Joint User Grouping and Beamforming for Low Complexity Massive MIMO Systems

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***Abstract*—In massive MIMO systems, to partition users into groups and serve the groups separately can significantly reduce the processing complexity. In the existing literature, user grouping is done by classifying the channel covariance matrices. Consequently, the inter-group interference due to user grouping and per group processing is not taken into account. In addition, those methods only work for a fixed number of groups, and the optimal group number is hard to determine. In this paper, a joint user grouping and beamforming strategy is proposed to jointly optimize the number of groups, the user grouping, and the beamforming. The scheme is derived by maximizing the total expected signal-to-interference-leakage-and-noise-ratio (SLNR) lower bound in the network via two-timescale stochastic optimization techniques. Numerical results demonstrate significant sum rate performance gain over the baseline scheme in the literature.**

***Index Terms*—Massive MIMO, User grouping, Beamforming, Two-timescale stochastic optimization**

I. Introduction

Massive MIMO can achieve very high performance gain through transmitting the signal over a large number of antennas. However, the large dimension of the antenna array significantly increases the complexity in terms of channel state information (CSI) feedback and beamforming. One of the important techniques for complexity reduction is to separate users into groups so that the base station (BS) can compute the beamformers for each group separately. Towards this end, the work [1] proposed user grouping based on the second order channel statistics, such that the users in the same group share similar channel statistics in the spatial domain, but users from different groups have almost orthogonal statistics. Following the user grouping, the work [2] and some follow up works [3] and [4] proposed two-layer precoding strategies to decompose the beamformer into a *pre-beamformer* specified for each group and an *inner precoder* adaptive to the instantaneous channel for each user. With the pre-beamforming, the channel dimension is reduced, and the BS can process each group of users with much lower complexity. It was shown in [1] and [2] that good performance can be achieved with reduced complexity in instantaneous processing under the scenario that users are geographically clustered.

However, it is in general challenging to find a good user partition. First, in some scenarios, users may be uniformly distributed in geographic locations, and hence, their signal subspace may overlap with one another. As a result, it may be infeasible to set users apart so that the signal subspaces of different groups are nearly orthogonal. In this case, smart pre-beamformers should be jointly designed to steer the group signal to the null signal subspace of the other groups. Second, it is difficult to determine the optimal number of groups to form. More groups may lead to more inter-group interference, whereas, fewer groups may result in higher complexity in per group processing.

In prior works on user grouping, [1] and [5] studied several subspace based user grouping algorithms. These algorithms are mostly based on the *similarity* between the channel covariance matrices, but the inter-group interference due to user grouping and per group processing is ignored. Note that, if the users are already clustered in their geographic locations, the subspace based user grouping may work well. However, if the users are uniformly distributed, such methods may result in large inter-group interference. In addition, the number of groups G is fixed in [1] and [5], and it is not clear how to choose a good parameter G. Intuitively, the optimal G should depend on the user topology as well as the beamforming strategy. For a given user grouping result, a pre-beamforming problem was studied in [4], where a trace quotient problem was formulated to balance the signal energy and the inter group interference. However, without a good user grouping, the pre-beamforming would not achieve the potential performance of massive MIMO. In general, user grouping and beamforming should depend on each other. Such observation motivates a joint design on user grouping and beamforming, which has not been well addressed in the literature.

This paper attempts to jointly design the user grouping and beamforming for low complexity per grouping processing in massive MIMO systems. We formulate an optimization problem to jointly determine the number of groups G, the user grouping, and the associated pre-beamforming, as maximizing the overall expected SLNR of the network. Two efficient techniques are developed to find the solution. First, the problem is decomposed into an inner precoding problem and a joint user grouping and pre-beamforming problem. It is shown that the inner precoding and pre-beamforming can be relaxed to matrix trace quotient problems, which can be solved efficiently. Second, to solve the grouping problem, we apply the merge-and-split algorithm inspired from the coalitional game theory to search for a suboptimal grouping solution with flexible group numbers. Numerical results demonstrate significant sum rate performance gain over the subspace based baseline algorithm in [1].

II. System Model

A. Channel Model

Consider a single cell massive MIMO system, where the BS equips with N_t antennas and serves $K \leq N_t$ single antenna users. Denote the downlink channel between the BS and the kth user as $\mathbf{h}_k^{\mathrm{H}}$, where $\mathbf{h}_k \in \mathbb{C}^{N_t}$ is modeled as

$$\mathbf{h}_k = \mathbf{R}_k^{\frac{1}{2}} \mathbf{h}^{\omega} \tag{1}$$

in which $\mathbf{h}^{\omega} \sim \mathcal{CN}(0, \mathbf{I})$ is a random vector with independent and identically distributed (i.i.d.) elements , $\mathbf{R}_k = \mathbb{E}\left\{\mathbf{h}_k \mathbf{h}_k^{\mathrm{H}}\right\}$ is the channel covariance matrix known by the BS with $\mathrm{tr}\left\{\mathbf{R}_k\right\} = l_k N_t$, and l_k denotes the large-scale fading coefficient. It is assumed that $\mathrm{rank}(\mathbf{R}_k) \ll N_t$. In particular, consider that the transmission signal from the BS has narrow *angular spread (AS)* $\triangle$, and there is a diffuse field of isotropic scatters around the users. The (p, q)th entry of the transmit covariance matrix $\mathbf{R}_k$ is given by [6]

$$[\mathbf{R}_k]_{(p,q)} = l_k \int_{-\pi}^{\pi} e^{j[\phi_p(\theta) - \phi_q(\theta)]} \mathcal{P}(\theta - \overline{\theta}_k) d\theta \tag{2}$$

where $\phi_p(\theta) - \phi_q(\theta)$ accounts for the phase difference between the pth and qth antenna elements over the angle of departure (AOD) θ at the BS, the function $\mathcal{P}(\theta - \overline{\theta})$ is the power angular spectrum (PAS) with respect to (w.r.t.) the AOD θ at the BS.

The rank deficiency property motivates the two-layer beamforming structure as follows.

B. Two-layer Beamforming

Consider that K users are partitioned into G groups, where the number G is to be determined. The BS delivers message $x_{g,i}$ to user i in group g using beamformer $\widetilde{\mathbf{w}}_{g,i} \in \mathbb{C}^{N_t}$ with the following structure

$$\widetilde{\mathbf{w}}_{g,i} = \mathbf{V}_g \mathbf{w}_{g,i} \tag{3}$$

where $\mathbf{V}_g \in \mathbb{C}^{N_t \times M}$ is the pre-beamformer that is assigned identically to the users in group g, $\mathbf{w}_{g,i} \in \mathbb{C}^M$ is the inner precoder for each user i in group g, and M is a system parameter that can be chosen as the number of dominant eigenvalues of the channel covariance matrices $\mathbf{R}_k$. In particular, the pre-beamformer $\mathbf{V}_g$ is designed in conjunction with user grouping and adaptive to the *global* CSI statistics, whereas, $\mathbf{w}_{g,i}$ is computed from the *local* instantaneous CSI within a group.

Considering equal power allocation among all the users, the received signal at user i in group g is given by

$$y_{g,i} = \sqrt{\frac{P}{K}} \mathbf{h}_{g,i}^{\mathrm{H}} \mathbf{V}_g \mathbf{w}_{g,i} x_{g,i} + \sqrt{\frac{P}{K}} \sum_{j \in S_g \setminus \{i\}} \mathbf{h}_{g,i}^{\mathrm{H}} \mathbf{V}_g \mathbf{w}_{g,j} s_{g,j} + \sqrt{\frac{P}{K}} \sum_{l \neq g} \sum_{j \in S_l} \mathbf{h}_{g,i}^{\mathrm{H}} \mathbf{V}_l \mathbf{w}_{l,j} x_{l,j} + n_{g,i}$$

where S_g denotes the set of users in group g, the term $n_{g,i} \sim \mathcal{CN}(0, 1)$ is the additive Gaussian noise with unit variance, and P is the noise normalized total transmit power.

Assume that the BS knows perfectly the M-dimensional equivalent channels $\{\mathbf{h}_{g,k}^{\mathrm{H}} \mathbf{V}_g\}$. Since the inner precoders $\mathbf{w}_{g,i}$ are computed in a M-dimensional subspace for each group individually, the complexity has been significantly reduced.

The goal of this paper is to investigate how to separate the users into groups $S_1, S_2, \ldots, S_G$ with associated beamformers $\mathbf{V}_g$ and $\mathbf{w}_{g,i}$, and a properly chosen group number G.

III. Joint User Grouping and Beamforming Design

There are many beamforming methods proposed for multiuser massive MIMO systems in the literature. In particular, we focus here on the minimum mean-square error (MMSE) beamforming [7], which is simple and considered to achieve good performance from low to high signal-to-noise ratio (SNR).

Let $\mathbf{H}^{\mathrm{H}} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_K]^{\mathrm{H}} \in \mathbb{C}^{K \times N_t}$ be the composite channel matrix. The transmit beamforming matrix that satisfies the MMSE criteria is given by [7]

$$\widetilde{\mathbf{W}}^{\mathrm{MMSE}} = \mathbf{H}(\mathbf{H}^{\mathrm{H}} \mathbf{H} + \alpha \mathbf{I})^{-1}$$

where $\alpha = K/P$. From the result in [8], the kth normalized $\widetilde{\mathbf{w}}_k^{\mathrm{MMSE}}$ column of $\widetilde{\mathbf{W}}^{\mathrm{MMSE}}$ can be written as $\widetilde{\mathbf{w}}_k^{\mathrm{MMSE}} = e^{j\theta} \widetilde{\mathbf{w}}_k^{\mathrm{SLNR}}$, where $\widetilde{\mathbf{w}}_k^{\mathrm{SLNR}}$ is a vector obtained from the SLNR maximization criterion as follows

$$\widetilde{\mathbf{w}}_k^{\mathrm{SLNR}} = \arg \max_{\|\widetilde{\mathbf{w}}_k\|^2 = 1} \mathrm{SLNR}_k \triangleq \frac{|\mathbf{h}_k^{\mathrm{T}} \widetilde{\mathbf{w}}_k|^2}{\sum_{j \neq k} |\mathbf{h}_j \widetilde{\mathbf{w}}_k|^2 + \alpha}. \tag{4}$$

In this section, we exploit the above SLNR property to enable joint user grouping and beamforming for low complexity per group processing in massive MIMO.

A. Formulation via Stochastic Optimization

The two-layer beamforming structure in (3) imposes two additional constraints on the beamformer $\widetilde{\mathbf{w}}_{g,i}$: (i) the users in the same group g have the same pre-beamformer $\mathbf{V}_g$, which only depends on the channel statistics and the grouping result, and (ii) the inner precoder depends on the intra-group CSI. As a result, we cannot directly maximize the instantaneous SLNR in (4), but instead, the expected value $\mathbb{E}\{\sum_{i \in S_g} \mathrm{SLNR}_{g,i}\}$ for each group. A mathematical tool to tackle these structural constraints is multi-timescale stochastic optimization [9], [10] described as follows.

Let $\mathbf{H}_g^{\mathrm{H}} = [\{\mathbf{h}_k\}_{k \in S_g}]^{\mathrm{H}}$ be the composite channel matrix for group S_g. Let $\mathcal{S} = \{S_1, S_2, \ldots, S_g, \ldots\}$ be a user partition, where $S_i \neq \emptyset$, $S_i \bigcap S_j = \emptyset$ for $i \neq j$, and $\bigcup_g S_g = \{1, 2, \ldots, K\}$. The SLNR criteria in (4) under joint user grouping and two-layer beamforming is given by the following stochastic optimization problem

$$\begin{aligned} \max_{\mathcal{S}, \{\mathbf{V}_g\}, \{\mathbf{w}_{g,i}(\cdot)\}} \quad & \sum_{g=1}^{|\mathcal{S}|} \mathbb{E}\left\{\sum_{i \in S_g} \mathrm{SLNR}_{g,i}\right\} \\ \text{subject to} \quad & \|\mathbf{w}_{g,i}\|^2 = 1, \quad \forall i \in S_g, \forall g \\ & \mathbf{V}_g^{\mathrm{H}} \mathbf{V}_g = \mathbf{I}, \quad \forall g \end{aligned} \tag{5}$$

where

$$\mathrm{SLNR}_{g,i} = \frac{|\mathbf{h}_{g,i}^{\mathrm{H}}\mathbf{V}_g\mathbf{w}_{g,i}|^2}{\sum_{j\in S_g\setminus\{i\}}|\mathbf{h}_{g,j}^{\mathrm{H}}\mathbf{V}_g\mathbf{w}_{g,i}|^2 + \sum_{l\neq g, j\in S_l}|\mathbf{h}_{l,j}^{\mathrm{H}}\mathbf{V}_g\mathbf{w}_{g,i}|^2 + \alpha}$$

in which, the expectation is taken over the channel statistics of all the users in the network, and $\mathbf{w}_{g,i}(\mathbf{H}_g, \mathbf{V}_g)$ is a function to be optimized that depends on the pre-beamformer $\mathbf{V}_g$ and the intra-group channel realizations $\mathbf{H}_g$, the pre-beamformer $\mathbf{V}_g$ depends on the global channel statistics, and finally, the grouping variable $\mathcal{S}$ is to be optimized to achieve the maximum total expected SLNR.

Using the decomposition technique for two-timescale stochastic optimization in [10], the problem in (5) can be decomposed into a series of *inner precoding problems* for each channel realization $\mathbf{H}_g$ in each group g

$$\Gamma_g(\mathbf{V}_g;\mathbf{H}_g) = \max_{\{\mathbf{w}_{g,i};i\in S_g\}} \mathbb{E}\left\{\sum_{i\in S_g}\mathrm{SLNR}_{g,i}\big|\mathbf{H}_g\right\} \quad (6)$$
$$\text{subject to} \quad \|\mathbf{w}_{g,i}\|^2 = 1, \quad \forall i \in S_g$$

and a *joint user grouping and pre-beamforming problem*

$$\max_{\mathcal{S},\{\mathbf{V}_g\}} \sum_{g=1}^{|\mathcal{S}|}\mathbb{E}\left\{\Gamma_g(\mathbf{V}_g;\mathbf{H}_g)\right\} \quad (7)$$
$$\text{subject to} \quad \mathbf{V}_g^{\mathrm{H}}\mathbf{V}_g = \mathbf{I}, \qquad g = 1,2,\ldots,|\mathcal{S}|.$$

B. The Inner Precoding

For each group g, given the intra-group channel realization $\mathbf{H}_g$, the expectation is evaluated over the channels from outside the group g. Hence, it is very difficult to obtain the explicit expression of the objective function (6). To circumvent this challenge, we relax the inner problem by deriving a lower bound on $\mathbb{E}\left\{\mathrm{SLNR}_{g,i}\big|\mathbf{H}_g\right\}$ in terms of the second order statistics $\mathbf{R}_i$ for $i \notin S_g$ as follows.

Lemma 1 (Lower bound of the conditional SLNR): The following holds

$$\mathbb{E}\left\{\mathrm{SLNR}_{g,i}\big|\mathbf{H}_g\right\} \geq \frac{\mathbf{w}_{g,i}^{\mathrm{H}}\widetilde{\mathbf{G}}_{g,i}\mathbf{w}_{g,i}}{\mathbf{w}_{g,i}^{\mathrm{H}}\mathbf{T}_{g,i}\mathbf{w}_{g,i}} \quad (8)$$

where $\widetilde{\mathbf{G}}_{g,i} \triangleq \mathbf{V}_g^{\mathrm{H}}\mathbf{h}_{g,i}\mathbf{h}_{g,i}^{\mathrm{H}}\mathbf{V}_g$ and

$$\mathbf{T}_{g,i} = \sum_{j\in S_g\setminus\{i\}}\widetilde{\mathbf{G}}_{g,j} + \mathbf{V}_g^{\mathrm{H}}\Bigg(\sum_{l\neq g, j\in S_l}\mathbf{R}_{l,j}\Bigg)\mathbf{V}_g + \alpha\mathbf{I}.$$

As a result, the inner problem in (6) can be relaxed as the maximization on the expected SLNR lower bound in (8). This yields a standard trace quotient problem [11], and the maximizer $\mathbf{w}_{g,i}^{\star}$ of (8) is given by

$$\mathbf{w}_{g,i}^{\star} = \sqrt{\varphi_{g,i}}\mathbf{T}_{g,i}^{-1}\mathbf{V}_g^{\mathrm{H}}\mathbf{h}_{g,i} \quad (9)$$

where the coefficient $\varphi_{g,i}$ is to normalize $\mathbf{w}_{g,i}^{\star}$ such that $\|\mathbf{w}_{g,i}^{\star}\| = 1$.

C. The Joint User Grouping and Pre-beamforming

Using the inner precoding solution $\mathbf{w}_{g,i}^{\star}$ in (9), the maximum value of the conditional SLNR lower bound (8) is obtained, but the unconditional SLNR $\mathbb{E}\left\{\Gamma_g(\mathbf{V}_g;\mathbf{H}_g)\right\} = \mathbb{E}\left\{\mathbb{E}\left\{\mathrm{SLNR}_{g,i}\big|\mathbf{H}_g\right\}\right\}$ in (7) is still hard to compute. To compromise, we derive the lower bound of $\mathbb{E}\left\{\Gamma_g(\mathbf{V}_g;\mathbf{H}_g)\right\}$ in the following lemma.[1]

Lemma 2 (Lower bound of the expected SLNR): Under inner precoding (9), the expected SLNR can be lower bounded as

$$\mathbb{E}\left\{\Gamma_g(\mathbf{V}_g;\mathbf{H}_g)\right\} \geq \frac{\mathrm{tr}\left\{\mathbf{V}_g^{\mathrm{H}}\mathbf{Q}_g\mathbf{V}_g\right\}}{\mathrm{tr}\left\{\mathbf{V}_g^{\mathrm{H}}\mathbf{Q}_{-g}\mathbf{V}_g\right\}} \quad (10)$$

where

$$\mathbf{Q}_g = \bar{\mathbf{R}}_g - \frac{1}{M}\sum_{i\in S_g}\sum_{m=1}^{K_g-1}\lambda_m(\mathbf{R}_{g,i})\mathbf{I} \quad (11)$$

$$\mathbf{Q}_{-g} = \sum_{l\neq g}\bar{\mathbf{R}}_l + \frac{\alpha}{M}\mathbf{I} \quad (12)$$

in which $\bar{\mathbf{R}}_g \triangleq \sum_{i\in S_g}\mathbf{R}_{g,i}$ denotes the *aggregate covariance matrix* of the users in group g, $\lambda_m(A)$ denotes the mth largest eigenvalue of matrix A, and $K_g = |S_g|$ denotes the number of users in group g.

Using Lemma 2, we relax the maximization problem (7) by maximizing the lower bound of the objective function (7). The relaxed joint user grouping and pre-beamforming problem can be written as

$$\max_{\mathcal{S},\{\mathbf{V}_g\}} \sum_{g=1}^{|\mathcal{S}|}\frac{\mathrm{tr}\left\{\mathbf{V}_g^{\mathrm{H}}\mathbf{Q}_g\mathbf{V}_g\right\}}{\mathrm{tr}\left\{\mathbf{V}_g^{\mathrm{H}}\mathbf{Q}_{-g}\mathbf{V}_g\right\}} \quad (13)$$
$$\text{subject to} \quad \mathbf{V}_g^{\mathrm{H}}\mathbf{V}_g = \mathbf{I}, \qquad g = 1,2,\ldots,|\mathcal{S}|.$$

Remark 1 (Comparison to subspace based grouping): Mathematically, the subspace based grouping in [1] and [5] is to minimize the overall the *chordal* distance $\sum_{g=1}^{G}\sum_{i\in S_g}\|\mathbf{U}_i\mathbf{U}_i^{\mathrm{H}} - \bar{\mathbf{U}}_g\bar{\mathbf{U}}_g^{\mathrm{H}}\|_F^2$, where $\mathbf{U}_i$ is a $N_t \times M$ matrix that consists of M dominant eigenvector of $\mathbf{R}_i$, $\bar{\mathbf{U}}_g$ consists of M dominant eigenvectors of $\sum_{i\in S_g}\mathbf{U}_i\mathbf{U}_i^{\mathrm{H}}$ (representing the group center), and the number of groups G is fixed. As a result, the inter-group interference is ignored. As a comparison, the objective (13) characterizes the expected SLNR, where the inter-group interference is captured. Moreover, the problem (13) also optimizes the number of groups G.

IV. Algorithms for Joint User Grouping and Pre-beamforming

The joint user grouping and pre-beamforming problem (13) is hard to solve, since the number of feasible user partitions is exponential to the number of users K. Note that, even for a fixed group number G in the simple subspace based grouping, the K-means algorithm proposed in [1] and [5] does not guarantee to find the optimal grouping. Therefore, we only focus on suboptimal solutions to (13).

[1]Related result can be found in [4], but the lower bound here is tighter.

Algorithm 1 Pre-beamforming Algorithm

1) Initialization: Obtain $\mathbf{Q}_g$ and $\mathbf{Q}_{-g}$ from (11)-(12). Choose a starting point $\mathbf{V}_g^{(0)}$.
2) Given $\mathbf{V}_g^{(n)}$, compute $\rho^{(n)} = \frac{\text{tr}\left\{\mathbf{V}_g^{(n)\text{H}}\mathbf{Q}_g\mathbf{V}_g^{(n)}\right\}}{\text{tr}\left\{\mathbf{V}_g^{(n)\text{H}}\mathbf{Q}_{-g}\mathbf{V}_g^{(n)}\right\}}$.
3) Choose $\mathbf{V}_g^{(n+1)}$ to be the eigenvectors of $\mathbf{Q}_g - \rho^{(n)}\mathbf{Q}_{-g}$ corresponding to the M largest eigenvalues.
4) Repeat from Step 2 until $(\rho^{(n+1)} - \rho^{(n)})/\rho^{(n)} < \epsilon$.

A. Solution to the Pre-beamformer

Given the user grouping $\mathcal{S}$, the optimal pre-beamformer $\mathbf{V}_g^\star = \mathbf{V}_g^\star(\mathcal{S})$ is the solution that maximizes the matrix trace quotient tr $\left\{\mathbf{V}_g^{\text{H}}\mathbf{Q}_g\mathbf{V}_g\right\}$ /tr $\left\{\mathbf{V}_g^{\text{H}}\mathbf{Q}_{-g}\mathbf{V}_g\right\}$. Although such maximization problem is non-convex, there exists iterative algorithm that is proven to converge to the global optimal solution $\mathbf{V}_g^\star$ at *quadratic* convergence rate [11], [12]. The pre-beamforming algorithm is summarized in Algorithm 1.

A good property of Algorithm 1 is that the objective value $\rho^{(n)}$ in Step 2) strictly increases every step. Since the algorithm converges at quadratic rate, one can terminate the algorithm after very few steps, without losing too much precision to the optimal solution $\mathbf{V}_g^\star$.

B. Algorithm for User Grouping

Given a user grouping $\mathcal{S}$, define $v(S_g)$ as a function that quantifies the value of forming user group S_g. In particular, we choose

$$v(S_g) \triangleq \frac{\text{tr}\left\{(\mathbf{V}_g^\star)^{\text{H}}\mathbf{Q}_g\mathbf{V}_g^\star\right\}}{\text{tr}\left\{(\mathbf{V}_g^\star)^{\text{H}}\mathbf{Q}_{-g}\mathbf{V}_g^\star\right\}}$$

i.e., approximately the maximum SLNR for group S_g. From Section IV-A, $v(S_g)$ can be efficiently computed.

The joint user grouping and pre-beamforming problem in (13) is equivalent to maximizing the total grouping value $V_G = \sum_{g=1}^{|\mathcal{S}|} v(S_g)$ over all feasible user grouping $\mathcal{S}$. To avoid exhaustive enumeration, we propose an iterative algorithm based on two rules, *merge* and *split*. The merge-and-split algorithm was introduced and studied in [13] for coalition formation in cooperative game theory and applied to many applications in wireless communication systems [14], [15]. The merge and split algorithm for user grouping is summarized in Algorithm 2.

A good property of the merge-and-split algorithm is that it terminates in finite steps. This is because each merge or split operation strictly increases the total value V_G, but since there are only finite feasible user partitions $\mathcal{S}$, the merge-and-split iteration must reach a local maximum point in finite steps.

To implement the merge and split algorithm, a good initialization is important to accelerate the algorithm termination and avoid bad local optimal point. For example, if the initial grouping is the grand user set itself, i.e., $\mathcal{S} = \{S_1\} = \{1, 2, \ldots, K\}$, then it costs a huge complexity to try every combination of splitting S_1. On the other hand, if the initial grouping is to form each user a group itself, i.e., $\mathcal{S} = \{\{1\}, \{2\}, \ldots, \{K\}\}$, then the algorithm may suffer from bad local optimum. This is because, the initial inter-group interference too large due to the large number of groups, and consequently, merging any two groups cannot significantly reduce the inter-group interference but lose some signal energy. To avoid these, we choose the K-means algorithm to partition the users into G_0 groups based on the dominant subspace of the covariance matrices.

Algorithm 2 User Grouping via Merge and Split

1) Initialization: Choose a initial user grouping $\mathcal{S} = \{S_1, S_2, \ldots\}$.
2) **Merge:** Merge any two user groups S_i and S_j, if the merge yields a higher group value, i.e., $\{S_i, S_j\} \to S_i \bigcup S_j$ if $v(S_i \bigcup S_j) > v(S_i) + v(S_j)$.
3) **Split:** Split a group S_i into two subgroups $S_i^{(1)}$ and $S_i^{(2)}$, if the split yields a higher sum group value, i.e., $S_i \to \{S_i^{(1)}, S_i^{(2)}\}$ if $v(S_i) > v(S_i^{(1)}) + v(S_i^{(2)})$.
4) Repeat the merge and split operations until the user grouping does not change anymore.

K-means initialization:

1) Choose a parameter G_0 and randomly select G_0 users $k_1, k_2, \ldots k_{G_0}$. Let $\bar{\mathbf{U}}_g = \mathbf{U}_{k_g}$, for $g = 1, 2, \ldots, G_0$, where $\mathbf{U}_k$ consists of M dominant eigenvectors of the covariance matrix $\mathbf{R}_k$.
2) For each user k, choose a group g such that $\|\mathbf{U}_k\mathbf{U}_k^{\text{H}} - \bar{\mathbf{U}}_g\bar{\mathbf{U}}_g^{\text{H}}\|_F^2 \le \|\mathbf{U}_k\mathbf{U}_k^{\text{H}} - \bar{\mathbf{U}}_l\bar{\mathbf{U}}_l^{\text{H}}\|_F^2$ for all $l \ne g$. Let $k \to S_g$.
3) For each group g, let $\bar{\mathbf{U}}_g$ be a matrix that contains the M dominant eigenvectors of $\sum_{i \in S_g} \mathbf{U}_i\mathbf{U}_i^{\text{H}}$.
4) Repeat from 2) until the user grouping does not change.

Remark 2 (Complexity): Since the user grouping is updated in a slow timescale (e.g., once over thousands of channel realizations), the computational complexity could be ignored in the long run. The system can still benefit from the complexity reduction for inner precoding under the two-layer structure.

V. Numerical results

Consider a single cell massive MIMO system with users randomly and uniformly distributed in a 600×400[m] area. The BS equips with $N_t = 60$ antennas and is placed at the coordinate $(0, -200[\text{m}])$ as illustrated in Fig. 1. The extended WINNER channel model [16] under the *urban macro scenario* (light-of-sight (LOS) case) is adopted to model the path loss. The transmit covariance is modeled as (2), with AS $\triangle = 15$ degree and uniform PAS. The noise at the receiver is normalized such that the average path gain is 0 dB.

The performance of the proposed scheme is compared to the following **baseline** (subspace based method in [1]): users are grouped based on the chordal distance on the dominant eigen subspace of the covariance matrices using the K-means algorithm.

Fig. 2 shows the sum rate over the total number of users in the cell under total transmission power $P = 10$ dB. The

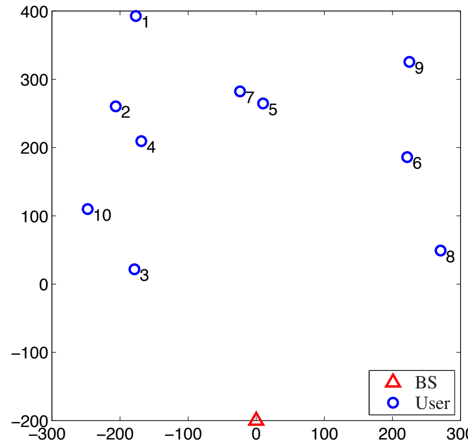

Figure 1. A sample of the user placement, where there are $K = 10$ users and the BS is at $(0, -200[\text{m}])$.

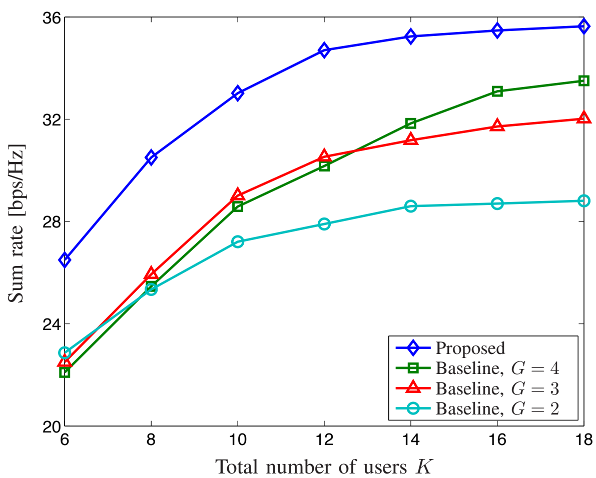

Figure 2. Sum rate over the total number of users in the cell under total transmission power $P = 10$ dB.

parameter M for two-layer beamforming is chosen as $M = 8$.[2] First of all, for the baseline in [1], it is better to have fewer groups when the number of users K is small, but more groups when K is large. However, the baseline cannot dynamically adjust the group number G. As a comparison, the proposed scheme always performs the best, because it not only adapts the number of groups to the total number of users and channel statistics, but also it is aware of the inter-group interference via the proposed joint user grouping and beamforming design. Second, for a large number of users, the marginal sum rate gain of adding more users to the network decreases. This is because the inter-group interference also increases due to the uniform placement of the users.

[2]The choice of M follows the guideline in [1], which is close to but smaller than the number of dominant eigenvalues of the channel covariance matrix.

VI. Conclusions

In this paper, a joint user grouping and beamforming strategy was proposed under the two-layer beamforming structure. The problem was formulated as the maximization of the total expected SLNR in the network. Using multi-timescale stochastic optimization techniques, the problem was decomposed into a series of inner precoding problems and a joint user grouping and pre-beamforming problem, where closed form solutions were derived for the inner problems and a merge-and-split algorithm was proposed for the outer problem. As a comparison to the conventional design where the number of groups is fixed in a heuristic way, under the proposed method, the number of groups, the user grouping, and the pre-beamformer are jointly optimized. Numerical results demonstrated significant performance gain in terms of sum rate over the baseline scheme in the literature.

Appendix

A. Proof of Lemma 1 (Sketch):

The lower bound can be derived as follows

$$\mathbb{E}\left\{\text{SLNR}_{g,i}\middle|\mathbf{H}_g\right\} \geq \frac{|\mathbf{h}_{g,i}^{\text{H}}\mathbf{V}_g\mathbf{w}_{g,i}|^2}{\sum\limits_{(l,j)\neq(g,i)} \mathbb{E}\left\{|\mathbf{h}_{l,j}^{\text{H}}\mathbf{V}_g\mathbf{w}_{g,i}|^2\middle|\mathbf{H}_g\right\}+\alpha}$$

due to the *Jesen's inequality* on the convex function $f(x) = \frac{a}{b+cx}$. The denominator can be derived as

$$\begin{aligned}
&\sum_{(l,j)\neq(g,i)} \mathbb{E}\left\{|\mathbf{h}_{l,j}^{\text{H}}\mathbf{V}_g\mathbf{w}_{g,i}|^2\middle|\mathbf{H}_g\right\}+\alpha \\
&\quad= \sum_{j\in S_g\setminus\{i\}} \mathbf{w}_{g,i}^{\text{H}}\mathbf{V}_g^{\text{H}}\mathbf{h}_{g,j}\mathbf{h}_{g,j}^{\text{H}}\mathbf{V}_g\mathbf{w}_{g,i} \\
&\qquad+ \sum_{l\neq g, j\in S_l} \mathbf{w}_{g,i}^{\text{H}}\mathbf{V}_g^{\text{H}}\mathbb{E}\left\{\mathbf{h}_{l,j}\mathbf{h}_{l,j}^{\text{H}}\middle|\mathbf{H}_g\right\}\mathbf{V}_g\mathbf{w}_{g,i}+\alpha \\
&\quad= \mathbf{w}_{g,i}^{\text{H}}\mathbf{V}_g^{\text{H}}\left(\sum_{j\in S_g\setminus\{i\}}\mathbf{G}_{g,j}+\sum_{l\neq g, j\in S_l}\mathbf{R}_{l,j}+\alpha\mathbf{I}\right)\mathbf{V}_g\mathbf{w}_{g,i}
\end{aligned}$$

where the first equality is due to the fact that the inner precoder $\mathbf{w}_{g,i}$ only depends on the in-group channel realization $\mathbf{H}_g$ but not the inter-group channels $\{\mathbf{h}_{l,j}\}$ for $l \neq g$, and moreover, $\mathbf{V}_g$ only depends on the channel statistics but not the channel realizations. The second equality is because the channels between users are independent, i.e., $\mathbb{E}\{\mathbf{h}_{l,j}\mathbf{h}_{l,j}^{\text{H}}|\mathbf{H}_g\} = \mathbb{E}\{\mathbf{h}_{l,j}\mathbf{h}_{l,j}^{\text{H}}\} = \mathbf{R}_{l,j}$, and $\mathbf{G}_{g,\cdot} \triangleq \mathbf{h}_{g,j}\mathbf{h}_{g,j}^{\text{H}}$.

With further straight-forward manipulations, the results in Lemma 1 can be obtained.

B. Proof of Lemma 2 (Sketch):

The lower bound can be derived as follows

$$\begin{aligned}
\mathbb{E}\left\{\Gamma_g(\mathbf{V}_g;\mathbf{H}_g)\right\} &= \mathbb{E}\left\{\max_{\{\mathbf{w}_{g,i}(\cdot)\}}\mathbb{E}\left\{\sum_{i\in S_g}\text{SLNR}_{g,i}(\mathbf{w}_{g,i})\middle|\mathbf{H}_g\right\}\right\} \\
&\geq \mathbb{E}\left\{\sum_{i\in S_g}\gamma_{g,i}^{\text{L}}(\mathbf{w}_{g,i}^{\text{ZF}};\mathbf{V}_g,\mathbf{H}_g)\right\} \qquad (14)
\end{aligned}$$

where $\gamma^{\rm L}_{g,i}(\mathbf{w}_{g,i};\mathbf{V}_g,\mathbf{H}_g) \triangleq \frac{\mathbf{w}^{\rm H}\widetilde{\mathbf{G}}_{g,i}\mathbf{w}}{\mathbf{w}^{\rm H}\mathbf{T}_{g,i}\mathbf{w}}$ takes the form of the lower bound in Lemma 1, in which $\widetilde{\mathbf{G}}_{g,i}$ and $\mathbf{T}_{g,i}$ are given in Lemma 1 for each user i in group g. The inequality is due to the fact that the zero-forcing (ZF) precoder $\mathbf{w}^{\rm ZF}_{g,i}$ is not optimal in maximizing $\gamma^{\rm L}_{g,i}(\mathbf{w};\mathbf{V}_g,\mathbf{H}_g)$.

The function $\gamma^{\rm L}_{g,i}(\mathbf{w}^{\rm ZF}_{g,i};\mathbf{V}_g,\mathbf{H}_g)$ can be evaluated as

$$\frac{|\mathbf{h}^{\rm H}_{g,i}\mathbf{V}_g\mathbf{w}^{\rm ZF}_{g,i}|^2}{\left(\mathbf{w}^{\rm ZF}_{g,i}\right)^{\rm H}\left[\mathbf{V}^{\rm H}_g\left(\sum_{l\neq g}\bar{\mathbf{R}}_l\right)\mathbf{V}_g\right]\mathbf{w}^{\rm ZF}_{g,i}+\alpha} \tag{15}$$

where $\bar{\mathbf{R}}_g \triangleq \sum_{i\in S_g}\mathbf{R}_{g,i}$ denotes the *aggregate covariance matrix* of users in group g.

1) The numerator of (15): In particular, the ZF precoder $\mathbf{w}^{\rm ZF}_{g,i}$ in (15) can be written as

$$\mathbf{w}^{\rm ZF}_{g,i} = \frac{\mathbf{P}_{g,i}\mathbf{V}^{\rm H}_g\mathbf{h}_{g,i}}{\|\mathbf{P}_{g,i}\mathbf{V}^{\rm H}_g\mathbf{h}_{g,i}\|}$$

where

$$\mathbf{P}_{g,i} = \mathbf{I} - \widetilde{\mathbf{H}}_{g,-i}(\widetilde{\mathbf{H}}^{\rm H}_{g,-i}\widetilde{\mathbf{H}}_{g,-i})^{-1}\widetilde{\mathbf{H}}^{\rm H}_{g,-i}$$

is a $M\times M$ projection matrix to project the equivalent channel $\mathbf{V}^{\rm H}_g\mathbf{h}_{g,i}$ for user i in group g to the null space of spanned by the vectors $\{\mathbf{V}^{\rm H}_g\mathbf{h}_{g,j} : j\in S_g, j\neq i\}$. Here $\widetilde{\mathbf{H}}_{g,-i} = \mathbf{V}^{\rm H}_g\mathbf{H}_{g,-i}$, $\mathbf{H}_{g,-i} = [\{\mathbf{h}_{g,j} : j\in S_g, j\neq i\}]$ is the $N_t\times(K_g-1)$ matrix containing the channel vectors in group g except user i, and $K_g = |S_g|$ is the number of users in group g.

Using the projection matrix property $\mathbf{P}_{g,i} = \mathbf{P}_{g,i}\mathbf{P}^{\rm H}_{g,i} = \mathbf{P}^{\rm H}_{g,i}$, the following holds

$$\begin{aligned}|\mathbf{h}^{\rm H}_{g,i}\mathbf{V}_g\mathbf{w}^{\rm ZF}_{g,i}|^2 &= \frac{|\mathbf{h}^{\rm H}_{g,i}\mathbf{V}_g\mathbf{P}_{g,i}\mathbf{V}^{\rm H}_g\mathbf{h}_{g,i}|^2}{\|\mathbf{P}_{g,i}\mathbf{V}^{\rm H}_g\mathbf{h}_{g,i}\|^2}\\ &= \|\mathbf{P}_{g,i}\mathbf{V}^{\rm H}_g\mathbf{h}_{g,i}\|^2 = \text{tr}\left\{\mathbf{h}^{\rm H}_{g,i}\mathbf{V}_g\mathbf{P}_{g,i}\mathbf{V}^{\rm H}_g\mathbf{h}_{g,i}\right\}.\end{aligned}$$

Therefore, we have

$$\begin{aligned}&\mathbb{E}\left\{|\mathbf{h}^{\rm H}_{g,i}\mathbf{V}_g\mathbf{w}^{\rm ZF}_{g,i}|^2\middle|\mathbf{H}_{g,-i}\right\}\\ &\quad= \mathbb{E}\left\{\text{tr}\left\{\mathbf{h}^{\rm H}_{g,i}\mathbf{V}_g\mathbf{P}_{g,i}\mathbf{V}^{\rm H}_g\mathbf{h}_{g,i}\right\}\middle|\mathbf{H}_{g,-i}\right\}\\ &\quad= \text{tr}\left\{\mathbf{P}_{g,i}\mathbf{V}^{\rm H}_g\mathbf{R}_{g,i}\mathbf{V}_g\mathbf{P}_{g,i}\right\}\\ &\quad\geq \sum_{m=K_g}^{M}\lambda_m(\mathbf{V}^{\rm H}_g\mathbf{R}_{g,i}\mathbf{V}_g)\\ &\quad= \text{tr}\left\{\mathbf{V}^{\rm H}_g\mathbf{R}_{g,i}\mathbf{V}_g\right\} - \textstyle\sum_{m=1}^{K_g-1}\lambda_m(\mathbf{V}^{\rm H}_g\mathbf{R}_{g,i}\mathbf{V}_g)\\ &\quad\geq \text{tr}\left\{\mathbf{V}^{\rm H}_g\mathbf{R}_{g,i}\mathbf{V}_g\right\} - \textstyle\sum_{m=1}^{K_g-1}\lambda_m(\mathbf{R}_{g,i})\end{aligned}$$

where the function $\lambda_m(A)$ yields the mth largest eigenvalue of A. In the inequality, the equality is achieved when the channel vectors in $\mathbf{H}_{g,-i}$ span the (K_g-1)-dimensional dominant eigen subspace of $\mathbf{V}^{\rm H}_g\mathbf{R}_{g,i}\mathbf{V}_g$, and consequently, the projector $\mathbf{P}_{g,i}$ projects the matrix $\mathbf{V}^{\rm H}_g\mathbf{R}_{g,i}\mathbf{V}_g$ onto the subspace spanned by the $M-K_g+1$ eigenvectors corresponding to the $M-K_g+1$ least eigenvalues.

Since the above result does not depend on channel realization $\mathbf{H}_{g,-i}$, we have $\mathbb{E}\left\{|\mathbf{h}^{\rm H}_{g,i}\mathbf{V}_g\mathbf{w}^{\rm ZF}_{g,i}|^2\right\} \geq \text{tr}\left\{\mathbf{V}^{\rm H}_g\mathbf{R}_{g,i}\mathbf{V}_g\right\} - \sum_{m=1}^{K_g-1}\lambda_m(\mathbf{R}_{g,i})$.

2) The denominator of (15): The first term in the denominator of (15) can be upper bounded as

$$\begin{aligned}\left(\mathbf{w}^{\rm ZF}_{g,i}\right)^{\rm H}\left[\mathbf{V}^{\rm H}_g\left(\sum_{l\neq g}\bar{\mathbf{R}}_l\right)\mathbf{V}_g\right]\mathbf{w}^{\rm ZF}_{g,i} &= \left\|\left(\sum_{l\neq g}\bar{\mathbf{R}}_l\right)^{\frac{1}{2}}\mathbf{V}_g\mathbf{w}^{\rm ZF}_{g,i}\right\|^2\\ &\leq \left\|\left(\sum_{l\neq g}\bar{\mathbf{R}}_l\right)^{\frac{1}{2}}\mathbf{V}_g\right\|_2^2\|\mathbf{w}^{\rm ZF}_{g,i}\|^2 \leq \text{tr}\left\{\mathbf{V}^{\rm H}_g\textstyle\sum_{l\neq g}\bar{\mathbf{R}}_l\mathbf{V}_g\right\}\end{aligned}$$

where $\|\cdot\|$ is the vector Euclidean norm, and $\|\cdot\|_2$ is the matrix Euclidean norm (spectral norm).

With further straight-forward manipulations, the results in Lemma 2 can be obtained.

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