of auxiliary random variables jointly distributed with the inputs and for which no cardinality bounds on the corresponding alphabets are known. The achievable region in [23, Corollary 1] is also no smaller than the region in [10, Section V] specialized to the case of unilateral source cooperation (see [23, Remark 2, point 6]). Although [23, Corollary 1] is, to the best of our knowledge, the largest known achievable region for the general memoryless CCIC with unilateral cooperation, its evaluation in general is quite involved as the rate region is specified by 9 jointly distributed auxiliary random variables and by 30 rate constraints. In [23] inner bounds were compared numerically to the 2×2 MIMO outer bound for the GCCIC; the 2×2 MIMO outer bound is loose in general compared to the bounds in [9], [11], [12]. Although it was noted in [23] that, for the simulated set of channel gains, the proposed bounds are not far away from one another, a performance guarantee in terms of (sum-)capacity to within a constant gap was not given. In this work we characterize the capacity to within a constant gap for several channel configurations, i.e., for the Gaussian Z- and S-channels and for the fully-connected GCCIC in some parameters regimes, which, roughly speaking, exclude the case of weak interference at both receivers.

3) *Non-causal cognitive radio channel*: The cognitive radio channel is commonly modeled following the pioneering work of Devroye *et al* [4] in which the superior capabilities of the cognitive source are modeled as perfect non-causal knowledge of PTx's message at CTx. For this non-causal model the capacity region in Gaussian noise is known exactly for some parameter regimes and to within 1 bit otherwise [24]. In this work we remove the ideal non-causal message knowledge assumption by considering a more realistic scenario where CTx causally learns the PTx's message through a noisy link. The study of the causal model stems from the question of whether cognitive radio can offer a substantial rate gain over the noncooperative IC. Since the answer was in the positive for the non-causal model [24], the next question is whether such gains can be attained in practical channels where message knowledge must be obtained through a noisy channel. This work answers this question in the positive. In particular, we identify the set of the channel parameters sufficient to attain, to within a constant gap, the ultimate performance limits of

cognitive radio as predicted by the non-causal model [24].

B. Contributions and paper organization

The rest of the paper is organized as follows. Section II describes the channel model, defines the concept of capacity to within a constant gap and of generalized degrees of freedom (gDoF), and summarizes known inner and outer bounds. Section III characterizes the capacity region of the symmetric GCCIC to within 1 bit for every set of the channel parameters, with the exclusion of a subset of the weak interference regime, for which the sum-capacity to within 3.16 bits is provided (see Theorem 1). Section IV considers the general GCCIC and characterizes its capacity region to within 2 bits when, roughly speaking, the interference is not weak at both receivers (see Theorem 2). In order to better understand the weak interference regime, we analyze the 'interference asymmetric' GCCIC in which one of the interfering links is absent which models different network topologies; we determine the capacity region to within 2 bits for the Z-channel in Section V (see Theorem 3), and to within 2 bits for the S-channel in Section VI (see Theorem 4). Section VII concludes the paper. Most of the proofs are reported in the Appendix. In particular, the Appendix contains the details of the relatively simple proposed achievable schemes, which can be used to provide design insights into practical schemes for future cognitive networks. For all system models considered, we compare the gDoF attained with causal unilateral cooperation with that of other known forms of cooperation to quantify when causal cognitive radio might be worth implementing in practice.

II. SYSTEM MODEL AND BACKGROUND

Throughout the paper we adopt the notation convention of [25]. In particular, $[n_1 : n_2]$ denotes the set of integers from n_1 to $n_2 \geq n_1$; $[x]^+ := \max\{0, x\}$ for $x \in \mathbb{R}$; $\log^+(x) := \max\{0, \log(x)\}$ for $x \in \mathbb{R}_+$; Y^j is a vector of length j with components (Y_1, \dots, Y_j) . The subscript c (in sans serif font) is used for quantities related to the cognitive pair, while the subscript p (in sans serif font) for those related to the primary pair. The subscript f (in sans serif font) is used to refer to generalized feedback information received at CTx. The subscript c (in roman font) is used to denote common messages, while the subscript p (in roman font) to denote private messages. The notation $\text{eq}(n)$ is used to denote the rightmost side of the equation number n .

A. The Gaussian noise channel

A single-antenna full-duplex GCCIC, shown in Fig. 1, is described by the input/output relationship

$$\begin{bmatrix} Y_f \\ Y_p \\ Y_c \end{bmatrix} = \begin{bmatrix} \sqrt{C} & \star \\ \sqrt{S_p} & \sqrt{I_c} e^{j\theta_c} \\ \sqrt{I_p} e^{j\theta_p} & \sqrt{S_c} \end{bmatrix} \begin{bmatrix} X_p \\ X_c \end{bmatrix} + \begin{bmatrix} Z_f \\ Z_p \\ Z_c \end{bmatrix} \quad (1)$$

where \star indicates the channel gain that does not affect the capacity region (because CTx can remove its transmit signal X_c from its channel output Y_f). The channel gains are constant, and therefore known to all nodes. Without loss of generality

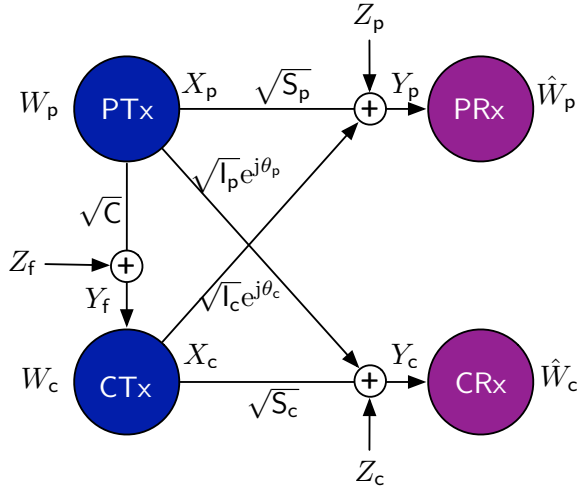


Fig. 1: The two-user Gaussian Causal Cognitive Interference Channel (GCCIC).

certain channel gains can be taken to be real-valued and non-negative because a node can compensate for the phase of one of its channel gains. The channel inputs are subject to a unitary average power constraint without loss of generality, i.e., $\mathbb{E}[|X_i|^2] \leq 1, i \in \{p, c\}$. The noises are independent circularly symmetric Gaussian random variables with, without loss of generality, zero mean and unit variance.

PTx has a message $W_p \in [1 : 2^{NR_p}]$ for PRx and CTx has a message $W_c \in [1 : 2^{NR_c}]$ for CRx, where $N \in \mathbb{N}$ denotes the codeword length and $R_p \in \mathbb{R}_+$ and $R_c \in \mathbb{R}_+$ the transmission rates for PTx and CTx, respectively. The messages W_p and W_c are independent and uniformly distributed on their respective domains. At time $i, i \in [1 : N]$, PTx maps its message W_p into a channel input symbol $X_{p,i}(W_p)$ and CTx maps its message W_c and its past channel observations into a channel input symbol $X_{c,i}(W_c, Y_f^{i-1})$. At time N , PRx makes an estimate of its intended message based on all its channel observations as $\widehat{W}_p(Y_p^N)$, and similarly CRx outputs $\widehat{W}_c(Y_c^N)$. The capacity region is the convex closure of all non-negative rate pairs (R_p, R_c) such that $\max_{u \in \{c, p\}} \mathbb{P}[\widehat{W}_u \neq W_u] \rightarrow 0$ as $N \rightarrow +\infty$.

The noncooperative Gaussian IC is obtained as a special case of the GCCIC by setting $C = 0$ and the Gaussian non-causal cognitive IC in the limit for $C \rightarrow +\infty$. A GCCIC is said to be a Z-channel if $I_p = 0$, i.e., the CRx does not experience interference from PTx, and an S-channel if $I_c = 0$, i.e., the PRx does not experience interference from CTx.

Capacity region to within a constant gap. The capacity region of the GCCIC is said to be known to within GAP bits if we can show an inner bound region \mathcal{I} and an outer bound region \mathcal{O} such that

$$(R_p, R_c) \in \mathcal{O} \implies ([R_p - \text{GAP}]^+, [R_c - \text{GAP}]^+) \in \mathcal{I}.$$

Generalized Degrees of Freedom (gDoF). The knowledge of the capacity region to within a constant gap implies an exact

capacity characterization at high SNR. The gDoF is a performance measure introduced in [16] for the noncooperative IC to capture the high SNR behavior of the sum-capacity as a function of the relative strengths of direct and interference links. The gDoF represents a more refined characterization of the sum-capacity at high SNR compared to the classical DoF. In order to quantify the gain of causal unilateral source cooperation compared to the noncooperative IC, we shall use the gDoF as a performance measure. Let $S > 1$ and parameterize

$$S_p := S^1, \text{ primary direct link,} \quad (2a)$$

$$S_c := S^1, \text{ cognitive direct link,} \quad (2b)$$

$$I_p := S^{\alpha_p}, \alpha_p \geq 0, \text{ interference at CRx from PTx,} \quad (2c)$$

$$I_c := S^{\alpha_c}, \alpha_c \geq 0, \text{ interference at PRx from CTx,} \quad (2d)$$

$$C := S^\beta, \beta \geq 0, \text{ cooperation link,} \quad (2e)$$

where α_p and α_c measure the strength of the interference links compared to the direct link, while β the strength of the cooperation link compared to the direct link. We remark that the parameterization in (2), with direct links of the same strength, is used only for evaluation of the gDoF. Moreover, in order to capture different network topologies, we focus on

- 1) interference-symmetric channel: $\alpha_p = \alpha_c = \alpha$;
- 2) Z-channel: $\alpha_p = 0, \alpha_c = \alpha$;
- 3) S-channel: $\alpha_p = \alpha, \alpha_c = 0$.

The case $\alpha_p = \alpha_c = 0$ is not interesting since in this case the GCCIC reduces to two parallel point-to-point links for which cooperation is useless. For the above three cases, the system is parameterized by the triplet (S, α, β) , where S is referred to as the (direct link) SNR, α as the interference exponent and β as the cooperation exponent.¹ The gDoF is defined as

$$d(\alpha, \beta) := \lim_{S \rightarrow +\infty} \frac{\max\{R_p + R_c\}}{2 \log(1 + S)} \quad (3)$$

where the maximization is intended over all possible achievable rate pairs (R_c, R_p) . Without cooperation, the gDoF $d(\alpha, 0)$ reduces to the gDoF characterized in [16] while for $\beta \rightarrow +\infty$ to the gDoF that can be evaluated from the capacity characterization to within 1 bit of [24]. Here we are interested in determining under which condition on the cooperation exponent β we have $d(\alpha, \beta) > d(\alpha, 0)$ since a strict improvement in gDoF implies an unbounded gain in terms of sum-capacity as the SNR grows to infinity.

B. Known outer bounds for the GCCIC

In the literature several outer bounds are known for bilateral source cooperation [9], [11], [12], [13]. Here we specialize some of them for the GCCIC in (1). We let $\mathbb{E}[X_p X_c^*] = \rho$, for some $\rho \in \mathbb{C}$ such that $|\rho| \leq 1$. An outer bound region

¹In principle the system performance also depends on the phases of the interfering links (θ_c, θ_p) . However, as far as gDoF and sum-capacity to within a constant gap are concerned, the phases (θ_c, θ_p) only matter if the IC channel matrix $\begin{bmatrix} \sqrt{S_p} & \sqrt{I_c} e^{j\theta_c} \\ \sqrt{I_p} e^{j\theta_p} & \sqrt{S_c} \end{bmatrix}$ is rank deficient, in which case one received signal is a noisier version of the other and the overall channels behave, sum-capacity-wise, as a Multiple Access Channel (MAC).

$$R_c \leq \log(1 + S_c) \quad (4a)$$

$$R_p \leq \min \left\{ \log \left(1 + (\sqrt{S_p} + \sqrt{I_c})^2 \right), \log(1 + C + S_p) \right\} \quad (4b)$$

$$R_p + R_c \leq \min \left\{ r^{(CS)}, r^{(DT)}, r^{(PV)} \right\} \quad (4c)$$

$$r^{(CS)} \leq \log(1 + S_c) + \min \left\{ \log \left(1 + (\sqrt{S_p} + \sqrt{I_c})^2 \right), \log(1 + C + S_p) \right\} \quad (4d)$$

$$r^{(DT)} \leq \min \left\{ \log \left(\frac{1 + \max\{I_c, S_c\}}{1 + I_c} \right) + \log \left(1 + (\sqrt{S_p} + \sqrt{I_c})^2 \right), \right. \\ \left. \log \left(\frac{1 + C + \max\{S_p, I_p\}}{1 + I_p} \right) + \log \left(1 + (\sqrt{S_c} + \sqrt{I_p})^2 \right) \right\} \quad (4e)$$

$$r^{(PV)} \leq \log \left(\left(1 + \left(\frac{\sqrt{S_p}}{\sqrt{\max\{1, I_p\}}} + \sqrt{I_c} \right)^2 \right) \left(1 + \left(\frac{\sqrt{S_c}}{\sqrt{\max\{1, I_c\}}} + \sqrt{I_p} \right)^2 \right) \right) + \Delta \quad (4f)$$

$$\Delta := \log \left(\frac{1 + \left(\frac{\sqrt{S_c}}{\sqrt{\max\{1, I_c\}}} + \frac{\sqrt{I_p}}{\sqrt{\max\{1, C\}}} \right)^2}{(1 + C) \frac{1 + \left(\frac{\sqrt{S_c}}{\sqrt{\max\{1, I_c\}}} + \sqrt{I_p} \right)^2}{1 + \left(\frac{\sqrt{S_c}}{\sqrt{\max\{1, I_c\}}} + \sqrt{I_p} \right)^2}} \right) \quad (4g)$$

for the GCCIC is reported in (4) at the top of next page and is obtained by upper bounding over $(\rho, \theta_c, \theta_p)$ each mutual information term in the bounds in [9], [11], [12] (the details can be found in Appendix A). In particular, the bounds on the individual rates in (4a) and (4b) are cut-set bounds, and the sum-rate upper bound in (4c) is the minimum of three quantities obtained as follows: from the cut-set bounds on the individual rates we obtain (4d), from [12] we obtain (4e), and from [11] we obtain (4f).

The upper bound in (4) for $C \rightarrow +\infty$ reduces to the upper bound for the Gaussian non-causal cognitive IC in [24, Theorem III.1], which unifies previously known outer bounds for the weak ($S_c > I_c$) and strong ($S_c \leq I_c$) interference regimes. The region in [24, Theorem III.1] is known to be achievable to within 1 bit in all parameter regimes. However, in weak interference ($S_c > I_c$), the capacity region of the Gaussian non-causal cognitive IC is known exactly and is given by

$$R_p \leq \log \left(1 + \frac{S_p + |\gamma_c|^2 I_c + 2|\gamma_c| \sqrt{S_p I_c}}{1 + (1 - |\gamma_c|^2) I_c} \right) \quad (5a)$$

$$R_c \leq \log(1 + (1 - |\gamma_c|^2) S_c) \quad (5b)$$

union over all $|\gamma_c| \leq 1$. Therefore, the region in (5) is an outer bound for the GCCIC for $S_c > I_c$.

From the sum-rate upper bound in (4c), with the parameterization in (2), we can immediately obtain the following gDoF upper bound

$$d \leq \frac{1}{2} \min \left\{ d^{(CS)}(\alpha_c, \beta) + d^{(CS)}(\alpha_p, 0), \right. \quad (6a)$$

$$\left. \min \{ d^{(DT)}(\alpha_c, 0), d^{(DT)}(\alpha_p, \beta) \}, \right. \quad (6b)$$

$$\left. d^{(PV)}(\alpha_p, \alpha_c, \beta) \right\} \quad (6c)$$

where

$$d^{(CS)}(\alpha, \beta) := \max\{1, \min\{\alpha, \beta\}\} \quad (6d)$$

$$d^{(DT)}(\alpha, \beta) := \max\{\beta, \alpha, 1\} - \alpha + \max\{\alpha, 1\} \quad (6e)$$

$$d^{(PV)}(\alpha_p, \alpha_c, \beta) := \max\{1 - \alpha_p, \alpha_c\} \\ + \max\{1 - \alpha_c + \beta, \alpha_p\}. \quad (6f)$$

The proof follows by using the upper bound in (4c) in the gDoF definition in (3) (the details can be found in Appendix A). The achievability for the interference-symmetric ($\alpha_p = \alpha_c = \alpha$) and the interference-asymmetric cases (either $\alpha_p = 0, \alpha_c = \alpha$ or $\alpha_p = \alpha, \alpha_c = 0$) will follow from the constant gap results in the next sections.

C. Known inner bounds for the general memoryless CCIC

To the best of our knowledge, the largest known achievable region for the general memoryless IC with generalized feedback, or bilateral source cooperation, is the *superposition+binning* region from [10, Section V]. In this scheme, adapted to the case of unilateral source cooperation, the PTx's message is split into four parts: the *noncooperative common message* and the *noncooperative private message* are sent as in the Han-Kobayashi's scheme for the noncooperative IC [7]; the *cooperative common message* and the *cooperative private message* are decoded at CTx in a given slot and retransmitted in the next slot by using a decode-and-forward based block-Markov scheme. The CTx's message is split into two parts: the *noncooperative common message* and the *noncooperative private message* that are sent as in the Han-Kobayashi's scheme for the noncooperative IC [7]. The common messages are decoded at both destinations while non-intended private messages are treated as noise. For cooperation, the two sources 'beam form' the PTx's cooperative common message to the destinations as in a distributed MIMO system, and the CTx precodes its private messages against the interference created

by the PTx's cooperative private message as in a MIMO broadcast channel. The achievable region in [10, Section V] is quite complex to evaluate because it is a function of 11 auxiliary random variables and is described by about 30 rate constraints per source-destination pair. In this work we will use a small subset of these 11 auxiliary random variables in each parameter regime (see Appendices B and C) and show that the corresponding schemes are to within a constant gap from the outer bound in (4).

As noted in the Introduction, the largest known achievable region for the IC with unilateral source cooperation is, to the best of our knowledge, the region in [23, Corollary 1]. The difference between [23, Corollary 1] and the region in [10, Section V] adapted to the case of unilateral source cooperation is, see [23, Remark 2, point 6]: "in [10, Section V] binning is done sequentially and conditionally, while [23, Corollary 1] utilizes joint binning technique. [...] In [23, Corollary 1] uses joint backward decoding at the receivers, while two-step decoding is used in [10, Section V]." As far as (sum-)capacity to within a constant gap is concerned, the results in this paper show that these differences are not fundamental for approximate optimality.

Next, in Section III we consider the *symmetric* GCCIC, where the direct links have the same strength and the interfering links have the same strength. We characterize its capacity to within a constant gap in the strong interference regime and in the weak interference regime when the strength of the cooperation link is greater than a threshold. When the system experiences weak interference and the strength of the cooperation link is below a threshold the sum-capacity to within a constant gap is characterized. This preliminary analysis allows us to identify the key features of the proposed achievable schemes in the strong and weak interference regimes. Moreover, it sets the stage for the constant gap derivation for the general GCCIC in some parameter regimes in Section IV and for the whole capacity characterization to within a constant gap for the general Z-channel in Section V, and for the general S-channel in Section VI.

III. THE CAPACITY REGION TO WITHIN A CONSTANT GAP FOR THE SYMMETRIC GCCIC

The symmetric GCCIC is defined by $S_p = S_c = S$ and $I_p = I_c = I = S^\alpha$. Following the naming convention of the noncooperative IC, we say that the symmetric GCCIC has strong interference if $S \leq I$, that is $1 \leq \alpha$, and weak interference otherwise. Our main result for the symmetric GCCIC is as follows:

Theorem 1 *When $S \leq I$, the capacity region of the symmetric GCCIC is achievable to within 1 bit using schemes based on superposition coding. Depending on the strength of the cooperation link, the approximately optimal strategies might or might not require to engage in cooperation. The achievable rate regions for this regime are given in (7), in (9) and in (11) below. When $S > I$ and $C \geq \Delta_{th}$, with $\Delta_{th} := \left(S + I + 2\sqrt{IS \frac{1}{1+I}}\right) (1+I)$, the capacity region of the symmetric GCCIC is achievable to within 1 bit using a scheme*

based on superposition coding and DPC whose achievable rate region is given in (18) below. When $S > I$ and $C < \Delta_{th}$, the sum-capacity of the symmetric GCCIC is achievable to within 3.16 bits using schemes based on superposition coding whose achievable sum-rates are given by (13), (15) and (17) below depending on the strength of the interference links compared to the cooperation link.

The result in Theorem 1 provides a capacity characterization to within a constant gap for the symmetric GCCIC for a set of channel parameters, which excludes the case of weak interference ($S > I$) when the cooperation link is not strong enough ($C < \Delta_{th}$). For this set of parameters, in fact, Theorem 1 provides only an approximate sum-capacity result. As we shall see later in more details, the main difficulty that arises in this regime to deal with approximate capacity characterization is due to the lack of outer bounds of the type $2R_p + R_c/R_p + 2R_c$, which, to the best of our knowledge, are not available in the literature and whose derivation is outside the scope of this work.

The rest of the section is devoted to the proof of Theorem 1. In order to highlight the key steps in the proof, we use the gDoF as starting point for our discussion. The gDoF upper bound for the symmetric GCCIC is obtained by setting $\alpha_p = \alpha_c = \alpha$ in (6). Fig. 2 shows the gDoF and the gap (per user) for the symmetric GCCIC for the different regions in the (α, β) plane, where the whole set of parameters has been partitioned into multiple sub-regions depending upon different levels of cooperation (β) and interference (α) strengths. In regimes 1, 3, 4 and 5 of Fig. 2 the gDoF attained by the symmetric GCCIC is the same as that achieved by the noncooperative IC given by [16]

$$d^{IC}(\alpha) = \min\{\max\{1 - \alpha, \alpha\}, \max\{1 - \alpha/2, \alpha/2\}, 1\}.$$

Thus, unilateral cooperation provides a strict gDoF gain over the noncooperative IC in regimes 2 and 6 of Fig. 2. For reference, the gDoF on the non-causal cognitive IC can be evaluated from [24] as

$$d^{CIC}(\alpha) = \max\{1 - \alpha/2, \alpha/2\}.$$

In general we have

$$d(\alpha, 0) = d^{IC}(\alpha) \leq d(\alpha, \beta) \leq d^{CIC}(\alpha) = \lim_{\beta \rightarrow +\infty} d(\alpha, \beta).$$

From Fig. 2, in regime 2 with $\beta \geq \alpha - 1$, in regimes 3 and 4, and in regime 6 with $\beta \geq \min\{\alpha, 1 - \alpha\}$, causal unilateral source cooperation attains the ultimate gDoF limit of the non-causal cognitive IC.

At a high level, the approximately optimal coding schemes are as follows. In the strong interference and weak cooperation regime both users employ a noncooperative common message. In the strong interference and strong cooperation regime, PTx's common message becomes cooperative and is forwarded to PRx with the help of CTx. In the weak interference regime, each user splits its message into a common and a private part; for CTx the two message parts are noncooperative while for PTx are cooperative; PTx's cooperative common message is the 'cloud center' of a superposition coding scheme, and PTx's cooperative private message is the 'known interference'

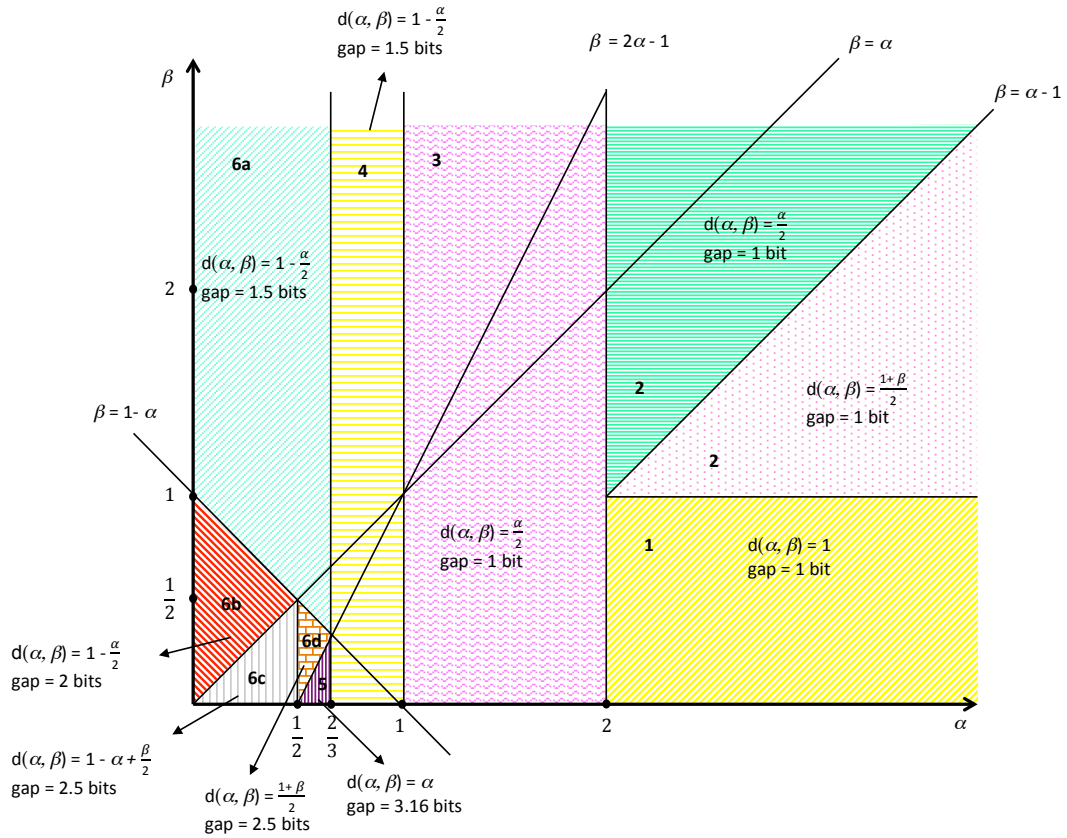


Fig. 2: Optimal gDoF and constant gap for the symmetric GCCIC in the different regimes of (α, β) .

against which CTx's message is precoded in a DPC-based scheme. Binning/DPC is used in the weak interference and strong cooperation regime where CTx can easily decode the signal from PTx because of strong cooperation, but CRx cannot because of weak interference; therefore in this regime it makes sense that the best use of CTx's knowledge of PTx's message is to treat it as a 'known state' to precode its message against it.

We shall now discuss each regime of Fig. 2 separately.

A. Regime 1 (strong interference): same gDoF as in the noncooperative IC, and capacity region to within 1 bit with a noncooperative scheme

Regime 1 corresponds to very strong interference ($\alpha \geq 2$) and weak cooperation ($\beta \leq 1$). In the noncooperative IC with very strong interference it is exactly optimal to use only (noncooperative) common messages in order to achieve the whole capacity region; since the interference is very strong, it can be decoded by treating the intended signal as noise, after which each receiver is left with an interference-free point-to-point channel from its transmitter; this noncooperative strategy achieves

$$\mathcal{I}^{\text{III-A}} : R_c \leq \log(1 + S), \quad (7a)$$

$$R_p \leq \log(1 + S), \quad (7b)$$

or $d \leq (1 + 1)/2 = 1$. Since the cooperation link is weak in regime 1, the amount of data PTx could communicate

to CTx for cooperation is very limited. As a result in this regime unilateral cooperation does not improve the gDoF performance compared to the noncooperative case. In other words, in regime 1, cooperation provides a 'beam forming gain' but not a gDoF gain. To see this, the cut-set upper bounds on individual rates in (4a) and (4b), in the symmetric case for $\beta \leq 1 \iff C \leq S$, give the following upper bounds on the individual rates

$$\mathcal{O}^{\text{III-A}} : R_c \leq \log(1 + S), \quad (8a)$$

$$R_p \leq \log(1 + S + C) \leq \log(1 + 2S) \leq \log(1 + S) + \log(2). \quad (8b)$$

From the upper bound on R_p in (8b), we see that unilateral cooperation can at most double the SNR on the primary direct link, which can at most increase the rate by 1 bit compared to the noncooperative case. As a result, the gDoF with unilateral cooperation is $d = 1$ and the rate pair in (7) is optimal to within 1 bit, i.e., $\max\{\text{eq}(8a) - \text{eq}(7a), \text{eq}(8b) - \text{eq}(7b)\} \leq \max\{0, \log(2)\} = 1$ bit.

B. Regime 2 (strong interference): improved gDoF compared to the noncooperative IC, and capacity region to within 1 bit with a cooperative scheme

In regime 2 the interference is very strong ($\alpha \geq 2$) and the cooperation is strong ($\beta > 1$). Similarly to the noncooperative IC in very strong interference regime, the transmitters send a common message only. As opposed to regime 1, where both

messages were sent noncooperatively, here the PTx takes advantage of the strong cooperation link and sends its message to PRx with the help of the CTx. In order to enable cooperation, a block Markov coding scheme is used as follows. Transmission is over a frame of $B \gg 1$ slots. In slot $t \in [1 : B]$, the PTx sends its old (cooperative common) message $W_{p,t-1}$ and superposes to it the new (cooperative common) message $W_{p,t}$, while the CTx forwards the primary old (cooperative common) message $W_{p,t-1}$ and superposes to it its (noncooperative common) message $W_{c,t}$. At the end of slot t , CTx decodes the new message $W_{p,t}$ after subtracting the contribution of the old message $W_{p,t-1}$. The destinations wait until the whole frame has been received and then proceed to jointly backward decode all messages. The details can be found in Appendix B-B and the achievable region is given in (52), which in the symmetric GCCIC in very strong interference reduces to

$$\mathcal{I}^{\text{III-B}} : R_c \leq \log(1+S), \quad (9a)$$

$$R_p \leq \log(1+C), \quad (9b)$$

$$R_p + R_c \leq \log(1+S+1). \quad (9c)$$

The region in (9) is strictly larger than the noncooperative capacity region in very strong interference given by (7) for $S(1+S) \leq 1$, or $\alpha \geq 2$, and $C > S$, or $\beta > 1$, which is precisely the definition of regime 2. The sum-capacity from (9) can take two possible values, depending on which one among the MAC sum-rate bound in (9c) and the sum of the bounds on the individual rates in (9a)-(9b) is the most stringent. In particular, the following sum-rate is achievable

$$R_p + R_c \leq \begin{cases} \log(1+C) + \log(1+S) & \text{if } C(1+S) \leq 1 \\ \log(1+S+1) & \text{if } C(1+S) > 1 \end{cases},$$

that is, $d \leq (\beta+1)/2$ if $\beta+1 \leq \alpha$ and $d \leq \alpha/2$ otherwise (in both cases the gDoF is larger than $d^{\text{IC}} = 1$).

From the outer bound region obtained from the cut-set upper bounds on the individual rates in (4a) and (4b) and the sum-rate upper bound in (4e), under the condition $\beta > 1 \iff C > S$, we have that any achievable rate pair must satisfy

$$\mathcal{O}^{\text{III-B}} : R_c \leq \log(1+S), \quad (10a)$$

$$R_p \leq \log(1+S+C) \leq \log(1+C) + \log(2), \quad (10b)$$

$$\begin{aligned} R_p + R_c &\leq \log\left(1 + \left(\sqrt{S} + \sqrt{1}\right)^2\right) \\ &\leq \log(1+S+1) + \log(2), \end{aligned} \quad (10c)$$

since $(\sqrt{x} + \sqrt{y})^2 \leq 2(x+y)$, $\forall(x,y) \in \mathbb{R}_+^2$. The upper bound in (10) and the achievable region in (9) are to within 1 bit of one another since

$$\text{GAP} \leq \max\left\{\text{eq}(10a) - \text{eq}(9a), \text{eq}(10b) - \text{eq}(9b), \frac{\text{eq}(10c) - \text{eq}(9c)}{2}\right\} \leq \log(2).$$

This shows that the whole capacity region, and therefore the gDoF $d = \min\{\beta+1, \alpha\}/2$ too, is achievable to within 1 bit in regime 2.

C. Regime 3 (strong interference): same gDoF as in the noncooperative IC, and capacity region to within 1 bit with a cooperative scheme

Regime 3 corresponds to strong but not very strong interference ($\alpha \in [1, 2)$). Note that there are no restrictions on the cooperation exponent β in this regime. Similarly to regimes 1 and 2, here we use only common messages – a strategy that is capacity achieving in the corresponding noncooperative IC. The difference between regime 1 and regime 3 is that stripping decoding is no longer optimal and the receivers must instead jointly decode the intended and non-intended messages as in a MAC. By taking the largest between the achievable region developed for regime 2 in (9) and the noncooperative achievable region for this regime (i.e., common messages only), which has $R_p \leq \log(1+S)$ as a bound on the primary rate rather than $R_p \leq \log(1+C)$, we obtain the following achievable region

$$\mathcal{I}^{\text{III-C}} : R_c \leq \log(1+S), \quad (11a)$$

$$R_p \leq \log(1 + \max\{C, S\}), \quad (11b)$$

$$R_p + R_c \leq \log(1+S+1), \quad (11c)$$

which implies $d \leq \min\{1 + \max\{1, \beta\}, \max\{1, \alpha\}\}/2 = \alpha/2$, i.e., the sum-rate bound in (11c) is the tightest. In regime 3, no matter how strong the cooperation link is, cooperation does not increase the gDoF of the noncooperative IC.

From the outer bound region obtained from the cut-set upper bounds on the individual rates in (4a) and (4b) and the sum-rate upper bound in (4e), we have that any achievable rate pair must satisfy

$$\mathcal{O}^{\text{III-C}} : R_c \leq \log(1+S), \quad (12a)$$

$$\begin{aligned} R_p &\leq \log(1+S+C) \\ &\leq \log(1 + \max\{C, S\}) + \log(2), \end{aligned} \quad (12b)$$

$$\begin{aligned} R_p + R_c &\leq \log\left(1 + \left(\sqrt{S} + \sqrt{1}\right)^2\right) \\ &\leq \log(1+S+1) + \log(2). \end{aligned} \quad (12c)$$

It is easy to see that the regions in (12) and (11) are to within 1 bit of one another.

D. Regime 4 (weak interference): same gDoF as in the noncooperative IC

Regime 4 corresponds to moderately weak interference ($\alpha \in [2/3, 1)$). In this regime, rate splitting is needed to achieve the capacity to within 1 bit in the noncooperative IC [16]. Therefore we propose to use here the noncooperative scheme that consists of two messages for each user: the noncooperative common and the noncooperative private. The power of the noncooperative private message (which is treated as noise at the non-intended receiver) is such that it is received at or below the receiver noise floor [16]. As shown in [16], in the moderately weak interference regime the sum-rate upper bound of [14, Theorem 1] can be achieved to within 1 bit per user, that is, the following sum-rate is achievable

$$\begin{aligned} R_p + R_c &\leq \log(1+S+1) + \log(1+S) \\ &\quad - \log(1+1) - 2\log(2), \end{aligned} \quad (13)$$

or $d \leq \frac{\max\{1, \alpha\} + (1 - \alpha)}{2} = 1 - \alpha/2$. The cooperative sum-rate upper bound in (4e) can be further upper bounded as

$$R_p + R_c \leq \log(1 + S + I) + \log(1 + S) - \log(1 + I) + \log(2). \quad (14)$$

Therefore, the gap is at most $\text{GAP} \leq \frac{\text{eq}(14) - \text{eq}(13)}{2} \leq 3/2 \log(2)$ and is achieved by the noncooperative scheme with rate splitting as in [16].

In order to claim capacity to within a constant gap in the weak interference regime, we must derive an upper bound that reduces to, or is to within a constant gap of, the capacity outer bound in [16, Theorem 3] when $C = 0$. The outer bound region in [16, Theorem 3] is characterized by bounds on the individual rates, bounds on the sum-rate, and by bounds on $2R_c + R_p$ and $R_c + 2R_p$. Therefore, unless outer bounds on $2R_c + R_p$ and $R_c + 2R_p$ for the cooperative case are developed, it is not possible to claim optimality to within a finite gap of the upper bound in (4) for small C . Developing outer bounds on $2R_c + R_p$ and $R_c + 2R_p$ for the general IC with source cooperation is an important open problem, which is outside the scope of this work. An interesting question that could be answered by such a line of research is as follows. In [26], the authors interpreted the bounds on $2R_c + R_p$ and $R_c + 2R_p$ as a measure of the amount of ‘resource holes’, or inefficiency, due to the distributed nature of the noncooperative IC. In [26], the authors showed that with output feedback from a destination to its source, such ‘resource holes’ are no longer present; in other words, feedback enables coordination among the sources which results in a full utilization of the channel resources. An interesting open question is whether unilateral cooperation enables sufficient coordination among the sources for full utilization of the channel resources. In the limiting case where unilateral cooperation equals non-causal cognition, we know from [24] that the capacity region does not have bounds on $2R_c + R_p$ and $R_c + 2R_p$, i.e., there are no ‘resource holes’. Therefore the question can be rephrased as: is there a minimum strength of the cooperation link C above which unilateral causal cooperation results in no ‘resource holes’ in weak interference, i.e., bounds on $2R_c + R_p$ and $R_c + 2R_p$ are not needed to (approximately) characterize the capacity region?

E. Regime 5 (weak interference): same gDoF as in the noncooperative IC

In regime 5 the interference is moderately weak ($\alpha \in [1/2, 2/3)$) and the cooperation is fairly weak ($0 \leq \beta < 2\alpha - 1$). The gDoF upper bound gives $d = \alpha$ as for the noncooperative IC. Hence in this regime we use the scheme that is approximately optimal for the sum-capacity of the noncooperative IC, with noncooperative common and private messages and with power splits as in [16]. The noncooperative scheme achieves

$$R_p + R_c \leq 2 \log \left(1 + I + \frac{S}{\max\{1, I\}} \right) - 2 \log(2). \quad (15)$$

The cooperative sum-rate upper bound in (4f) can be further upper bounded as

$$R_p + R_c \leq 2 \log \left(1 + I + \frac{S}{\max\{1, I\}} \right) + 2 \log(2) + \Delta', \quad (16)$$

where Δ' is the latest Δ in (4g) in the regime $\beta < 2\alpha - 1 \iff C < I^2/S \iff \frac{S}{I} < \frac{1}{C}$ within the weak interference regime $1 \leq \frac{S}{I}$, that is,

$$\begin{aligned} \Delta' &= \max_{1 \leq \frac{S}{I} < \frac{1}{C}} \log \left(\frac{(1 + C) \left(1 + \left(\sqrt{\frac{S}{I}} + \sqrt{\frac{1}{C}} \right)^2 \right)}{1 + \left(\sqrt{\frac{S}{I}} + \sqrt{I} \right)^2} \right) \\ &\leq \max_{1 \leq \frac{S}{I} < \frac{1}{C}} \log \left(\frac{(1 + C) \left(1 + 2\frac{S}{I} + 2\frac{1}{C} \right)}{1 + \frac{S}{I} + I} \right) \\ &= \max_{1 \leq \frac{S}{I} < \frac{1}{C}} \log \left(\frac{(1 + C) \left(1 + 4\frac{1}{C} \right)}{1 + \frac{1}{C}(1 + C)} \right) \\ &= \log \left(\max \left\{ \frac{(1 + C) 5}{2 + C}, \frac{(1 + C) 4}{1 + C} \right\} \right) \leq \log(5), \end{aligned}$$

where in the derivation we used $1 \leq C$ (note that for $C < 1$ the outer bounds in (4) are to within a constant gap of the corresponding bounds for $C = 0$). Therefore, the gap (per user) is at most $\text{GAP} \leq \frac{\text{eq}(16) - \text{eq}(15)}{2} \leq \frac{(2+2)\log(2) + \log(5)}{2} \approx 3.16 \log(2)$ and is achieved by the noncooperative scheme.

The observations we made for regime 4, regarding possible extensions to the whole capacity region in the general case, apply to regime 5 as well.

F. Regime 6 (weak interference): improved gDoF compared to the noncooperative IC

In regime 6, the interference is quite weak ($\alpha < 2/3$) and the cooperation exponent satisfies $\beta \geq [2\alpha - 1]^+$. Since the interference is weak, we split the messages into a common part and a private part, as for the noncooperative IC. For the CTx the two messages are noncooperative, but for the PTx the common message is cooperative and the private message is noncooperative. In other words, in regime 6 we extend the scheme used in regime 2 by adding a private message for each transmitter. The cooperation mechanism is based on decode-and-forward: at any given time slot of a block Markov coding scheme CTx decodes the primary common message, which PTx and CTx ‘beam form’ to the receivers in the next slot. The new common and private messages of each user are superposed to the old primary cooperative common message. The details of the achievable scheme are reported in Appendix B-C, where we show that the sum-rate in (55), namely

$$R_p + R_c \leq \min \left\{ \log \left(1 + \frac{S}{2I} \right) + \log \left(\frac{S + I + 1}{2} \right), \log \left(1 + \frac{S}{2I} \right) + \log \left(\frac{1 + C}{1 + C} \right) + \log \left(\frac{S + I^2 + 1}{2} \right) \right\}, \quad (17)$$

is achievable. Depending on which expression attains the minimum, we obtain the four subregions, indicated as from

6a to 6d, into which regime 6 is subdivided. In particular, for subregions 6a and 6b the tightest outer bound is the one in (4e), while for subregions 6c and 6d the tightest sum-rate outer bound is the one in (4f). Note that the outer bound in (4f) reduces to the more involved part of the W-curve of [16] for $\alpha < 2/3$ when $\beta = 0$. In Appendix B-D we show that this scheme is optimal to within 2.5 bits.

The achievable scheme used for regime 6 (defined as $\alpha < 2/3$) is also optimal to within a constant gap for the most part of regime 4 (defined as $\alpha \in [2/3, 1)$). In particular, as a consequence of the gap derivation in Appendix B-D, the achievable scheme for regime 6 and the outer bound in (4e) are to within a constant gap of one another when the interference is weak ($\alpha \leq 1$) and the cooperation satisfies $\beta \geq \min\{\alpha, 1-\alpha\}$.

The largest gap in regime 6 is of 2.5 bits in sub-regimes 6c and 6d, where the tightest sum-rate outer bound is the one in (4f). This gap may be decreased in several ways. For example, one can develop tighter bounds than the one in (4f), or develop more involved coding schemes. An example of the latter method can be found next, where we consider a DPC-based achievable scheme for the weak interference regime / regimes 4 and 6.

G. Regimes 4 and 6 (weak interference) with strong cooperation: capacity to within 1 bit with a cooperative scheme

We return on an observation made earlier, namely, that when the cooperation link gain C is sufficiently large, we expect the performance of the GCCIC to approach that of the non-causal cognitive IC. We next show that a DPC-based scheme is optimal to within 1 bit for the whole capacity region in the weak interference regime when the cooperation gain C is sufficiently strong, and we give a sufficient condition to quantify what ‘sufficiently strong C ’ means.

In the DPC-based achievable scheme in Appendix C-C, the primary private message is cooperative, while in the scheme used previously for regime 6 in Appendix B-C it was noncooperative. Here we propose that CTx, with knowledge of PTx’s primary private message, uses DPC to rid CRx of the interference due to the primary private message. In particular, PTx sends $X_p = \gamma_p S + \sqrt{1-|\gamma_p|^2} U_p$, for some $|\gamma_p|^2 \leq 1$, where S carries the PTx’s old private cooperative message and U_p carries the PTx’s new private cooperative message in a block Markov coding scheme. CTx sends $X_c = \gamma_c S + \sqrt{1-|\gamma_c|^2} U_c$, for some $|\gamma_c|^2 \leq 1$, where U_c carries the CTx’s private noncooperative message. In a given time slot, CTx knows PTx’s old private cooperative message S and decodes PTx’s new private cooperative message U_p from its channel output. CTx then precodes its private noncooperative message against the ‘known interference’ S ; thanks to DPC, CRx decodes U_c as if the interference S was not present [8], while treating U_p as noise. PRx does backward decoding in order to recover its message while treating U_c as noise. This DPC-based scheme is similar to the capacity achieving scheme for the non-causal cognitive IC in weak interference [27], [28], except for the fact that now CTx must decode PTx’s message in U_p , and that CRx’s equivalent noise variance includes the

interference due to U_p . To overcome this last problem, inspired by [16], we choose the power split γ_p in such a way that the interference created by U_p at CRx is at the same level of the noise. With this choice of parameters the achievable region in (62), specialized to the symmetric case, becomes

$$\mathcal{I}^{\text{III-G}} : R_p \leq \log \left(1 + \frac{C}{1+I} \right), \quad (18a)$$

$$R_p \leq \log \left(1 + \frac{S + |\gamma_c|^2 I + 2|\gamma_c| \sqrt{IS \frac{1}{1+I}}}{1 + (1 - |\gamma_c|^2) I} \right), \quad (18b)$$

$$R_c \leq \log \left(1 + \frac{(1 - |\gamma_c|^2) S}{1 + \frac{1}{1+I}} \right), \quad (18c)$$

for all $|\gamma_c| \leq 1$. Under the condition

$$\begin{aligned} \frac{C}{1+I} &\geq \max_{|\gamma_c| \leq 1} \frac{S + |\gamma_c|^2 I + 2|\gamma_c| \sqrt{IS \frac{1}{1+I}}}{1 + (1 - |\gamma_c|^2) I} \iff \\ C &\geq \left(S + I + 2\sqrt{IS \frac{1}{1+I}} \right) (1+I) \iff \beta \geq 1 + \alpha \end{aligned} \quad (19)$$

the constraint in (18a) is redundant.

The achievable region under the condition in (19) must next be compared to an outer bound. We use here as an outer bound the capacity region of the non-causal cognitive IC given in (5). By comparing (5a) with (18b), and (5b) with (18c), it is easy to see that for every value of $|\gamma_c| \leq 1$ the two regions are at most $\text{GAP} \leq \log \left(1 + \frac{1}{1+I} \right) \leq \log(2) = 1$ bit away. This capacity result to within a constant gap holds for a sufficiently large C and it agrees with the intuition that the GCCIC should perform more and more as the non-causal cognitive IC as C increases.

If we only consider the sum-capacity, in Appendix C-D we show that the scheme in (62), of which the scheme in (18) is a special case, achieves the sum-capacity upper bound in (4e) to within 1 bit when the channel gains satisfy $C \geq S$, that is, $\beta \geq 1$, which is smaller than the gap of 1.5 bits we found with the superposition-based scheme. Note that the condition $C \geq S$ for sum-capacity approximate optimality is less restrictive than the one in (19) (which is approximately $C \geq 4S(1+I)$) needed for the approximate optimality of the whole rate region.

We have now concluded the proof of Theorem 1. Before concluding this Section, we compare the gDoF performance of the symmetric GCCIC with that of other channel models so as to determine when unilateral cooperation may be worth implementing in practical systems.

H. Comparisons

When the gDoF, or high SNR throughput, is the desired performance metric, we can make the following observations:

- Causal unilateral source cooperation does not improve on the gDoF of the noncooperative IC when

$$\alpha \in \left[\frac{2}{3}, 2 \right] \text{ or } \beta \leq \min \{ 1, [2\alpha - 1]^+ \}$$

as shown by the green and yellow-shaded regions in Fig. 3, that is, the regimes 1, 3, 4 and 5 in Fig. 2. For

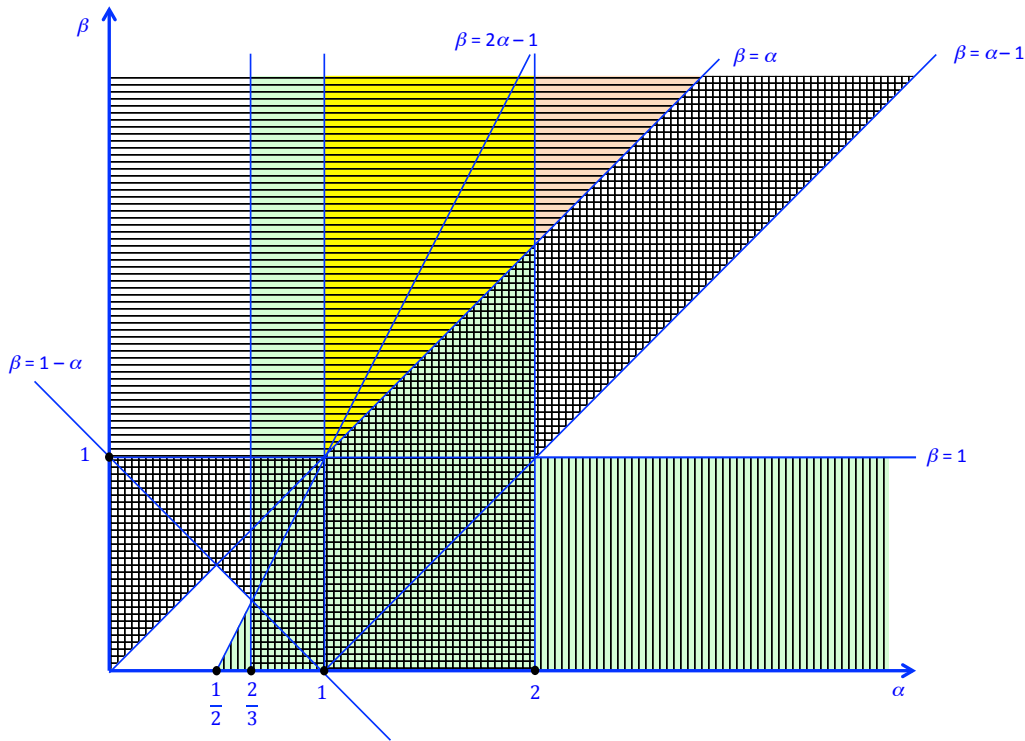


Fig. 3: Regions in which the gDoF of the symmetric GCCIC is equal to that of the noncooperative IC (green and yellow regions), of the relay channel (red and yellow regions), of the non-causal cognitive IC (region with horizontal lines), and of bilateral source cooperation (region with vertical lines). Note that the different regions can overlap.

this set of parameters, unilateral cooperation might not be worth implementing in practical systems since the same gDoF is achieved without explicit cooperation, i.e., unilateral cooperation only provides a power gain.

- In the regime $1 \leq \alpha \leq \beta$, unilateral cooperation attains the gDoF of the classical relay channel given by $d^{\text{RC}} = \max\{1, \min\{\alpha, \beta\}\} = \alpha$, as shown by the red and yellow-shaded regions in Fig. 3, i.e., parts of the regime 2 and regime 3 in Fig. 2 where $d = \alpha/2$, which correspond to a subset of the strong interference where the cooperation link is greater than the interference link. For this set of parameters cognitive radio might not be worth implementing in practical systems since the rate $R_c = 0$ for the cognitive pair is approximately sum-capacity optimal. There are however other rate pairs (R_c, R_p) attaining the optimal sum-rate with $R_c > 0$.
- The gDoF of the GCCIC is equal to that of the non-causal cognitive IC, given by $d = \max\{1 - \alpha/2, \alpha/2\}$, everywhere except in the regimes 5, 6c and 6d in Fig. 2, and for $\alpha \geq \max\{2, \beta + 1\}$, as shown by the horizontal-line-shaded region in Fig. 3. For this set of parameters unilateral cooperation attains the ultimate performance limits of non-causal cognitive radio and therefore represents the ideal channel condition for cognitive radio.
- The gDoF of unilateral cooperation equals that of bilateral cooperation, with cooperation links of the same strength as considered in [11], when $\beta \leq 1$ or $\beta \in [\alpha - 1]^+, \alpha$ except in the regimes 6c and 6d in Fig. 2 as shown

by the vertical-line-shaded region in Fig. 3. For this set of parameters unilateral cooperation attains the same gDoF of bilateral cooperation but with less resources and therefore represents a better trade-off in practical systems.

- For the symmetric case, our analysis suggests that superposition coding is approximately optimal if either the interference is strong or the cooperation is strong; when both interference and cooperation are weak, then cooperation based on DPC coding is approximately optimal. Even when superposition coding is approximately optimal in weak interference, DPC coding might lead to a smaller gap. The DPC-based scheme is more complex to implement in practice than superposition coding; hence there might be an interesting practical trade-off between complexity and constant gap.

IV. THE CAPACITY REGION TO WITHIN A CONSTANT GAP FOR THE GENERAL GCCIC

We now focus on the general GCCIC, which is more complex to analyze due to the fact that one has to deal with 5 different channel parameters. Following the naming convention of the noncooperative IC, we say that the general GCCIC has strong interference if $\{S_p \leq I_p, S_c \leq I_c\}$, weak interference if $\{S_p > I_p, S_c > I_c\}$, and mixed interference otherwise. Moreover, we say that the general GCCIC has strong cooperation if $C > S_p$ and weak cooperation otherwise. As we shall see later in more details, this section provides a capacity characterization to within a constant gap for the general GCCIC when, roughly speaking, the two receivers do not

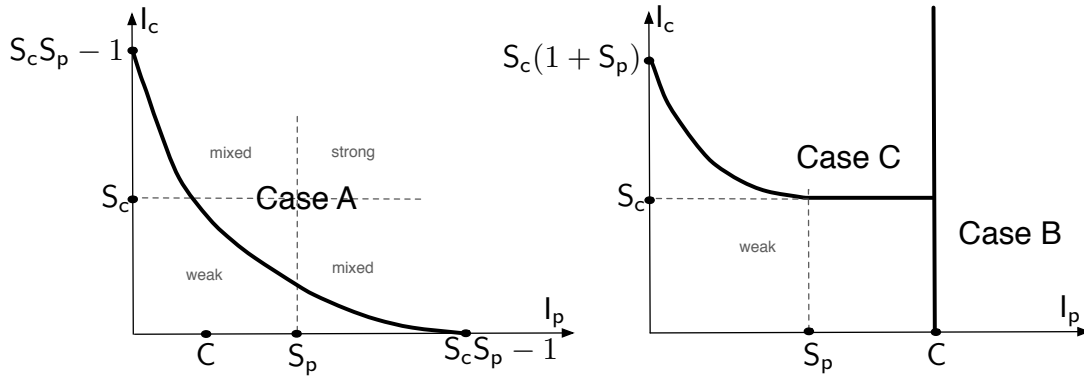


Fig. 4: The regimes identified by Theorem 2 where capacity is known to within a constant gap (indicated as “Case A”, “Case B” and “Case C”).

experience weak interference simultaneously. As highlighted for the symmetric GCCIC, for this regime, outer bounds of the type $2R_p + R_c/R_p + 2R_c$ seem to be needed, whose derivation is outside the scope of this work. In particular, our main result for the general GCCIC is as follows:

Theorem 2 When $C \leq S_p$ and $S_c S_p \leq (1+I_p)(1+I_c)$ (denoted as “Case A” in Fig. 4), the capacity region of the general GCCIC is achievable to within 2 bits using a noncooperative scheme based on superposition coding, whose achievable rate region is 1 bit away from the region in (21) below. When $S_p < C \leq I_p$ (denoted as “Case B” in Fig. 4), the capacity region of the general GCCIC is achievable to within 1 bit using a cooperative scheme based on superposition coding, whose achievable rate region is given in (24) below. In this strategy only the common message of PTx is sent cooperatively. When $\max\{S_p, I_p\} < C$ and $S_c \leq \alpha I_c$ with $\alpha = \min\left\{1, \frac{1+2I_p}{1+I_p+S_p}\right\}$ (denoted as “Case C” in Fig. 4), the capacity region of the general GCCIC is achievable to within 1.8 bits using a cooperative scheme based on superposition coding, whose achievable rate region is given in (27) below. When $S_c > I_c$ and $C \geq \left(S_p + I_c + 2\sqrt{S_p I_c \frac{I_p}{1+I_p}}\right)(1+I_p)$, the capacity region of the GCCIC is achievable to within 1 bit with a cooperative scheme based on DPC and superposition coding, whose achievable rate region is the generalization of the region in (18). This strategy involves private messages only.

The rest of the section is devoted to the proof of Theorem 2. We divide the whole set of parameters depending on the strength of the cooperation link C compared to the direct link S_p and the interference link I_p . Fig. 4 shows the regimes of Theorem 2 for which we have an approximate capacity result (indicated as “Case A”, “Case B” and “Case C” as in Theorem 2). As it can be noted from Fig. 4, our capacity characterization to within a constant gap roughly excludes the weak interference regime. The case when $S_c > I_c$ and $C \geq \left(S_p + I_c + 2\sqrt{S_p I_c \frac{I_p}{1+I_p}}\right)(1+I_p)$ is a straightforward generalization of the condition in (19) for the symmetric case studied in Section III-G and shall therefore not be further discussed. We shall now discuss each case separately.

A. The case $C \leq S_p$: when unilateral cooperation may not be useful

We start our discussion with a simple observation. Under the condition $C \leq S_p$ we can further bound (4) as

$$\mathcal{O}^{\text{IV-A}} : R_c \leq \log(1+S_c), \quad (20a)$$

$$R_p \leq \log(1+S_p) + \log(2), \quad (20b)$$

$$R_p + R_c \leq \log^+\left(\frac{1+S_c}{1+I_c}\right) + \log(1+S_p+I_c) + \log(2), \quad (20c)$$

$$R_p + R_c \leq \log^+\left(\frac{1+S_p}{1+I_p}\right) + \log(1+S_c+I_p) + \log(4). \quad (20d)$$

The bounds in (20) are to within 1 bit of

$$\mathcal{I}^{\text{IV-A}} : R_c \leq \log(1+S_c), \quad (21a)$$

$$R_p \leq \log(1+S_p), \quad (21b)$$

$$R_p + R_c \leq \log(1+S_p+I_c) + \log^+\left(\frac{1+S_c}{1+I_c}\right), \quad (21c)$$

$$R_p + R_c \leq \log(1+S_c+I_p) + \log^+\left(\frac{1+S_p}{1+I_p}\right), \quad (21d)$$

which is achievable to within 1 bit for the noncooperative IC when the ‘ $R_1 + 2R_2, 2R_1 + R_2$ ’-type of bounds in [16, Theorem 3] are redundant²; with the notation adopted in this paper, one can easily show that these bounds are redundant if

$$S_c S_p \leq (1+I_p)(1+I_c). \quad (22)$$

Hence we can immediately conclude that the noncooperative scheme of [16] is optimal to within 2 bits in the regime

²By using the ‘worst noise covariance argument’ as in [12], one can show that the upper bound in [16, Theorem 3], which was derived for the noncooperative IC in weak interference, is actually valid for all channel parameters if one replaces $\log\left(\frac{1+\text{SNR}_i}{1+\text{INR}_j}\right)$ with $\log^+\left(\frac{1+\text{SNR}_i}{1+\text{INR}_j}\right)$, $i \neq j$, $i = 1, 2$. By using the notation of [16], the steps of the proof are as follows

$$\begin{aligned} n(R_1 + 2R_2 - 3\epsilon) &\leq I(X_1^n; Y_1^n) + I(X_2^n; Y_2^n) + I(X_2^n; Y_2^n) \\ &\leq I(X_1^n; Y_1^n, S_1^n) + I(X_2^n; Y_2^n, Y_1^n, X_1^n) \\ &\quad + I(X_1^n; X_2^n; Y_2^n) - I(X_1^n; Y_2^n | X_2^n) \\ &= \underbrace{I(X_1^n; Y_1^n, S_1^n) + I(X_2^n; Y_2^n | X_1^n) - I(X_1^n; Y_2^n | X_2^n)}_{=h(Y_1^n | S_1^n) - h(Z_1^n)} \\ &\quad + \underbrace{I(X_2^n; Y_2^n | X_1^n, Y_1^n)}_{\text{use worst noise covariance}} + I(X_1^n; X_2^n; Y_2^n). \end{aligned}$$

identified by (22) when the cooperation link gain satisfies $C \leq S_p$. Notice that the regime in (22), depicted in Fig. 4 on the left, includes the strong interference regime and most of the mixed interference regime; in other words, it roughly excludes the weak interference regime.

The capacity result that we just proved is the generalization of the symmetric capacity result of Theorem 1 in Regime 1 and part of Regime 3 of Fig. 2 (i.e., in the symmetric case the condition in (22) simplifies to $S \leq 1 + I$, which at high SNR corresponds to $1 \leq \alpha$, and the condition $C \leq S$ at high SNR corresponds to $\beta \leq 1$). As for Theorem 1 in the corresponding regime, a noncooperative scheme is approximately optimal.

When $S_c S_p > (1 + I_p)(1 + I_c)$ and $C \leq S_p$ (which in the symmetric case corresponds to $1 > \alpha$ and $\beta \leq 1$ and for which we could only show a sum-capacity result to within a constant gap in Theorem 1) we expect that, in order to show an approximate capacity result, upper bounds on $R_p + 2R_c$ and $2R_p + R_c$ must be derived.

B. The case $S_p < C \leq I_p$: when unilateral cooperation is useful

For $S_p < C \leq I_p$ we further bound (4) as

$$\mathcal{O}^{\text{IV-B}} : R_c \leq \log(1 + S_c), \quad (23a)$$

$$R_p \leq \log(1 + C) + \log(2), \quad (23b)$$

$$R_p + R_c \leq \log^+ \left(\frac{1 + S_c}{1 + I_c} \right) + \log(1 + S_p + I_c) + \log(2), \quad (23c)$$

$$R_p + R_c \leq \log(1 + S_c + I_p) + 2 \log(2). \quad (23d)$$

In this regime, unilateral cooperation helps increasing the rate of the primary user. In the symmetric case, the upper bound in (23) reduces to the part of Regime 2 and 3 of Fig. 2 for $1 < \beta \leq \alpha$; we therefore consider the generalization of the achievable scheme we used for Regime 2 of Fig. 2 to the case of general channel gains. Here PTx takes advantage of the strong cooperation link and sends its message with the help of the CTx. The sum-rate upper bound in (23d) suggests that CRx should decode the PTx's message in addition to its intended message, that is, PTx should use a (cooperative) common message only. The sum-rate upper bound in (23c), suggests that PRx should decode CTx's message only when $I_c > S_c$, that is, CTx should use both a (noncooperative) common and a (noncooperative) private message. This is exactly the strategy described in Appendix B-B and the resulting achievable region is given in (52), namely

$$\mathcal{I}^{\text{IV-B}} : R_c \leq \log(1 + S_c), \quad (24a)$$

$$R_p \leq \log(1 + C), \quad (24b)$$

$$R_p + R_c \leq \log(1 + S_p + I_c) + \log^+ \left(\frac{1 + S_c}{1 + I_c} \right), \quad (24c)$$

$$R_p + R_c \leq \log(1 + S_c + I_p). \quad (24d)$$

By comparing the upper bound in (23) with the achievable region in (24) we conclude that the capacity region is known to within 1 bit for a general GCCIC where the channel gains satisfy $S_p < C \leq I_p$. Notice that we did not impose any condition on the strength of I_c compared to S_c , i.e., in

other words this gap result holds regardless of whether the interference at PRx is strong ($I_c \geq S_c$) or weak ($I_c < S_c$).

C. The case $\max\{S_p, I_p\} < C$ and $S_c \leq I_c$: when unilateral cooperation is useful

For this case we further bound (4) as

$$\mathcal{O}^{\text{IV-C}} : R_c \leq \log(1 + S_c), \quad (26a)$$

$$R_p \leq \log(1 + C) + \log(2), \quad (26b)$$

$$R_p + R_c \leq \log(1 + S_p + I_c) + \log(2), \quad (26c)$$

$$R_p + R_c \leq \log \left(\frac{1 + 2C}{1 + I_p} \right) + \log(1 + S_c + I_p) + \log(2). \quad (26d)$$

In this regime, unilateral cooperation helps increasing both the rate of the primary user and the sum-capacity. In the symmetric case, the upper bound in (26) reduces to the part of Regime 2 and 3 of Fig. 2 for $1 < \alpha < \beta$. Here PTx takes advantage of the strong cooperation link and sends its message with the help of the CTx. The sum-rate upper bound in (26c) suggests that PRx should decode the CTx's message in addition to its intended message, that is, CTx should use a (noncooperative) common message only; this is so because the condition $S_c \leq I_c$ corresponds to strong interference at the PRx. The sum-rate upper bound in (26d), suggests that PTx should use both a (cooperative) common and a (cooperative) private message; this is so because here we do not specify which one among S_p and I_p is the largest, and therefore the interference at CRx could be either strong or weak. This is exactly the strategy described in Appendix C-E, which is based on superposition coding only (as the cognitive common message is not precoded against the interference of the primary private message); both the common and the private messages of PTx are cooperative; this scheme can be thought of as the extension of the scheme used in Section IV-B so as to include a private message for PTx in case the interference at CRx is weak.

The achievable region is given in (64). With the possible suboptimal choices $|\gamma_p|^2 = \frac{1}{1 + I_p}$, $|\gamma_c|^2 = \frac{1}{1 + S_c}$ inspired by [16], the achievable region in (64) becomes

$$\mathcal{I}^{\text{IV-C}} : R_c \leq \log \left(\frac{1 + \frac{I_p}{1 + I_p} + S_c}{1 + \frac{I_p}{1 + I_p} + \frac{S_c}{1 + S_c}} \right), \quad (27a)$$

$$R_p \leq \log(1 + C), \quad (27b)$$

$$R_p + R_c \leq \log(1 + S_p + I_c), \quad (27c)$$

$$R_p + R_c \leq \log \left(1 + \frac{C}{1 + I_p} \right) + \log \left(\frac{1 + S_c + I_p}{1 + \frac{I_p}{1 + I_p} + \frac{S_c}{1 + S_c}} \right), \quad (27d)$$

$$R_p + R_c \leq \log \left(1 + \frac{S_p}{1 + I_p} + \frac{I_c}{1 + S_c} \right) + \log \left(\frac{1 + S_c + I_p}{1 + \frac{I_p}{1 + I_p} + \frac{S_c}{1 + S_c}} \right), \quad (27e)$$

$$R_p + 2R_c \leq \log \left(1 + \frac{S_p}{1 + I_p} + I_c \right) + \log \left(\frac{1 + S_c + I_p}{1 + \frac{I_p}{1 + I_p} + \frac{S_c}{1 + S_c}} \right). \quad (27f)$$

$$\begin{aligned}
\text{eq.(26c)} - \text{eq.(27e)} &\leq \log(1 + S_p + I_c) + \log(2) - \log\left(1 + \frac{S_p}{1 + I_p} + \frac{I_c}{1 + S_c}\right) - \log\left(\frac{1 + S_c + I_p}{1 + \frac{I_p}{1 + I_p} + \frac{S_c}{1 + S_c}}\right) \\
&\leq \log(1 + S_p + I_c) - \log\left(1 + \frac{S_p + I_c}{1 + \max\{I_p, S_c\}}\right) - \log(1 + S_c + I_p) + \log(6) \\
&= \log\left(\frac{1 + S_p + I_c}{1 + \max\{I_p, S_c\} + S_p + I_c} \frac{1 + \max\{I_p, S_c\}}{1 + \max\{I_p, S_c\} + \min\{I_p, S_c\}}\right) + \log(6) \leq \log(6) \quad (25)
\end{aligned}$$

By comparing the upper bounds in (26) with the inner bounds in (27) it can be shown that they are at most

$$\begin{aligned}
\text{GAP} &\leq \max\left\{\log(3), \log(2), \frac{\log(2)}{2}, \frac{\log(12)}{2}, \frac{\log(6)}{2}\right\} \\
&= \frac{\log(12)}{2} \approx 1.8 \text{ bits},
\end{aligned}$$

bits away when the condition in (66) holds for the considered choice of parameters, namely

$$S_c \frac{1 + I_p + S_p}{1 + 2I_p} \leq I_c \quad (28)$$

so that the bound on $R_p + 2R_c$ in (27) can be dropped. Notice that the sum-rate bound in (26c) and the one in (27e) are the same up to a constant gap, which is given by (25) at the top of the page.

The condition in (28) is similar to the condition in (22), which we derived in order to claim that bounds of the form $R_p + 2R_c/2R_p + R_c$ were redundant in the noncooperative achievable region in the weak interference regime. In general, as can be noticed from the analysis so far, the weak interference regime is more challenging than the other regimes. In the next sections we concentrate on two special GCCIC where one of the interfering links is absent: the case where CRx does not experience interference (i.e., the so-called Z-channel for which $I_p = 0$), and the case where PRx does not experience interference (i.e., the so-called S-channel for which $I_c = 0$), for which we shall prove a constant gap result also in the weak interference regime. As we shall see, DPC-based schemes appear to be needed for approximate optimality in weak interference.

V. THE CAPACITY REGION TO WITHIN A CONSTANT GAP FOR THE Z-CHANNEL

Our main result for the Z-channel is as follows:

Theorem 3 *The capacity region of the Z-channel (i.e., the link $PTx \rightarrow CRx$ is non-existent) is characterized to within 2 bits as follows. When $C \leq S_p$, the capacity region of the Z-channel is achievable to within 2 bits using a noncooperative scheme based on superposition coding, whose achievable rate region is 1 bit away from the region in (31) below. When $C > S_p$ and $S_c \leq I_c$, the capacity region of the Z-channel is achievable to within 1 bit using a cooperative scheme based on superposition coding whose achievable rate region is given in (33) below. This scheme uses a private message for PTx and a common message for CTx. Finally, when $C > S_p$ and*

$S_c > I_c$, the capacity region of the Z-channel is achievable to within 1 bit using a cooperative scheme based on DPC and superposition coding whose achievable rate region is given in (35) below. This scheme uses only private messages for both PTx and CTx.

The rest of the section is devoted to the proof of Theorem 3, that is, the upper bound

$$R_c \leq \log(1 + S_c), \quad (29a)$$

$$R_p \leq \log\left(1 + \left(\sqrt{S_p} + \sqrt{I_c}\right)^2\right), \quad (29b)$$

$$R_p \leq \log(1 + C + S_p), \quad (29c)$$

$$R_p + R_c \leq \log^+\left(\frac{1 + S_c}{1 + I_c}\right) + \log\left(1 + \left(\sqrt{S_p} + \sqrt{I_c}\right)^2\right), \quad (29d)$$

from (4) by setting $I_p = 0$, can be achieved to within a constant gap. The region in (29) without the bound in (29c) (i.e., the only one that depends on C) is the capacity upper bound for the non-causal cognitive IC in [24, Theorem III.1], which unifies previously known outer bounds for the weak ($S_c > I_c$) and strong ($S_c \leq I_c$) interference regimes and is achievable to within 1 bit. Hence, we interpret the bound in (29c) as the ‘cost’ of causal cooperation on the Z-channel.

Moreover, as we shall see later in more details, the capacity region of the Z-channel, differently from that of the general GCCIC, does not have bounds of the type $2R_p + R_c/R_p + 2R_c$. This important feature allows for the characterization to within a constant gap of the whole capacity region for any value of the channel parameters.

For the proof of Theorem 3, we consider separately different parameter regimes. Given the result in Theorem 2, we only need to consider the case $I_c \leq S_c(1 + S_p)$ (since $S_c S_p - 1 < S_c(1 + S_p)$). In the symmetric case, the regime $I_c \leq S_c(1 + S_p)$ is equivalent to $I \leq S(1 + S)$, or $\alpha \leq 2$ at high SNR, that is, we need to focus on the case where the Z-channel does not exhibit very strong interference.

A. *Case $C \leq S_p$: when unilateral cooperation might not be useful*

For the case $C \leq S_p$ we further outer bound the capacity upper bound in (29) as

$$\mathcal{O}^{\text{V-A}} : R_c \leq \log(1 + S_c), \quad (30a)$$

$$R_p \leq \log(1 + S_p) + \log(2), \quad (30b)$$

$$R_p + R_c \leq \log^+\left(\frac{1 + S_c}{1 + I_c}\right) + \log(1 + S_p + I_c) + \log(2). \quad (30c)$$

The region in (30) is at most 1 bit away from

$$\mathcal{I}^{\text{V-A}} : R_c \leq \log(1 + S_c), \quad (31a)$$

$$R_p \leq \log(1 + S_p), \quad (31b)$$

$$R_p + R_c \leq \log^+ \left(\frac{1 + S_c}{1 + I_c} \right) + \log(1 + S_p + I_c), \quad (31c)$$

which is achievable to within 1 bit by a noncooperative scheme [16]. Therefore, for this set of parameters we have that the outer bound in (30) is achievable to within 2 bits.

The difference between the case $C \leq S_p$ for the Z-channel and the corresponding case for the general channel in Theorem 2 in Section IV-A is that here we do not need to impose the condition in (22) to claim the redundancy of the bounds on $R_p + 2R_c/2R_p + R_c$ in the noncooperative achievable region. This is so because those bounds do not matter, up to a constant gap of 1 bit, in the corresponding noncooperative IC [16].

B. Case $C > S_p$, $S_c \leq I_c$ (i.e., strong interference at PRx): when unilateral cooperation is useful

In this case, we further outer bound the region in (29) as

$$\mathcal{O}^{\text{V-B}} : R_c \leq \log(1 + S_c), \quad (32a)$$

$$R_p \leq \log(1 + C) + \log(2), \quad (32b)$$

$$R_p + R_c \leq \log(1 + S_p + I_c) + \log(2). \quad (32c)$$

In this regime, we use the same strategy employed for the general GCCIC in the same regime, i.e., for $C > S_p$ and $I_c \geq S_c$ in Fig. 4 Case C, by setting $I_p = 0$. Here PTx takes advantage of the strong cooperation link and sends its message with the help of the CTx. Moreover, since the PTx does not create interference at the CRx ($I_p = 0$), it sends a (cooperative) private message only. On the other hand, since the interference at the PRx is strong, the CTx sends a (noncooperative) common message only. This is exactly the strategy described in Appendix C-E and the resulting achievable region is given by (64) (this is the same achievable region we used in Section IV-C). In (64), we further set $I_p = 0$ and $|\gamma_p| = 1$ so that the PTx sends a private message only. With the possible suboptimal choice $|\gamma_c|^2 = \frac{1}{1+S_c}$, the achievable region in (64) becomes

$$\mathcal{I}^{\text{V-B}} : R_c \leq \log \left(\frac{1 + S_c}{1 + \frac{S_c}{1+S_c}} \right), \quad (33a)$$

$$R_p \leq \log(1 + C), \quad (33b)$$

$$R_p + R_c \leq \log(1 + S_p + I_c), \quad (33c)$$

$$R_p + R_c \leq \log \left(1 + S_p + \frac{I_c}{1 + S_c} \right) + \log \left(\frac{1 + S_c}{1 + \frac{S_c}{1+S_c}} \right). \quad (33d)$$

Notice that the bound on $R_p + 2R_c$ in (64f) is always redundant because of the condition in (65) since here we set $|\gamma_p| = 1$; this implies that the difference between this case for the Z-channel and the corresponding case for the general channel in Theorem 2 in Section IV-C is that here we do not need to impose the condition in (28) to claim the redundancy of the bound on $R_p + 2R_c$ in the achievable region.

It is not difficult to see that the outer bound in (32) and the inner bound in (33) are at most 1 bit away.

C. Case $C > S_p$, $S_c > I_c$ (i.e., weak interference at PRx): when unilateral cooperation is useful

For this case, an outer bound for the Z-channel is given by the capacity of the non-causal cognitive IC in weak interference in (5) together with the cut-set bound in (4b), i.e.,

$$\mathcal{O}^{\text{V-C}} : R_c \leq \log(1 + (1 - |\gamma_c|^2)S_c), \quad (34a)$$

$$R_p \leq \log \left(1 + \frac{S_p + |\gamma_c|^2 I_c + 2|\gamma_c| \sqrt{S_p I_c}}{1 + (1 - |\gamma_c|^2)I_c} \right), \quad (34b)$$

$$R_p \leq \log(1 + C) + \log(2), \quad (34c)$$

union over all $|\gamma_c| \leq 1$. Since $C > S_p$, PTx takes advantage of the strong cooperation link and sends its message with the help of the CTx. Moreover, since the PTx does not create interference at the CRx ($I_p = 0$), it sends a (cooperative) private message only. The outer bound in (34b) suggests that the PRx should treat as noise the message of the CTx, while the bound in (34a) tells us that the CRx should decode its own message without experiencing interference. In order to model this last observation, we use a DPC-based scheme. In this strategy the CTx precodes its message against the ‘known interference’ so that the CRx decodes its own message as if the interference was not present [8]. This is exactly the strategy described in Appendix C-C and the resulting achievable region is given by (62) with $I_p = 0$. We further set $|\gamma_p| = 0$ in (62) and we obtain

$$\mathcal{I}^{\text{V-C}} : R_c \leq \log(1 + (1 - |\gamma_c|^2)S_c), \quad (35a)$$

$$R_p \leq \log \left(1 + \frac{S_p + |\gamma_c|^2 I_c}{1 + (1 - |\gamma_c|^2)I_c} \right), \quad (35b)$$

$$R_p \leq \log(1 + C), \quad (35c)$$

for all $|\gamma_c| \leq 1$. By simple computations, the achievable region in (35) can be shown to be at most 1 bit away from the upper bound in (34).

Note that here we used a DPC-based scheme in order to determine the capacity to within a constant gap in weak interference, while in Section IV-C for the general GCCIC we only used superposition coding.

D. Comparisons

We conclude this section by comparing the performance of unilateral cooperation on the Z-channel with other forms of cooperation. Moreover, we also consider whether the absence of an interfering link is beneficial in the GCCIC. We shall use as performance metric the gDoF, or high SNR throughput. In order to reduce the number of parameters, we restrict our attention to the case where the direct links have the same strength. For future reference, the gDoF of the noncooperative Z-channel is given by [29]

$$d^{\text{IC-Z}} = \min\{\max\{1 - \alpha/2, \alpha/2\}, 1\}$$

and that of the non-causal cognitive Z-channel, which can be evaluated from [24], is

$$d^{\text{CIC-Z}} = \max\{1 - \alpha/2, \alpha/2\}.$$

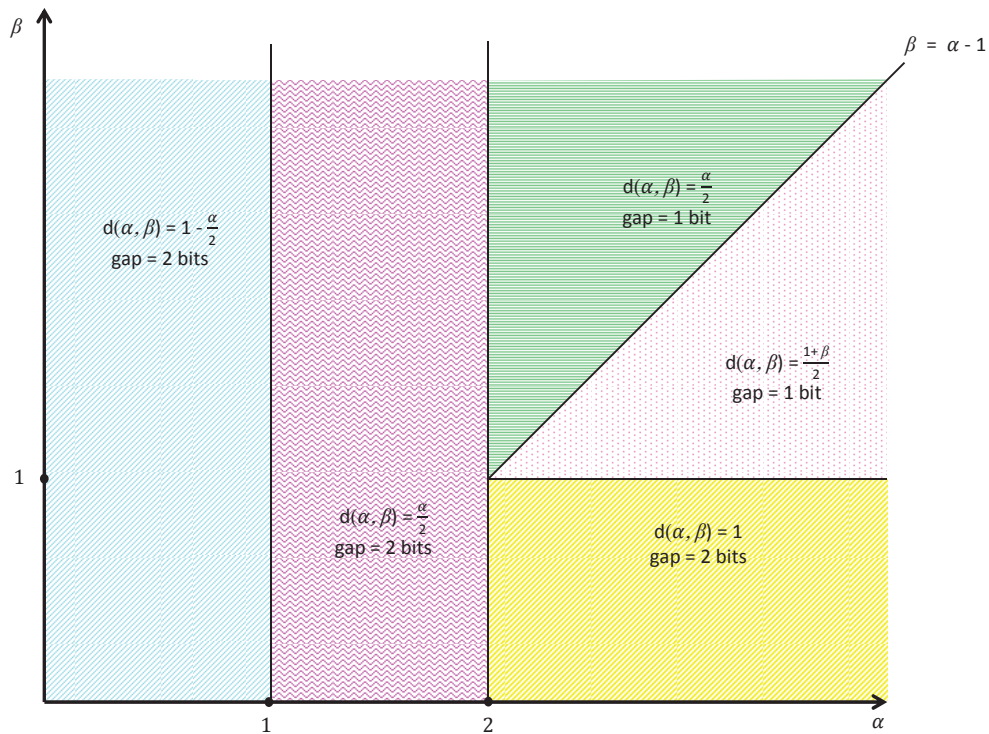


Fig. 5: Optimal gDoF and constant gap for the Z-channel in the different regimes of (α, β) .

Fig. 5 shows the gDoF and the gap for the Z-channel for different regions in the (α, β) plane. The whole set of parameters has been partitioned into multiple sub-regions depending upon different level of cooperation (β) and interference (α) strengths.

When comparing unilateral cooperation with other channel models in terms of gDoF we observe:

- For the noncooperative IC, it is well known that removing an interference link cannot degrade the performance and the sum-capacity is known exactly for all channel parameters [29]. The same cannot be said in full generality for the cooperative channel because “useful cooperative information” can flow through the interference link. Thus for the Z-channel, cooperation only improves the gDoF with respect to the noncooperative case in the regime $\alpha \geq 2$ and $\beta \geq 1$, i.e., in very strong interference and strong cooperation (the gDoF achieved with and without cooperation is the same in the green and yellow regions in Fig. 6).
- For the Z-channel, unilateral cooperation attains the gDoF of the classical relay channel when $1 \leq \alpha \leq \beta$, as shown by the red and yellow-shaded regions in Fig. 6.
- The Z-channel achieves the same gDoF of the non-causal cognitive channel everywhere except in $\alpha > \max\{2, \beta + 1\}$ (region with horizontal lines in Fig. 6).
- The gDoF of unilateral cooperation equals the gDoF upper bound of bilateral cooperation [11] when $\beta \leq \max\{1, \alpha\}$ (region with vertical lines in Fig. 6) that corresponds to the case where the cooperation link is weaker than the best between the direct link and the

interference link. In this case bilateral cooperation might not be worth implementing in practice. Notice that here we compare the (provably achievable) gDoF for the case of unilateral cooperation to an upper bound for bilateral cooperation. To the best of our knowledge, it has not been shown that the gDoF upper bound for the Z-channel with bilateral source cooperation is achievable, which we expect to be.

- By comparing Fig. 2 and Fig. 5 we observe that the gDoF of the Z-channel is always greater or equal than that of the interference-symmetric GCCIC. This is due to the fact that the PTx does not cooperate in sending the cognitive signal. Therefore by removing the link between PTx and CRx we rid CRx of only an interfering signal and this leads to an improvement in gDoF.

The regimes where the Z-channel strictly outperforms the interference-symmetric GCCIC are when $0 \leq \alpha \leq \frac{2}{3}$ and $\beta \leq \min\{\alpha, 1 - \alpha\}$ (region with vertical lines in Fig. 9), i.e., weak interference and fairly weak cooperation. This regime can be thought of as the one where interference is the most harmful for the interference-symmetric GCCIC.

VI. THE CAPACITY REGION TO WITHIN A CONSTANT GAP FOR THE S-CHANNEL

Our main result for the S-channel is as follows:

Theorem 4 *The capacity region of the S-channel (i.e., the link $CTx \rightarrow PRx$ is non-existent) is characterized to within 2 bits as follows. When $C \leq \max\{I_p, S_p\}$, the capacity region of the S-channel is achievable to within 2 bits using a noncooperative*

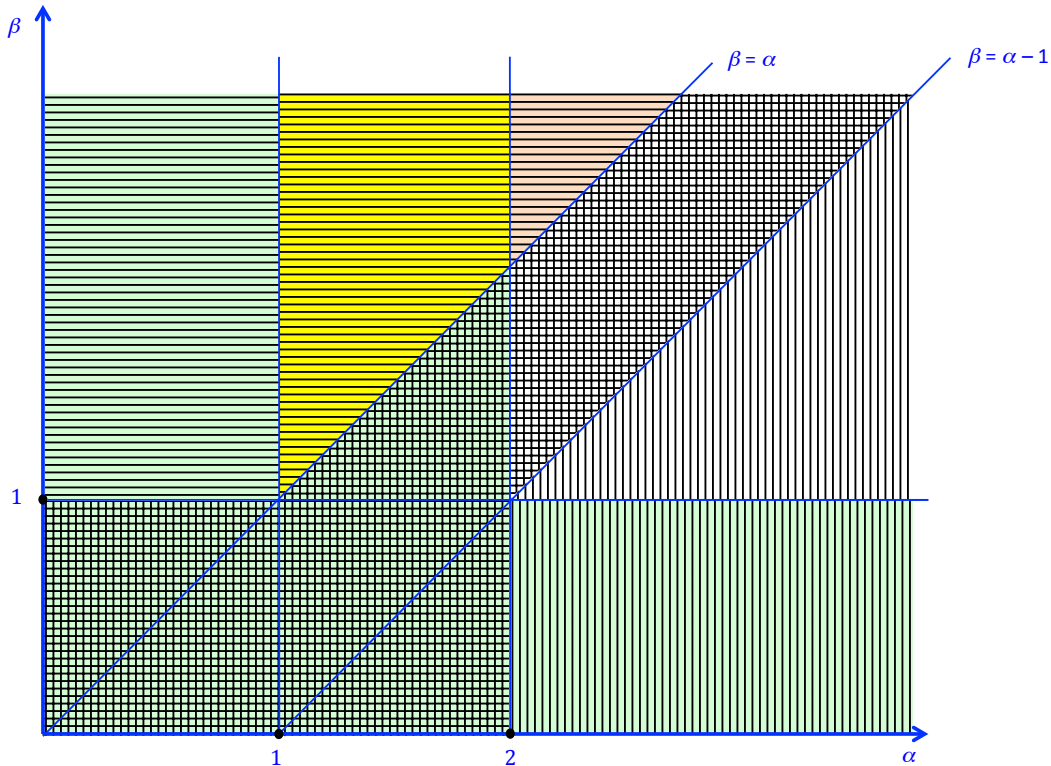


Fig. 6: Regions in which the gDoF of the Z-channel is equal to that of the noncooperative IC (green and yellow regions), of the relay channel (red and yellow regions), of the non-causal cognitive IC (region with horizontal lines), and of bilateral source cooperation (region with vertical lines). Note that the different regions can overlap.

scheme based on superposition coding, whose achievable rate region is 1 bit away from the region in (38) below. When $C > \max\{I_p, S_p\}$, the capacity region of the S-channel is achievable to within 2 bits using a cooperative scheme based on DPC and superposition coding. In particular, if $C \geq \Delta_{th}$ with $\Delta_{th} = \min\{I_p, S_p\}(1 + \max\{I_p, S_p\})$, the proposed scheme uses only private messages for both PTx and CTx and the achievable rate region is given in (40) below, while if $C < \Delta_{th}$ the scheme uses common and private messages for PTx and the achievable rate region is given in (42) below.

As for the Z-channel, also the capacity region of the S-channel, differently from that of the general GCCIC, does not have bounds of the type $2R_p + R_c/R_p + 2R_c$. This feature allows for the characterization to within a constant gap of the whole capacity region for any value of the channel parameters.

The rest of the section is devoted to the proof of Theorem 4. We distinguish two cases, depending on whether the following upper bound

$$R_c \leq \log(1 + S_c), \quad (36a)$$

$$R_p \leq \log(1 + S_p), \quad (36b)$$

$$R_p + R_c \leq \log\left(1 + (\sqrt{S_c} + \sqrt{I_p})^2\right) + \log\left(\frac{1 + C + \max\{I_p, S_p\}}{1 + I_p}\right), \quad (36c)$$

from (4) with $I_c = 0$, can be achieved with a noncooperative scheme or not. Note that the bounds on R_p and R_c in (36) are

the capacity region of the corresponding non-causal cognitive IC; therefore we interpret the sum-rate bound in (36) as the ‘cost’ for causally learning the primary message at the CTx through a noisy channel.

For the proof of Theorem 4, we consider separately different parameter regimes. Given the result in Theorem 2, we should only consider the case $I_p \leq S_c S_p - 1$ when $C \leq S_p$, and $I_p \leq C$ when $C > S_p$. However, here we will use a DPC-based scheme for the case $\max\{S_p, I_p\} < C$ for which we only used superposition coding in Section IV-C.

A. Case $C \leq \max\{I_p, S_p\}$: when unilateral cooperation might not be useful

For the case $C \leq \max\{I_p, S_p\}$ we can further bound (36) as

$$\mathcal{O}^{\text{VI-A}}: R_c \leq \log(1 + S_c), \quad (37a)$$

$$R_p \leq \log(1 + S_p), \quad (37b)$$

$$R_p + R_c \leq \log(1 + S_c + I_p) + \log^+\left(\frac{1 + S_p}{1 + I_p}\right) + 2\log(2). \quad (37c)$$

The region in (37) is at most 1 bit away from

$$\mathcal{I}^{\text{VI-A}}: R_c \leq \log(1 + S_c), \quad (38a)$$

$$R_p \leq \log(1 + S_p), \quad (38b)$$

$$R_p + R_c \leq \log(1 + S_c + I_p) + \log^+\left(\frac{1 + S_p}{1 + I_p}\right), \quad (38c)$$

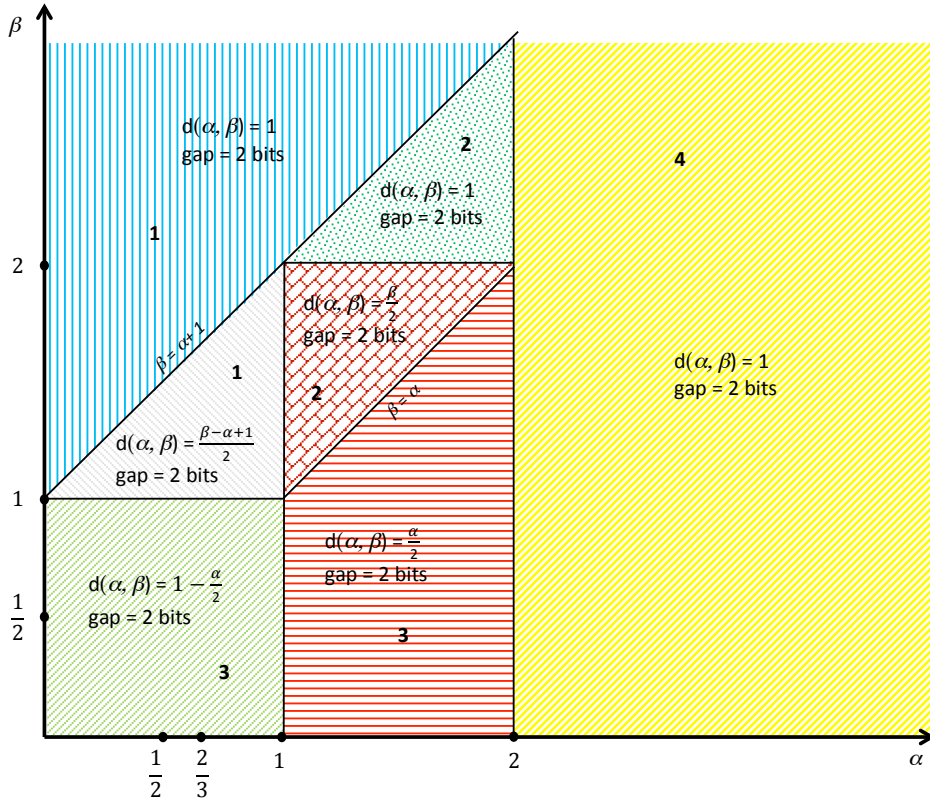


Fig. 7: Optimal gDoF and constant gap for the S-channel in the different regimes of (α, β) .

which is achievable to within 1 bit by a noncooperative scheme [16]. Therefore we conclude that for $C \leq \max\{I_p, S_p\}$ a noncooperative scheme is optimal to within 2 bits.

As for the Z-channel, the difference between this case and the corresponding case for the general GCCIC in Theorem 2 is that here we do not need to impose extra conditions to claim the redundancy of the bounds on $R_p + 2R_c/2R_p + R_c$ in the noncooperative achievable region since those bounds do not matter, up to a constant gap, in the noncooperative IC [16].

B. Case $C > \max\{I_p, S_p\}$: when unilateral cooperation is useful

When $C > \max\{I_p, S_p\}$, a sufficient condition for the sum-rate upper bound in (36) to be redundant is that

$$1 + S_p \leq \frac{1 + C + \max\{I_p, S_p\}}{1 + I_p} \Leftrightarrow C \geq \min\{I_p, S_p\}(1 + \max\{I_p, S_p\}). \quad (39)$$

For the set of parameters in (39), we use the achievable region in (62) from Appendix C-C, adapted to the S-channel case by setting $I_c = 0$, and with $|\gamma_c| = 0$, $C(1 - |\gamma_p|^2) = S_p$, to obtain the following achievable region

$$\mathcal{I}^{\text{VI-B}} : R_c \leq \log\left(1 + \frac{S_c}{1 + \frac{S_p I_p}{C}}\right), \quad (40a)$$

$$R_p \leq \log(1 + S_p). \quad (40b)$$

By comparing the rate bounds in (40) with those in (36), we see that when (39) holds the gap is at most 1 bit since

$$\begin{aligned} \log(1 + S_c) - \log\left(1 + \frac{S_c}{1 + \frac{S_p I_p}{C}}\right) &\leq \log\left(1 + \frac{S_p I_p}{C}\right) \\ &\leq \log\left(1 + \frac{\min\{I_p, S_p\} \max\{I_p, S_p\}}{\min\{I_p, S_p\}(1 + \max\{I_p, S_p\})}\right) \leq \log(2). \end{aligned}$$

This shows that, when the condition in (39) holds, not only the upper bound is achievable to within 1 bit but we can also achieve to within 1 bit the ultimate capacity of the corresponding non-causal cognitive channel. This result agrees with the intuition that, as the strength of the cooperation link increases, the performance of the causal cognitive channel should approach that of the corresponding non-causal model. The condition in (39) can thus be interpreted as a sufficient condition on the strength of the cooperation link to achieve the capacity region of the corresponding non-causal model to within a constant gap.

We are now left with the case

$$\left\{ \max\{I_p, S_p\} < C, C < \min\{I_p, S_p\}(1 + \max\{I_p, S_p\}) \right\} \subseteq \{S_p < C < S_p(1 + I_p)\}. \quad (41)$$

In the regime $S_p < C < S_p(1 + I_p)$ we use the DPC-based achievable scheme in Appendix C-F. In this scheme CTx sends a private message only since X_c is not received at PRx; PTx sends a private and a common message, both with the help of CTx. The PTx's common message is forwarded by CTx

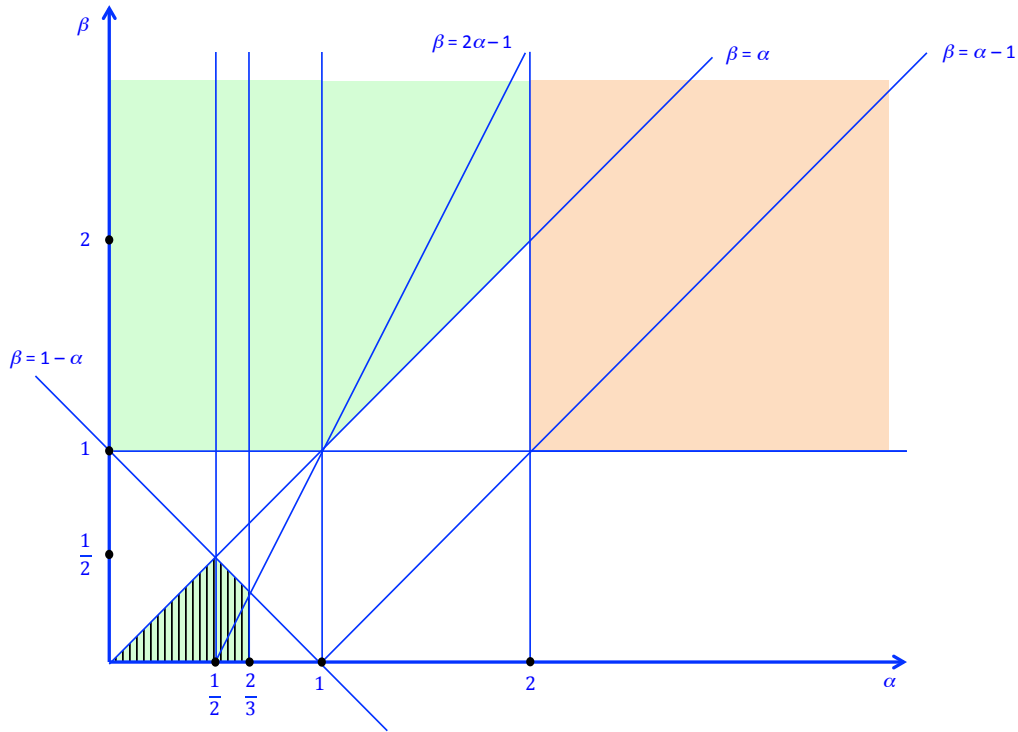


Fig. 9: Regions in which the S-channel outperforms the symmetric GCCIC (green region), the symmetric GCCIC outperforms the S-channel (red region), the Z-channel outperforms the symmetric GCCIC (region with vertical lines). Note that the different regions can overlap.

- The S-channel achieves the same gDoF of the non-causal cognitive IC everywhere except in $\alpha \leq 2$ and $\beta \leq \min\{2, \alpha + 1\}$ (region with horizontal lines in Fig. 8).
- The gDoF of unilateral cooperation equals the gDoF upper bound of bilateral cooperation when $\alpha \geq 2$ and $\beta \leq 1$ or when $\alpha \leq 2$ and $\beta \leq \min\{2, \alpha + 1\}$ (region with vertical lines in Fig. 8).
- The S-channel outperforms the interference-symmetric GCCIC when either $0 \leq \alpha \leq \frac{2}{3}$ and $\beta \leq \min\{\alpha, 1 - \alpha\}$ or when $\alpha \leq 2$ and $\beta \geq \max\{1, \alpha\}$ (green region in Fig. 9). On the other hand, the interference-symmetric GCCIC outperforms the S-channel in very strong interference and strong cooperation, i.e., $\alpha \geq 2$ and $\beta \geq 1$. This is due to the fact that the information for the PRx can no longer be routed through the CTx since $\sqrt{I_c}e^{j\theta_c} = 0$ (red region in Fig. 9).

VII. CONCLUSIONS

In this work we considered the CCIC, a network with two source-destination pairs sharing the same channel. In contrast to the noncooperative IC, in the CCIC the CTx exploits information about the PTx from its own channel observations. This scenario represents a more practically relevant model for cognitive radio than the non-causal cognitive IC, where the CTx is assumed to have a priori knowledge of the PTx's message. In particular, we believe that it is applicable in some practical heterogeneous deployments for 4G cellular networks.

We proposed achievable schemes that match known outer bounds to within a constant gap if, roughly speaking, the chan-

nel does not exhibit weak interference at both destinations. We characterized the capacity region to within a constant gap for the case where one interfering link is absent, which includes cases of weak interference. From our analysis a practical guideline for system design is that superposition coding is approximately optimal when the interference at the primary receiver is strong and that binning / dirty paper coding is approximately optimal when the interference at the primary receiver is weak. We identified the set of parameters where causal cooperation achieves the same gDoF of the noncooperative IC and of the relay channel. We also highlighted under which channel conditions the gDoF achieved with bilateral source cooperation and with non-causal cognition equals that achieved with only unilateral causal cooperation.

APPENDIX A CAPACITY REGION UPPER BOUND AND GDOF UPPER BOUND

In this work we use known outer bounds from [9], [12], [11]. These bounds were developed for the case of bilateral source cooperation. Here we adapt them to the case of unilateral source cooperation.

A. Cut-set upper bounds

The cut-set upper bound for a relay channel with gain S on the link from the source to the destination, gain C on the link from the source to the relay, and gain I on the link from the

relay to the destination is upper bounded by [25]

$$\begin{aligned} & \max_{|\rho| \leq 1} \min \left\{ \log \left(1 + S + I + 2|\rho|\sqrt{SI} \right), \right. \\ & \quad \left. \log \left(1 + (1 - |\rho|^2) (C + S) \right) \right\} \\ & \leq \min \left\{ \log \left(1 + (\sqrt{S} + \sqrt{I})^2 \right), \log (1 + C + S) \right\} \\ & =: r^{(\text{RC})}(S, I, C). \end{aligned} \quad (43)$$

The behavior of the rate $r^{(\text{RC})}(S, I, C)$ in (43) at high SNR, with $I = S^\alpha$, $C = S^\beta$, is given by (6d).

For an IC with cooperative sources, the rate of a given source cannot be larger than the rate that this source can achieve when the other source acts as a pure relay. Therefore, for the GCCIC, we have

$$R_p \leq r^{(\text{RC})}(S_p, I_c, C) \quad (44)$$

$$R_c \leq r^{(\text{RC})}(S_c, I_p, 0) \quad (45)$$

which are the upper bounds on the individual rates in (4a) and (4b), which imply the sum-rate upper bound in (4d).

B. Sum-rate bounds from [12]

From [12] we have

$$\begin{aligned} R_p + R_c & \leq \max_{|\rho| \leq 1} \log \left(\frac{1 + (1 - |\rho|^2) (C + \max\{I_p, S_p\})}{1 + (1 - |\rho|^2) I_p} \right) \\ & \quad + \log \left(1 + I_p + S_c + 2|\rho|\sqrt{S_c I_p} \right) \\ & \leq \log \left(\frac{1 + C + \max\{I_p, S_p\}}{1 + I_p} \right) \\ & \quad + \log \left(1 + (\sqrt{I_p} + \sqrt{S_c})^2 \right). \end{aligned}$$

By swapping the role of the users, we obtain a similar sum-rate upper bound, and the combination of the two gives the sum-rate upper bound in (4e).

The function

$$\begin{aligned} r^{(\text{DT})}(S, I, C) & := \log \left(\frac{1 + C + \max\{S, I\}}{1 + I} \right) \\ & \quad + \log \left(1 + (\sqrt{I} + \sqrt{S})^2 \right) \end{aligned}$$

with $I = S^\alpha$, $C = S^\beta$, has the high SNR behavior given by (6e).

C. Sum-rate bound from [11]

From [11] we have the sum-rate upper bound reported in (4f), whose behavior at high SNR, with the parameterization in (2), gives (6f).

APPENDIX B

ACHIEVABLE SCHEMES BASED ON SUPERPOSITION CODING ONLY

A. Superposition-only Achievable Scheme

We specialize the ‘superposition only’ achievable scheme in [10, Theorem IV.1] to the case of unilateral cooperation. In [10, Theorem IV.1], the network comprises four nodes numbered from 1 to 4; nodes 1 and 2 are sources and nodes 3

and 4 destinations; source node $j \in [1 : 2]$, with input to the channel X_j and output from the channel Y_j , has a message W_j for node $j + 2$; destination node $j \in [3 : 4]$ has channel output Y_j from which it decodes the message W_{j-2} .

Both users do rate splitting, where only the common message of user 1 is cooperative, while all other messages are noncooperative. We set $Q = V_2$, $Y_1 = \emptyset$, $T_2 = X_2$, $U_1 = \emptyset$, $T_1 = X_1$ in [10, Theorem IV.1], i.e., then $R_1 = R_{11n} + R_{10c}$, $R_2 = R_{22n} + R_{20n}$, to obtain a scheme that comprises: a cooperative common message (carried by the pair (Q, V_1) at rate R_{10c}) for user 1, a noncooperative private message (carried by X_1 at rate R_{11n}) for user 1, a noncooperative common message (carried by U_2 at rate R_{20n}) for user 2 and a noncooperative private message (carried by X_2 at rate R_{22n}) for user 2. Here Q carries the ‘past cooperative common message’, and V_1 the ‘new cooperative common message’ in a block Markov encoding scheme.

The set of possible input distributions is

$$P_{Q, V_1, X_1, U_2, X_2} = P_Q P_{V_1, X_1 | Q} P_{U_2, X_2 | Q}. \quad (46)$$

A schematic representation of the achievable scheme is given in Fig. 10, where an arrow indicates superposition coding.

Regarding encoding. Source 2 cooperates with source 1 by using decode-and-forward in a block Markov coding scheme. In a given slot the old cooperative common message of source 1 is carried by Q , to which the new cooperative common message of source 1 is superposed and carried by V_1 , to which the noncooperative private message of source 1 is superposed and carried by X_1 . After source 2 decodes the new cooperative common message of source 1 carried by V_1 , with knowledge of Q and by treating the noncooperative private message of source 1 in X_1 as noise, it superposes its noncooperative common message carried by U_2 to the old cooperative common message of source 1 carried by Q , and then it superposes its noncooperative private message carried by X_2 . In this scheme the common messages are jointly (backward) decoded at all destinations while treating the non-intended private message as noise.

Regarding decoding. There are three decoding nodes in the network and therefore three groups of rate constraints. These are:

- Node 2/CTx decodes V_1 from its channel output with knowledge of (Q, U_2, X_2) . Successful decoding is possible if (see [10, eq(6a)])

$$R_{10c} \leq I(Y_2; V_1 | U_2, X_2, Q). \quad (47a)$$

- Node 3/PRx jointly decodes (Q, V_1, X_1, U_2) from its channel output, with knowledge of some message indices in V_1 , by treating X_2 as noise. Successful decoding is possible if (see [10, eq(6b)-(6f)])

$$R_{10c} + R_{20n} + R_{11n} \leq I(Y_3; Q, V_1, X_1, U_2) \quad (47b)$$

$$R_{20n} + R_{11n} \leq I(Y_3; X_1, U_2 | Q, V_1) \quad (47c)$$

$$R_{11n} \leq I(Y_3; X_1 | Q, V_1, U_2). \quad (47d)$$

- Node 4/CRx jointly decodes (Q, V_1, U_2, X_2) from its channel output, with knowledge of some message index

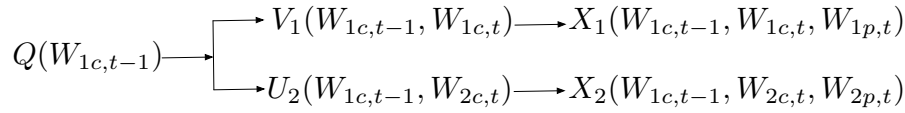


Fig. 10: Achievable scheme based on superposition coding.

in V_1 , by treating X_1 as noise. Successful decoding is possible if (see [10, eq(7b)-(7f)])

$$R_{10c} + R_{20n} + R_{22n} \leq I(Y_4; Q, V_1, X_2, U_2) \quad (47e)$$

$$R_{20n} + R_{22n} \leq I(Y_4; X_2, U_2 | Q, V_1) \quad (47f)$$

$$R_{22n} \leq I(Y_4; X_2 | Q, V_1, U_2). \quad (47g)$$

The achievable region, after Fourier-Motzkin elimination, is given by [10, Theorem IV.1]

$$R_1 \leq \text{eq}(47b) \quad (48a)$$

$$R_1 \leq \text{eq}(47a) + \text{eq}(47d) \quad (48b)$$

$$R_2 \leq \text{eq}(47f) \quad (48c)$$

$$R_1 + R_2 \leq \text{eq}(47b) + \text{eq}(47g) \quad (48d)$$

$$R_1 + R_2 \leq \text{eq}(47e) + \text{eq}(47d) \quad (48e)$$

$$R_1 + R_2 \leq \text{eq}(47a) + \text{eq}(47c) + \text{eq}(47g) \quad (48f)$$

$$R_1 + 2R_2 \leq \text{eq}(47c) + \text{eq}(47g) + \text{eq}(47e) \quad (48g)$$

for all distributions that factor as (46).

Remark 1. The rate bound in (48g) is redundant if

$$\min\{\text{eq}(48d), \text{eq}(48e), \text{eq}(48f)\} + \text{eq}(48c) \leq \text{eq}(48g)$$

that is, if for the considered input distribution we have

either $\text{eq}(47b) + \text{eq}(47f) \leq \text{eq}(47c) + \text{eq}(47e)$

$$\iff I(Y_3; Q, V_1) \leq I(Y_4; Q, V_1), \quad (49a)$$

or $\text{eq}(47d) + \text{eq}(47f) \leq \text{eq}(47c) + \text{eq}(47g)$

$$\iff I(Y_4; U_2 | Q, V_1) \leq I(Y_3; U_2 | Q, V_1), \quad (49b)$$

or $\text{eq}(47a) + \text{eq}(47f) \leq \text{eq}(47e)$

$$\iff I(Y_2; V_1 | U_2, X_2, Q) \leq I(Y_4; Q, V_1). \quad (49c)$$

Remark 2. If the private message of user 1 carried by X_1 is also decoded at Node 2 (a strategy that could be leading to a larger region than the one in (48) when the link between PTx and CTx is very large), then successful decoding at the cooperating source is possible if

$$R_1 = R_{10c} + R_{11n} \leq I(Y_2; V_1, X_1 | U_2, X_2, Q), \quad (50a)$$

$$R_{11n} \leq I(Y_2; X_1 | V_1, U_2, X_2, Q). \quad (50b)$$

If we now do Fourier-Motzkin elimination of the region in (47), by replacing the constraint in (47a) with those in (50), we obtain a new achievable region where the bounds that depend on (47a) in (48) change as follows: the bound in (48b) is replaced by (50a), and the one in (48f) by $R_1 + R_2 \leq \text{eq}(47e) + \text{eq}(50b)$. In Appendix C we shall further improve on this scheme by using DPC to cancel the ‘known interference’ due to the private message decoded at the cooperating source.

B. Achievable Scheme 1: message 1 is common, and message 2 is split

By identifying Node1 with the PTx (i.e., $X_p = X_1$), Node2 with the CTx (i.e., $X_c = X_2, Y_f = Y_2$), Node3 with the PRx (i.e., $Y_p = Y_3$) and Node4 with the CRx (i.e., $Y_c = Y_4$), by further setting $Q = \emptyset, V_1 = X_1$ (i.e., $R_{11n} = 0, R_1 = R_{10c}$) in the scheme in (48) in Appendix B-A, the following region is achievable

$$R_p \leq I(Y_f; X_p | U_2, X_c) \quad (51a)$$

$$R_c \leq I(Y_c; U_2, X_c | X_p) \quad (51b)$$

$$R_p + R_c \leq I(Y_p; U_2, X_p) + I(Y_c; X_c | U_2, X_p) \quad (51c)$$

$$R_p + R_c \leq I(Y_c; X_p, U_2, X_c) \quad (51d)$$

for all input distributions that factor as $P_{X_p, U_2, X_c} = P_{X_p} P_{X_c, U_2}$.

In Gaussian noise, we choose X_p, U_2, L_2 to be i.i.d. $\mathcal{N}(0, 1)$, and $X_c = \gamma_c U_2 + \sqrt{1 - |\gamma_c|^2} L_2$ for $|\gamma_c| \leq 1$. With this choice of inputs, the channel outputs are

$$Y_f = \sqrt{C} X_p + Z_f$$

$$Y_p = \sqrt{S_p} X_p + \sqrt{I_c} e^{j\theta_c} \left(\gamma_c U_2 + \sqrt{1 - |\gamma_c|^2} L_2 \right) + Z_p$$

$$Y_c = \sqrt{S_c} \left(\gamma_c U_2 + \sqrt{1 - |\gamma_c|^2} L_2 \right) + \sqrt{I_p} e^{j\theta_p} X_p + Z_c$$

and the achievable region in (51) reduces to

$$R_p \leq \log(1 + C)$$

$$R_c \leq \log(1 + S_c)$$

$$R_p + R_c \leq \log(1 + S_p + I_c) + \log \left(\frac{1 + (1 - |\gamma_c|^2) S_c}{1 + (1 - |\gamma_c|^2) I_c} \right)$$

$$R_p + R_c \leq \log(1 + S_c + I_p)$$

for all $|\gamma_c| \leq 1$. If $S_c \leq I_c$ we choose $|\gamma_c| = 1$ otherwise $|\gamma_c| = 0$ to obtain

$$R_p \leq \log(1 + C) \quad (52a)$$

$$R_c \leq \log(1 + S_c) \quad (52b)$$

$$R_p + R_c \leq \log(1 + S_p + I_c) + \log^+ \left(\frac{1 + S_c}{1 + I_c} \right) \quad (52c)$$

$$R_p + R_c \leq \log(1 + S_c + I_p). \quad (52d)$$

C. Achievable Scheme 2: both messages are split

For the GCCIC we identify Node1 with the PTx (i.e., $X_p = X_1$), Node2 with the CTx (i.e., $X_c = X_2, Y_f = Y_2$), Node3 with the PRx (i.e., $Y_p = Y_3$) and Node4 with the CRx (i.e., $Y_c = Y_4$) in the scheme in (48) in Appendix B-A.

In Gaussian noise, in order to comply with (46), we choose $Q = \emptyset$, V_1, L_1, U_2, L_2 i.i.d. $\mathcal{N}(0, 1)$ and we let

$$X_c = \gamma_c U_2 + \sqrt{1 - |\gamma_c|^2} L_2 : |\gamma_c|^2 \leq 1$$

$$X_p = \gamma_p V_1 + \sqrt{1 - |\gamma_p|^2} L_1 : |\gamma_p|^2 \leq 1.$$

With this choice of inputs the channel outputs are given by

$$\begin{aligned} Y_f &= \sqrt{C} \left(\gamma_p V_1 + \sqrt{1 - |\gamma_p|^2} L_1 \right) + Z_f \\ Y_p &= \sqrt{S_p} \left(\gamma_p V_1 + \sqrt{1 - |\gamma_p|^2} L_1 \right) \\ &\quad + \sqrt{l_c} e^{j\theta_c} \left(\gamma_c U_2 + \sqrt{1 - |\gamma_c|^2} L_2 \right) + Z_p \\ Y_c &= \sqrt{S_c} \left(\gamma_c U_2 + \sqrt{1 - |\gamma_c|^2} L_2 \right) \\ &\quad + \sqrt{l_p} e^{j\theta_p} \left(\gamma_p V_1 + \sqrt{1 - |\gamma_p|^2} L_1 \right) + Z_c. \end{aligned}$$

Inspired by [16] for the noncooperative IC in weak interference, we set $(1 - |\gamma_c|^2)l_c = (1 - |\gamma_p|^2)l_p = 1$ (here we assume $1 \leq \min\{l_p, l_c\}$) so that the scheme in (48) in Appendix B-A results in the following achievable region

$$R_p \leq \log \left(\frac{1 + S_p + l_c}{2} \right) \quad (53a)$$

$$R_p \leq \log \left(\frac{1 + C}{1 + C/l_p} \right) + \log \left(1 + \frac{S_p/l_p}{2} \right) \quad (53b)$$

$$R_c \leq \log \left(1 + \frac{S_c}{2} \right) \quad (53c)$$

$$R_p + R_c \leq \log \left(\frac{1 + S_p + l_c}{2} \right) + \log \left(1 + \frac{S_c/l_c}{2} \right) \quad (53d)$$

$$R_p + R_c \leq \log \left(\frac{1 + S_c + l_p}{2} \right) + \log \left(1 + \frac{S_p/l_p}{2} \right) \quad (53e)$$

$$\begin{aligned} R_p + R_c &\leq \log \left(\frac{1 + C}{1 + C/l_p} \right) + \log \left(\frac{1 + l_c + S_p/l_p}{2} \right) \\ &\quad + \log \left(1 + \frac{S_c/l_c}{2} \right) \quad (53f) \end{aligned}$$

$$\begin{aligned} R_p + 2R_c &\leq \log \left(\frac{1 + l_c + S_p/l_p}{2} \right) + \log \left(1 + \frac{S_c/l_c}{2} \right) \\ &\quad + \log \left(\frac{1 + S_c + l_p}{2} \right). \quad (53g) \end{aligned}$$

Note that the sum-rate in (53d) and the first upper bound in (4e) differ by at most 3 bits, and the sum-rate in (53e) and the second upper bound in (4e) by at most 4 bits when $C \leq \max\{S_p, l_p\}$.

For the symmetric case, i.e., $S_c = S_p = S$, $l_c = l_p = l$, the following sum-rate is achievable from (53)

$$R_p + R_c \leq \max \min \left\{ \begin{aligned} &\min\{\text{eq}(48a), \text{eq}(48b)\} + \text{eq}(48c), \\ &\text{eq}(48d), \text{eq}(48e), \text{eq}(48f), \\ &\frac{\min\{\text{eq}(48a), \text{eq}(48b)\} + \text{eq}(48g)}{2} \end{aligned} \right\} \quad (54a)$$

$$\min\{\text{eq}(48a), \text{eq}(48b)\} + \text{eq}(48c), \quad (54b)$$

$$\frac{\min\{\text{eq}(48a), \text{eq}(48b)\} + \text{eq}(48g)}{2} \quad (54c)$$

with

$$\text{eq}(48a) = \text{eq}(53a) = \log \left(\frac{S + l + 1}{2} \right)$$

$$\text{eq}(48b) = \text{eq}(53b) = \log \left(\frac{1 + C}{1 + \frac{C}{l}} \right) + \log \left(1 + \frac{S}{2l} \right)$$

$$\text{eq}(48c) = \text{eq}(53c) = \log \left(1 + \frac{S}{2} \right)$$

$$\text{eq}(48d) = \text{eq}(53d) = \log \left(\frac{S + l + 1}{2} \right) + \log \left(1 + \frac{S}{2l} \right)$$

$$\text{eq}(48e) = \text{eq}(53e) = \log \left(1 + \frac{S}{2l} \right) + \log \left(\frac{S + l + 1}{2} \right)$$

$$\begin{aligned} \text{eq}(48f) = \text{eq}(53f) &= \log \left(\frac{1 + C}{1 + \frac{C}{l}} \right) + \log \left(\frac{\frac{S}{l} + l + 1}{2} \right) \\ &\quad + \log \left(1 + \frac{S}{2l} \right) \end{aligned}$$

$$\begin{aligned} \text{eq}(48g) = \text{eq}(53g) &= \log \left(\frac{\frac{S}{l} + l + 1}{2} \right) + \log \left(1 + \frac{S}{2l} \right) \\ &\quad + \log \left(\frac{S + l + 1}{2} \right). \end{aligned}$$

We next show that the sum-rate in (54) is equal to the term in (54b). In order to show that the term in (54a) is redundant, consider the following facts:

- $\text{eq}(48a) + \text{eq}(48c)$ is always greater than $\text{eq}(48d)$ because $S \geq \frac{S}{l}$, since we assume $l \geq 1$;
- $\text{eq}(48b) + \text{eq}(48c)$ is always greater than $\text{eq}(48f)$ since $2l + Sl \geq S + l^2 + l \iff S \geq l$, which is always satisfied since we are in the weak interference regime.

In order to show that the term in (54c) is redundant, consider the following facts:

- the bound $\frac{\text{eq}(48a) + \text{eq}(48g)}{2}$ is always bigger than $\text{eq}(48d)$ since we assume $l \geq 1$ and it is therefore redundant;
- the bound $\frac{\text{eq}(48b) + \text{eq}(48g)}{2}$ is equal to $\frac{\text{eq}(48e) + \text{eq}(48f)}{2}$ and hence it is redundant.

Therefore we conclude that in the weak interference regime $1 \leq l \leq S$ the sum-rate in (54) is equal to (54b) and, since $\text{eq}(48e)$ is equal to $\text{eq}(48d)$, is given by

$$R_p + R_c \leq \min \left\{ \log \left(1 + \frac{S}{2l} \right) + \log \left(\frac{S + l + 1}{2} \right), \quad (55a) \right.$$

$$\left. \log \left(1 + \frac{S}{2l} \right) + \log \left(\frac{1 + C}{1 + C} \right) + \log \left(\frac{S + l^2 + l}{2} \right) \right\}. \quad (55b)$$

For future use, the term in (55b) is the smallest term if

$$\begin{aligned} (S + l + 1)(l + C) &\geq S + l^2 + l + SC + Cl^2 + Cl \\ &\iff S \geq C(l + 1). \end{aligned}$$

D. Constant gap result for the sum-capacity of the symmetric GCCIC in Regime 6 of Fig. 2

We analyze the regime $l_p = l_c = l < S_p = S_c = S$.

Parameter Range: $S(S + l) > l^2(l + 1)$ and $C \geq \frac{l^2}{S}$. In order to find the tightest upper bound we need to split this region in different subregions, namely:

- Regime 6a) $S < C(l+1)$: here the tightest gDoF upper bound gives

$$d(\alpha, \beta) \leq 1 - \frac{\alpha}{2};$$

- Regime 6b) $S \geq C(l+1)$ and $C \geq l$: here the tightest gDoF upper bound gives

$$d(\alpha, \beta) \leq 1 - \frac{\alpha}{2};$$

- Regime 6c) $S \geq C(l+1)$, $l^2 \leq S$ and $C < l$: here the tightest gDoF upper bound gives

$$d(\alpha, \beta) \leq 1 - \alpha + \frac{\beta}{2};$$

- Regime 6d) $S \geq C(l+1)$, $l^2 > S$, $C < l$ and $S(S+1) > l^2(l+1)$: here the tightest gDoF upper bound gives

$$d(\alpha, \beta) \leq \frac{1+\beta}{2}.$$

Inner Bound: We use the achievable scheme developed in Appendix B-C whose achievable sum-rate is given in (55), which in the weak interference regime (i.e., $\alpha \leq 1$) implies that the following gDoF is achievable

$$\begin{aligned} d(\alpha, \beta) &\geq \frac{1}{2} \min\{[1-\alpha]^+ + \max\{1, \alpha\}, \\ &\quad [1-\alpha]^+ + \beta - \max\{\alpha, \beta\} + \max\{1, 2\alpha\}\} \\ &= \begin{cases} 1 - \alpha/2 & \text{for } \beta \geq \min\{\alpha, 1 - \alpha\} \\ 1 - \alpha + \beta/2 & \text{for } \beta < \alpha, \alpha \in [0, 1/2] \\ (1 + \beta)/2 & \text{for } \beta < 1 - \alpha, \alpha \in [1/2, 1] \end{cases}. \end{aligned} \quad (56)$$

This shows the achievability of the gDoF upper bound in Regime 6 of Fig. 2. Actually, the proposed scheme is gDoF optimal in the whole weak interference regime $\alpha \leq 1$ except for $\beta \leq \min\{1 - \alpha, [2\alpha - 1]^+\}$, where a noncooperative scheme is gDoF optimal.

Outer Bound: For the regime $\beta \geq \min\{\alpha, 1 - \alpha\}$, where $d(\alpha, \beta) \leq 1 - \alpha/2$ (regimes 6a and 6b), we use the upper bound in (4e); otherwise (regimes 6c and 6d) we use the upper bound in (4f).

Gap: We analyze separately the different sub regimes:

- Regime 6a) For the regime $S < C(l+1)$ within $l \leq S$

$$\begin{aligned} \text{GAP} &\leq \text{eq(4e)} - \text{eq(55a)} \\ &\leq \log\left(\frac{1+S}{1+l}\right) + \log\left(1 + (\sqrt{S} + \sqrt{l})^2\right) \\ &\quad - \log\left(1 + \frac{S}{2l}\right) - \log\left(\frac{S+l+1}{2}\right) \\ &\leq 2\log(2) + \max_{1 \leq l \leq S} \log\left(\frac{1}{1+l} \frac{1+S}{1+\frac{S}{2l}}\right) \\ &= 2\log(2) + \max_{1 \leq l} \log\left(\frac{2l}{1+l}\right) = 3\log(2). \end{aligned}$$

- Regime 6b) For the regime $S \geq C(l+1)$ and $C \geq l$

$$\begin{aligned} \text{GAP} &\leq \text{eq(4e)} - \text{eq(55b)} \\ &\leq \log\left(\frac{1+S}{1+l}\right) + \log\left(1 + (\sqrt{S} + \sqrt{l})^2\right) + \\ &\quad - \log\left(1 + \frac{S}{2l}\right) - \log\left(\frac{1+C}{1+C}\right) - \log\left(\frac{S+l^2+1}{2}\right) \\ &\leq \log\left(\frac{1+S}{1+l}\right) + \log\left(\frac{1+S+l}{S+l^2+1}\right) + 2\log(2) \\ &\quad + \log\left(\frac{2l}{2l+S}\right) + \log\left(\frac{2l}{1+l}\right) \\ &= 4\log(2) + \log\left(\frac{1+S}{2l+S}\right) + 2\log\left(\frac{l}{1+l}\right) \\ &\quad + \log\left(\frac{1+S+l}{S+l^2+1}\right) \leq 4\log(2) \end{aligned}$$

since $1+S+l < S+l^2+1$, $1 \leq l$, and where we upper bounded the gap by evaluating it for $C = l$, i.e., minimum possible value for C , since the function is decreasing in C .

- Regime 6c) For the regime $S \geq C(l+1)$, $C < l$ and $l^2 \leq S$

$$\begin{aligned} \text{GAP} &\leq \text{eq(4f)} - \text{eq(55b)} \\ &\leq \log\left(1 + \left(\frac{\sqrt{S}}{\sqrt{l}} + \sqrt{l}\right)^2\right) + \log(1+C) \\ &\quad + \log\left(1 + \left(\frac{\sqrt{S}}{\sqrt{l}} + \frac{\sqrt{l}}{\sqrt{C}}\right)^2\right) - \log\left(1 + \frac{S}{2l}\right) \\ &\quad - \log\left(\frac{1+C}{1+C}\right) - \log\left(\frac{S+l^2+1}{2}\right) \\ &\leq \log\left(1 + \frac{S}{l} + l\right) + \log(2l) + \log\left(2 + \frac{S}{l}\right) \\ &\quad - \log\left(1 + \frac{S}{2l}\right) - \log(S+l^2+1) + 3\log(2) \leq 5\log(2), \end{aligned}$$

where we upper bounded the gap by evaluating it for $C = l$, i.e., the maximum possible value for C , since the function is increasing in C .

- Regime 6d) For the regime $S \geq C(l+1)$, $C < l$, $l^2 > S$ and $S(S+1) \geq l^2(l+1)$

$$\text{GAP} \leq \text{eq(4f)} - \text{eq(55b)} \leq 5\log(2),$$

by following exactly the same steps as done for Regime 6c) above.

This shows the achievability of the sum-capacity upper bound to within a constant gap of 2.5 bits (per user) in this regime.

APPENDIX C

ACHIEVABLE SCHEMES BASED ON SUPERPOSITION CODING AND DPC

A. DPC-based Achievable Scheme

We specialize the ‘binning+superposition’ achievable scheme in [10, Section V]. In [10, Theorem V.1], the network

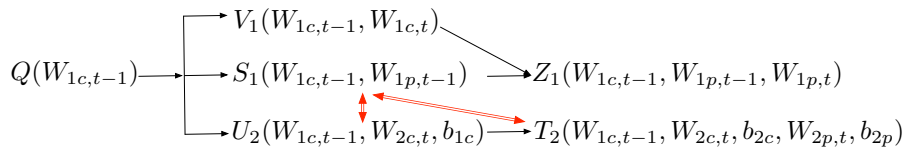


Fig. 11: Achievable scheme based on binning and superposition coding.

$$P_{Q,S_1,V_1,Z_1,X_1,U_2,T_2,X_2} = P_Q P_{V_1|Q} P_{S_1|Q} P_{Z_1|Q,S_1,V_1} P_{U_2,T_2|S_1,Q} P_{X_1|Q,S_1,V_1,Z_1} P_{X_2|Q,S_1,U_2,T_2}. \quad (57)$$

comprises four nodes numbered from 1 to 4; nodes 1 and 2 are sources and nodes 3 and 4 destinations; source node $j \in [1 : 2]$, with input to the channel X_j and output from the channel Y_j , has a message W_j for node $j+2$; destination node $j \in [3 : 4]$ has channel output Y_j from which it decodes message W_{j-2} .

Both users do rate splitting, where the messages of user 1 are cooperative while the messages of user 2 are noncooperative. In [10, Section V], we set $Y_1 = U_1 = T_1 = S_2 = V_2 = Z_2 = \emptyset$, i.e., then $R_1 = R_{11c} + R_{10c}$, $R_2 = R_{22n} + R_{20n}$, to obtain a scheme that comprises: a cooperative common message (carried by the pair (Q, V_1) at rate R_{10c}) for user 1, a cooperative private message (carried by the pair (S_1, Z_1) at rate R_{11c}) for user 1, a noncooperative common message (carried by U_2 at rate R_{20n}) for user 2 and a noncooperative private message (carried by T_2 at rate R_{22n}) for user 2. Here the pair (Q, S_1) carries the ‘past cooperative messages’, and the pair (V_1, Z_1) the ‘new cooperative messages’ in a block Markov encoding scheme. The channel inputs are functions of the auxiliary random variables, where X_1 is a function of (Q, S_1, V_1, Z_1) and X_2 is a function of (Q, S_1, U_2, T_2) .

The set of possible input distributions is given by (57) at the top of the page. A schematic representation of the achievable scheme is given in Fig. 11, where a black arrow indicates superposition coding and a red arrow indicates binning.

Regarding encoding. The codebooks are generated as follows: first the codebook Q is generated; then the codebook V_1 is superposed to Q ; independently of V_1 , the codebook S_1 is superposed to Q and then the codebook Z_1 is superposed to (Q, S_1, V_1) ; independently of (V_1, S_1, Z_1) , the codebook U_2 is superposed to Q and then the codebook T_2 is superposed to (Q, U_2) . With this random coding codebook generation, the pair (U_2, T_2) is independent of S_1 conditioned on Q . [10, Theorem V.1] involves several binning steps to allow for a large set of input distributions. Here, in order to simplify the scheme, we do not bin V_1 against S_1 ; the only binning steps are for (U_2, T_2) against S_1 . We use a block Markov coding scheme to convey the message of user 1 to user 2. In particular, at the end of any given time slot in a block Markov coding scheme, encoder 2 knows (Q, S_1, U_2, T_2) and decodes (V_1, Z_1) from its channel output; the decoded pair (V_1, Z_1) becomes the pair (Q, S_1) of the next time slot; then, at the beginning of each time slot, encoder 2, by binning, finds the new pair (U_2, T_2) that is jointly typical with (Q, S_1) ; for this to be possible, we must generate several (U_2, T_2) sequences for each message of user 2 so as to be able to find one pair to send with the correct joint distribution with (Q, S_1) ; this entails the

rate penalties in [10, eq(20)] for user 1 and then again [10, eq(20)] for user 2 by swapping the role of the subscripts 1 and 2, with $S_2 = Z_2 = V_2 = U_1 = T_1 = \emptyset$ and with V_1 independent of S_1 , i.e.,

$$R'_{20n} + R'_{22n} \geq I(S_1; U_2, T_2|Q) \quad (58a)$$

$$R'_{20n} \geq I(U_2; S_1|Q). \quad (58b)$$

Regarding decoding. There are three decoding nodes in the network and therefore three groups of rate constraints. These are:

- Node 2/CTx jointly decodes (V_1, Z_1) from its channel output with knowledge of the indices in (Q, S_1, U_2, T_2, X_2) . Successful decoding is possible if (i.e., use [10, eq(21)] by swapping the role of the subscripts 1 and 2, with $S_2 = Z_2 = V_2 = U_1 = T_1 = \emptyset$ and with V_1 independent of S_1)

$$R_{10c} + R_{11c} \leq I(Y_2; Z_1, V_1|U_2, T_2, X_2, S_1, Q) \quad (58c)$$

$$R_{11c} \leq I(Y_2; Z_1|U_2, T_2, X_2, S_1, Q, V_1). \quad (58d)$$

- Node 3/PRx jointly decodes (Q, S_1, U_2) from its channel output, with knowledge of some message indices in (V_1, Z_1) , by treating T_2 as noise. Successful decoding is possible if (see [10, eq(22)] where only the bounds in [10, eq(22a)], [10, eq(22f)], and [10, eq(22g)] remain after setting several auxiliary random variables to zero and removing the redundant constraints)

$$R_{10c} + R_{20n} + R_{11c} \leq I(Y_3; Q, V_1, S_1, Z_1, U_2) + (R'_{20n} - I(U_2; S_1|Q)) \quad (58e)$$

$$R_{20n} + R_{11c} \leq I(Y_3; S_1, Z_1, U_2|V_1, Q) + (R'_{20n} - I(U_2; S_1|Q)) \quad (58f)$$

$$R_{11c} \leq I(Y_3; S_1, Z_1|V_1, Q, U_2). \quad (58g)$$

- Node 4/CRx jointly decodes (Q, U_2, T_2) from its channel output, with knowledge of some message index in V_1 , by treating Z_1 as noise (recall that the pair (U_2, T_2) has been precoded/binning against S_1). Successful decoding is possible if (see [10, eq(22)], with the role of the users swapped, where only the bounds in [10, eq(22a)], [10, eq(22i)], and [10, eq(22k)] remain after setting several auxiliary random variables to zero and removing the

redundant constraints)

$$R_{10c} + R_{20n} + R_{22n} \leq I(Y_4; U_2, T_2, V_1, Q) + (R'_{20n} + R'_{22n}) \quad (58h)$$

$$R_{20n} + R_{22n} \leq I(Y_4; U_2, T_2 | V_1, Q) + (R'_{20n} + R'_{22n}) \quad (58i)$$

$$R_{22n} \leq I(Y_4; T_2 | V_1, Q, U_2) - R'_{22n}. \quad (58j)$$

From Remark 2 in Appendix B-A, after Fourier-Motzkin elimination of the achievable region in (58) where we take the constraints in (58a) and (58b) to hold with equality (i.e., $R'_{20n} = I(U_2; S_1 | Q)$, $R'_{22n} = I(S_1; T_2 | Q, U_2)$), we get

$$R_1 \leq \text{eq}(58e) \quad (59a)$$

$$R_1 \leq \text{eq}(58c) \quad (59b)$$

$$R_2 \leq \text{eq}(58i) \quad (59c)$$

$$R_1 + R_2 \leq \text{eq}(58e) + \text{eq}(58j) \quad (59d)$$

$$R_1 + R_2 \leq \text{eq}(58h) + \text{eq}(58g) \quad (59e)$$

$$R_1 + R_2 \leq \text{eq}(58h) + \text{eq}(58d) \quad (59f)$$

$$R_1 + 2R_2 \leq \text{eq}(58f) + \text{eq}(58j) + \text{eq}(58h) \quad (59g)$$

for all distributions that factor as (57).

Remark 3. As done in Remark 1 in Appendix B-A, the rate bound in (59g) is redundant if

$$\min\{\text{eq}(59d), \text{eq}(59e)\} + \text{eq}(59c) \leq \text{eq}(59g)$$

that is, if for the considered input distribution we have

$$\text{either } \text{eq}(58e) + \text{eq}(58i) \leq \text{eq}(58f) + \text{eq}(58h) \\ \iff I(Y_3; Q, V_1) \leq I(Y_4; Q, V_1), \quad (60a)$$

$$\text{or } \text{eq}(58g) + \text{eq}(58i) \leq \text{eq}(58f) + \text{eq}(58j) \\ \iff I(Y_4; U_2 | Q, V_1) - I(U_2; S_1 | Q) \\ \leq I(Y_3; U_2 | Q, V_1). \quad (60b)$$

B. DPC region for the Gaussian noise channel

We identify Node1 with the PTx (i.e., $X_p = X_1$), Node2 with the CTx (i.e., $X_c = X_2, Y_f = Y_2$), Node3 with the PRx (i.e., $Y_p = Y_3$) and Node4 with the CRx (i.e., $Y_c = Y_4$). For the Gaussian noise channel, in the achievable region in (59), we choose $Q = \emptyset$, we let S_1, V_1, Z_1, U_2, T_2' to be i.i.d. $\mathcal{N}(0, 1)$, and

$$X_p = |a_1|e^{j\theta_c}S_1 + b_1V_1 + c_1Z_1 : |a_1|^2 + |b_1|^2 + |c_1|^2 = 1,$$

$$X_c = |a_2|S_1 + b_2U_2 + c_2T_2' : |a_2|^2 + |b_2|^2 + |c_2|^2 = 1,$$

$$T_2 = T_2' + \lambda S_1$$

$$\lambda = \frac{S_c|c_2|^2}{S_c|c_2|^2 + 1 + |p|c_1|^2} \frac{\sqrt{|p|e^{j\theta_p}e^{j\theta_c}|a_1| + \sqrt{S_c}|a_2|}}{\sqrt{S_c}c_2},$$

where the choice of λ is so as to “pre-cancel” S_1 from Y_c in decoding T_2 , i.e., so as to have $I(Y_c; T_2 | V_1, Q, U_2) -$

$I(S_1; T_2 | Q, U_2) = I(Y_c; T_2 | V_1, Q, U_2, S_1)$. With these choices, the channel outputs are

$$Y_f = \sqrt{C} (|a_1|e^{j\theta_c}S_1 + b_1V_1 + c_1Z_1) + Z_f,$$

$$Y_p = (\sqrt{S_p}|a_1| + \sqrt{|c|}|a_2|)e^{j\theta_c}S_1 + \sqrt{S_p}(b_1V_1 + c_1Z_1) \\ + \sqrt{|c|}e^{j\theta_c}(b_2U_2 + c_2T_2') + Z_p,$$

$$Y_c = (\sqrt{|p|}e^{j\theta_p}e^{j\theta_c}|a_1| + \sqrt{S_c}|a_2|)S_1 + \sqrt{|p|}e^{j\theta_p}(b_1V_1 + c_1Z_1) \\ + \sqrt{S_c}(b_2U_2 + c_2T_2') + Z_c,$$

and the achievable region in (59) (notice that we have $I(S_1; U_2 | Q) = 0$ since U_2 is not precoded against S_1) becomes

$$R_p \leq I(Y_f; Z_1, V_1 | U_2, T_2, X_c, S_1, Q)$$

$$= \log(1 + C(|b_1|^2 + |c_1|^2)),$$

$$R_p \leq I(Y_p; Q, V_1, S_1, Z_1, U_2)$$

$$= \log\left(\frac{1 + S_p + |c| + 2\sqrt{S_p|c|}|a_1|^2|a_2|^2}{1 + |c|c_2|^2}\right),$$

$$R_c \leq I(Y_c; U_2, T_2 | V_1, Q) - I(S_1; T_2 | Q, U_2)$$

$$= I(Y_c; U_2 | V_1, Q) + I(Y_c; T_2 | V_1, Q, U_2, S_1)$$

$$= \log\left(1 + \frac{S_c|b_2|^2}{1 + |p|c_1|^2 + S_c|c_2|^2 + |\sqrt{|p|}e^{j\theta_p}e^{j\theta_c}|a_1| + \sqrt{S_c}|a_2||^2}\right) \\ + \log\left(1 + \frac{S_c|c_2|^2}{1 + |p|c_1|^2}\right),$$

$$R_p + R_c \leq I(Y_c; T_2 | V_1, Q, U_2) - I(S_1; T_2 | Q, U_2) \\ + I(Y_p; Q, V_1, S_1, Z_1, U_2)$$

$$= \log\left(\frac{1 + S_p + |c| + 2\sqrt{S_p|c|}|a_1|^2|a_2|^2}{1 + |c|c_2|^2}\right)$$

$$+ \log\left(1 + \frac{S_c|c_2|^2}{1 + |p|c_1|^2}\right),$$

$$R_p + R_c \leq I(Y_c; U_2, T_2, V_1, Q) - I(S_1; T_2 | Q, U_2) \\ + I(Y_p; S_1, Z_1 | V_1, Q, U_2)$$

$$= \log\left(1 + \frac{S_c|b_2|^2 + |p|b_1|^2}{1 + |p|c_1|^2 + S_c|c_2|^2 + |\sqrt{|p|}e^{j\theta_p}e^{j\theta_c}|a_1| + \sqrt{S_c}|a_2||^2}\right)$$

$$+ \log\left(1 + \frac{|\sqrt{S_p}|a_1| + \sqrt{|c|}|a_2||^2 + S_p|c_1|^2}{1 + |c|c_2|^2}\right)$$

$$+ \log\left(1 + \frac{S_c|c_2|^2}{1 + |p|c_1|^2}\right),$$

$$R_p + R_c \leq I(Y_c; U_2, T_2, V_1, Q) - I(S_1; T_2 | Q, U_2) \\ + I(Y_f; Z_1 | U_2, T_2, X_c, S_1, Q, V_1)$$

$$= \log\left(1 + \frac{S_c|b_2|^2 + |p|b_1|^2}{1 + |p|c_1|^2 + S_c|c_2|^2 + |\sqrt{|p|}e^{j\theta_p}e^{j\theta_c}|a_1| + \sqrt{S_c}|a_2||^2}\right)$$

$$+ \log\left(1 + \frac{S_c|c_2|^2}{1 + |p|c_1|^2}\right)$$

$$+ \log(1 + C|c_1|^2),$$

and

$$\begin{aligned}
R_p + 2R_c &\leq I(Y_c; T_2|V_1, Q, U_2) - I(S_1; T_2|Q, U_2) \\
&\quad + I(Y_c; U_2, T_2, V_1, Q) - I(S_1; T_2|Q, U_2) \\
&\quad + I(Y_p; S_1, Z_1, U_2|V_1, Q) \\
&= \log \left(1 + \frac{S_c |b_2|^2 + I_p |b_1|^2}{1 + I_p |c_1|^2 + S_c |c_2|^2 + |\sqrt{I_p} e^{j\theta_p} e^{j\theta_c} |a_1| + \sqrt{S_c} |a_2|^2} \right) \\
&\quad + 2 \log \left(1 + \frac{S_c |c_2|^2}{1 + I_p |c_1|^2} \right) \\
&\quad + \log \left(1 + \frac{|\sqrt{S_p} |a_1| + \sqrt{I_c} |a_2|^2 + S_p |c_1|^2 + I_c |b_2|^2}{1 + I_c |c_2|^2} \right).
\end{aligned}$$

Remark 4. Motivated by the observation in [16] that all terms that appear as noise should be at most at the level of the noise, we set

$$\begin{aligned}
|a_1| &= 0, \quad |b_1|^2 = \frac{I_p}{1 + I_p}, \quad |c_1|^2 = \frac{1}{1 + I_p}, \\
|a_2|^2 &= \frac{I_c}{1 + I_c} \frac{1}{1 + S_c}, \quad |b_2|^2 = \frac{I_c}{1 + I_c} \frac{S_c}{1 + S_c}, \quad |c_2|^2 = \frac{1}{1 + I_c},
\end{aligned}$$

so that the achievable region derived in this section is included into

$$R_p \leq \log(1 + C) \quad (61a)$$

$$R_p \leq \log(1 + S_p + I_c) - \log(2) \quad (61b)$$

$$R_c \leq \log(1 + S_c) - 2 \log(2) \quad (61c)$$

$$R_p + R_c \leq \log(1 + S_p + I_c) + \log \left(1 + \frac{S_c}{1 + I_c} \right) - 2 \log(2) \quad (61d)$$

$$\begin{aligned}
R_p + R_c &\leq \log(1 + I_p + S_c) - 3 \log(2) \\
&\quad + \log \left(1 + \frac{I_c}{1 + I_c} \frac{I_c}{1 + S_c} + \frac{S_p}{1 + I_p} \right) \quad (61e)
\end{aligned}$$

$$R_p + R_c \leq \log(1 + I_p + S_c) + \log \left(1 + \frac{C}{1 + I_p} \right) - 2 \log(2) \quad (61f)$$

for either

$$\begin{aligned}
I(Y_p; V_1) \leq I(Y_c; V_1) &\iff \frac{S_p |b_1|^2}{1 + S_p |c_1|^2 + I_c} \leq \frac{I_p |b_1|^2}{1 + I_p |c_1|^2 + S_c} \\
&\iff S_p(1 + S_c) \leq I_p(1 + I_c), \quad (61g)
\end{aligned}$$

or

$$\begin{aligned}
I(Y_c; U_2|V_1) \leq I(Y_p; U_2|V_1) \\
&\iff \frac{S_c |b_2|^2}{1 + S_c(1 - |b_2|^2) + I_p |c_1|^2} \leq \frac{I_c |b_2|^2}{1 + I_c(1 - |b_2|^2) + S_p |c_1|^2} \\
&\iff S_c \frac{1 + I_p + S_p}{1 + 2I_p} \leq I_c, \quad (61h)
\end{aligned}$$

so that the bound on $R_p + 2R_c$ is redundant (see conditions in (60)). In the regime $C > \max\{S_p, I_p\}$ (see Fig. 4 on the right) the gap would be 2 bits if one could neglect the sum-rate bound in (61e).

C. Achievable Scheme 3: both messages are private

From the general region in Section C-B, we set

$$\begin{aligned}
a_1 &= \gamma_p, \quad b_1 = 0, \quad c_1 = \sqrt{1 - |\gamma_p|^2}, \quad |\gamma_p| \leq 1, \\
a_2 &= \gamma_c, \quad b_2 = 0, \quad c_2 = \sqrt{1 - |\gamma_c|^2}, \quad |\gamma_c| \leq 1,
\end{aligned}$$

to obtain

$$R_p \leq \log(1 + C(1 - |\gamma_p|^2)) \quad (62a)$$

$$R_p \leq \log \left(\frac{1 + S_p + I_c + 2|\gamma_c||\gamma_p|\sqrt{S_p I_c}}{1 + (1 - |\gamma_c|^2)I_c} \right) \quad (62b)$$

$$R_c \leq \log \left(1 + \frac{(1 - |\gamma_c|^2)S_c}{1 + (1 - |\gamma_p|^2)I_p} \right) \quad (62c)$$

for all $(|\gamma_c|, |\gamma_p|) \in [0, 1]^2$.

From (62) the following sum-rate is achievable

$$\begin{aligned}
R_p + R_c &\leq \max_{(|\gamma_c|, |\gamma_p|) \in [0, 1]^2} \log \left(1 + \frac{(1 - |\gamma_c|^2)S_c}{1 + (1 - |\gamma_p|^2)I_p} \right) + \\
&\quad + \min \left\{ \log(1 + C(1 - |\gamma_p|^2)), \right. \\
&\quad \left. \log \left(1 + \frac{S_p + |\gamma_c|^2 I_c + 2|\gamma_c||\gamma_p|\sqrt{S_p I_c}}{1 + (1 - |\gamma_c|^2)I_c} \right) \right\}.
\end{aligned}$$

For the symmetric case, i.e., $S_c = S_p = S, I_c = I_p = I$, instead of solving analytically the optimization involved in the sum-rate maximization, which does not seem to lead to a closed-form expression, we choose to set $|\gamma_c| = 0$ and $(1 - |\gamma_p|^2) = 1$ if $C < \frac{S}{1+I}$ and $(1 - |\gamma_p|^2) = \frac{S}{C(1+I)}$ otherwise (i.e., these values are not necessarily optimal). With these choices the following sum-rate is achievable

$$R_p + R_c \leq \log \left(1 + \frac{S}{1+I} \right) + \log \left(\frac{1+S}{1 + \frac{S/C}{1+I}} \right) \quad (63a)$$

if $C \geq \frac{S}{1+I}$ and

$$R_p + R_c \leq \log \left(1 + \frac{S}{1+I} \right) + \log(1 + C) \quad (63b)$$

if $C < \frac{S}{1+I}$.

D. Constant gap result for the sum-capacity of the symmetric GCCIC in Regimes 4 and 6 of Fig. 2 for $\alpha < 1 \leq \beta$

With the DPC-based achievable scheme in Appendix C-C an achievable sum-rate is given by (63a), which we use to derive a smaller gap than those in Section III-D and Appendix B-D in the regime $I < S$ and $C \geq S$ (parts of regimes 4 and 6 of Fig. 2). The achievable sum-rate in (63a) implies

$$\begin{aligned}
d(\alpha, \beta) &\leq \lim_{S \rightarrow \infty} \frac{\log \left(1 + \frac{S}{1+I} \right) + \log \left(\frac{1+S}{1 + \frac{S/C}{1+I}} \right)}{2 \log(1 + S)} \\
&= \frac{1}{2} ([1 - \alpha]^+ + 1 - [1 - \beta]^+) \quad \alpha < 1 \leq \beta \quad \frac{2 - \alpha}{2}.
\end{aligned}$$

This shows the achievability of the gDoF upper bound by means of (63a).

By using the sum-capacity upper bound in (4e) under the condition $S \geq I$ and the achievable sum-rate in (63a) we obtain the following gap

$$\begin{aligned}
\text{GAP} &\leq \log \left(\frac{1+S}{1+I} \right) + \log(1 + S + I) + \log(2) \\
&\quad - \log \left(1 + \frac{S}{1+I} \right) - \log \left(\frac{1+S}{1 + \frac{S/C}{1+I}} \right) \\
&\leq \log \left(1 + \frac{S}{C} \right) + \log(2) \leq 2 \log(2),
\end{aligned}$$

using $S \leq C$. This example shows that an achievable scheme more complex than simple superposition coding, like a DPC-based one, can achieve a smaller gap.

E. Achievable Scheme 4: message 1 is split, and message 2 is common but not precoded

From the general region in Section C-B, we set

$$\begin{aligned} a_1 &= 0, \quad b_1 = \sqrt{1 - |\gamma_p|^2}, \quad c_1 = \gamma_p, \quad |\gamma_p| \leq 1, \\ a_2 &= \gamma_c, \quad b_2 = \sqrt{1 - |\gamma_c|^2}, \quad c_2 = 0, \quad |\gamma_c| \leq 1, \end{aligned}$$

to obtain

$$R_p \leq \log(1 + C) \quad (64a)$$

$$R_c \leq \log\left(1 + \frac{S_c(1 - |\gamma_c|^2)}{1 + |\gamma_p|^2 I_p + |\gamma_c|^2 S_c}\right) \quad (64b)$$

$$R_p + R_c \leq \log(1 + S_p + I_c) \quad (64c)$$

$$\begin{aligned} R_p + R_c &\leq \log(1 + |\gamma_p|^2 C) \\ &\quad + \log\left(1 + \frac{S_c(1 - |\gamma_c|^2) + I_p(1 - |\gamma_p|^2)}{1 + |\gamma_p|^2 I_p + |\gamma_c|^2 S_c}\right) \end{aligned} \quad (64d)$$

$$\begin{aligned} R_p + R_c &\leq \log(1 + |\gamma_p|^2 S_p + |\gamma_c|^2 I_c) \\ &\quad + \log\left(1 + \frac{S_c(1 - |\gamma_c|^2) + I_p(1 - |\gamma_p|^2)}{1 + |\gamma_p|^2 I_p + |\gamma_c|^2 S_c}\right) \end{aligned} \quad (64e)$$

$$\begin{aligned} R_p + 2R_c &\leq \log(1 + |\gamma_p|^2 S_p + I_c) \\ &\quad + \log\left(1 + \frac{S_c(1 - |\gamma_c|^2) + I_p(1 - |\gamma_p|^2)}{1 + |\gamma_p|^2 I_p + |\gamma_c|^2 S_c}\right). \end{aligned} \quad (64f)$$

In the rate region in (64), the constraint on $R_p + 2R_c$ becomes redundant if one of the conditions in (60) holds; in particular, if

$$\begin{aligned} I(Y_p; V_1) &\leq I(Y_c; V_1) \iff \\ \frac{(1 - |\gamma_p|^2)S_p}{1 + |\gamma_p|^2 S_p + I_c} &\leq \frac{(1 - |\gamma_p|^2)I_p}{1 + |\gamma_p|^2 I_p + S_c} \iff \\ \text{either } |\gamma_p| &= 1, \text{ or } S_p(1 + S_c) \leq I_p(1 + I_c), \end{aligned} \quad (65)$$

or if

$$\begin{aligned} I(Y_c; U_2|V_1) &\leq I(Y_p; U_2|V_1) \iff \\ \frac{(1 - |\gamma_c|^2)S_c}{1 + |\gamma_p|^2 I_p + |\gamma_c|^2 S_c} &\leq \frac{(1 - |\gamma_c|^2)I_c}{1 + |\gamma_p|^2 S_p + |\gamma_c|^2 I_c} \iff \\ \text{either } |\gamma_c| &= 1, \text{ or } S_c \frac{1 + |\gamma_p|^2 S_p}{1 + |\gamma_p|^2 I_p} \leq I_c. \end{aligned} \quad (66)$$

F. Achievable Scheme 5: message 1 is split, and message 2 is private; gap for the S-channel

From the region in Section C-B, we set $c_2 = 1$ to obtain

$$R_p \leq \log(1 + C(|c_1|^2 + |b_1|^2)) \quad (67a)$$

$$R_p \leq \log\left(1 + \frac{S_p}{1 + I_c}\right) \quad (67b)$$

$$R_c \leq \log\left(1 + \frac{S_c}{1 + I_p|c_1|^2}\right) \quad (67c)$$

$$\begin{aligned} R_p + R_c &\leq \log\left(\frac{1 + S_c + I_p}{1 + I_p(|a_1|^2 + |c_1|^2) + S_c}\right) \\ &\quad + \log\left(1 + \frac{S_c}{1 + I_p|c_1|^2}\right) + \log(1 + C|c_1|^2) \end{aligned} \quad (67d)$$

$$\begin{aligned} R_p + R_c &\leq \log\left(\frac{1 + S_c + I_p}{1 + I_p(|a_1|^2 + |c_1|^2) + S_c}\right) \\ &\quad + \log\left(1 + \frac{S_c}{1 + I_p|c_1|^2}\right) \\ &\quad + \log\left(1 + \frac{S_p(|a_1|^2 + |c_1|^2)}{1 + I_c}\right). \end{aligned} \quad (67e)$$

An achievable region for the S-channel is obtained by setting $I_c = 0$ in (67). Here we concentrate on the regime $S_p \leq C \leq (1 + I_p)S_p$ and evaluate the region in (67) for

$$|a_1|^2 = \frac{C - S_p}{(1 + I_p)S_p}, \quad |b_1|^2 = \frac{(1 + I_p)S_p - C}{(1 + I_p)S_p}, \quad |c_1|^2 = \frac{1}{1 + I_p}.$$

With these choices the region in (67) reduces to

$$R_p \leq \log(1 + S_p) \quad (68a)$$

$$R_c \leq \log\left(1 + \frac{S_c}{1 + \frac{I_p}{1 + I_p}}\right) \quad (68b)$$

$$R_p + R_c \leq \log\left(\frac{1 + S_c + I_p}{1 + S_c + \frac{I_p}{1 + I_p} \frac{C}{S_p}}\right) \quad (68c)$$

$$+ \log\left(1 + \frac{S_c}{1 + \frac{I_p}{1 + I_p}}\right) + \log\left(1 + \frac{C}{1 + I_p}\right) \quad (68d)$$

since the bound on R_p in (67a) would give $R_p \leq \log\left(1 + C \frac{2 + I_p - C/S_p}{1 + I_p}\right)$ which is redundant because

$$\begin{aligned} S_p \leq C \frac{2 + I_p - \frac{C}{S_p}}{1 + I_p} &\iff 1 - 2\frac{C}{S_p} + \left(\frac{C}{S_p}\right)^2 \leq I_p \left(\frac{C}{S_p} - 1\right) \\ &\iff S_p \leq C \leq (1 + I_p)S_p; \end{aligned}$$

notice also that the two sum-rate bounds in (67d) and in (67e) are the same.

We next match the achievable region in (68) to the outer bound

$$R_p \leq \log(1 + S_p) \quad (69a)$$

$$R_c \leq \log(1 + S_c) \quad (69b)$$

$$\begin{aligned} R_p + R_c &\leq \log\left(1 + (\sqrt{S_c} + \sqrt{I_p})^2\right) \\ &\quad + \log\left(\frac{1 + C + \max\{S_p, I_p\}}{1 + I_p}\right) \end{aligned} \quad (69c)$$

from (4) with $I_c = 0$. The bounds on R_p in (68) and (69) are the same, and the bounds on R_c in (68) and (69) are at most 1 bit apart. For the sum-rate, if $C/S_p \leq S_c$ (and recall that we focus on $S_p \leq C$) then

$$\begin{aligned}
\text{GAP} &\leq \log\left(1 + (\sqrt{S_c} + \sqrt{I_p})^2\right) \\
&+ \log\left(\frac{1+C+\max\{S_p, I_p\}}{1+I_p}\right) - \log\left(\frac{1+S_c+I_p}{1+S_c+\frac{I_p}{1+I_p}\frac{C}{S_p}}\right) \\
&- \log\left(1 + \frac{S_c}{1+\frac{I_p}{1+I_p}}\right) - \log\left(1 + \frac{C}{1+I_p}\right) \\
&\leq \log(2) + \log\left(\frac{1+S_c+\frac{I_p}{1+I_p}\frac{C}{S_p}}{1+\frac{S_c}{1+\frac{I_p}{1+I_p}}}\right) \\
&+ \log\left(\frac{1+C+\max\{S_p, I_p\}}{1+C+I_p}\right) \\
&\leq \log(2) + \log\left(\frac{1+S_c\left(1+\frac{I_p}{1+I_p}\right)}{1+\frac{S_c}{1+\frac{I_p}{1+I_p}}}\right) \\
&+ \log\left(\frac{1+2\max\{C, I_p\}}{1+C+I_p}\right) \\
&\leq \log(2) + 2\log(2) + \log(2) = 4\log(2),
\end{aligned}$$

while if $C/S_p > S_c$ then

$$\begin{aligned}
\text{GAP} &\leq \log(1+S_p) + \log(1+S_c) + \\
&- \log\left(\frac{1+S_c+I_p}{1+S_c+\frac{I_p}{1+I_p}\frac{C}{S_p}}\right) - \log\left(1 + \frac{S_c}{1+\frac{I_p}{1+I_p}}\right) \\
&- \log\left(1 + \frac{C}{1+I_p}\right) \\
&\leq \log\left(\frac{(1+S_p)(1+2C/S_p)}{1+I_p+C}\right) + \log\left(\frac{1+S_c}{1+\frac{S_c}{1+\frac{I_p}{1+I_p}}}\right) \\
&+ \log\left(\frac{1+I_p}{1+S_c+I_p}\right) \\
&\stackrel{1 \leq S_p \leq C}{\leq} \log\left(\max\left\{\frac{2(1+2C)}{1+C}, 3\right\}\right) + \log(2) + \log(1) \\
&= 3\log(2).
\end{aligned}$$

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