Initial Synchronization of DS-CDMA via Bursty Pilot Signals

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Abstract

We consider initial timing acquisition in DS-CDMA when propagation is affected by multipath and fading and where the base-station broadcasts a synchronization pilot signal in the form of bursts of modulated chips transmitted periodically and separated by long silent intervals. Subject to certain simplifying assumptions we derive the maximum likelihood (ML) estimator by solving a constrained maximization problem. Our ML timing estimator has constant complexity per observation sample. The relation to other estimation methods is addressed, and performance comparisons are provided by simulation. The proposed estimator yields good performance independently of the multipath-intensity profile of the channel, provided that the delay spread is not larger than a given maximum spread. Moreover, our estimator is fairly robust to the mismatch in the fading Doppler spectrum and provides good performance for both fast and slow fading.

Keywords: Synchronization, Timing Estimation, DS-CDMA Systems, Wireless Communications.

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1 Introduction and Motivation

In wireless mobile communication systems a mobile terminal (MT) must acquire the time reference of the base-station (BS) before starting communication. When data transmission occurs with a slot structure the basic time reference is slot timing. In third generation wireless communication systems [1] initial synchronization is facilitated by a pilot signal transmitted by the BS. In particular, all BSs broadcast a common *primary synchronization signal*. When the MT is switched on, it detects the presence and the timing of this signal. If the slot time reference of at least one BS is successfully acquired, the MT searches for some secondary synchronization signal carrying additional information (e.g., frame synchronization, BS identification). Once the BS is identified, the MT can send a call request on the BS random access channel.

In both frequency-division duplex and time-division duplex modes of UMTS [2, 3], the primary synchronization signal is bursty, i.e., it is non-zero only for a small fraction of time compared with the slot duration. In particular, the primary synchronization signal consists of a given burst of modulated chips separated by long silent intervals and repeated indefinitely. Motivated by this scheme, we consider the general problem of initial slot timing acquisition in a DS-CDMA system with a bursty pilot signal. In [4] a maximum likelihood (ML) timing estimator is derived by assuming that the channel multipath-intensity profile (MIP) is known at the receiver, and in [5, 6] the ML criterion is applied by modeling the channel as deterministic unknown and constant with time. Unfortunately the multipath intensity profile is not known *before* initial acquisition and, since the initial synchronization phase may last several slots, the algorithms based on the constant channel assumption might perform poorly in the presence of time-varying fading.

In this paper, we obtain a low-complexity slot timing estimator requiring a minimum amount of prior knowledge and yielding good performances on a wide range of channel MIPs and fading Doppler bandwidths. Since both the channel MIP and the noise plus interference power spectral density are not known at the receiver, we formulate a joint ML problem where all these parameters have to be estimated. We derive the ML estimator by solving a constrained maximization problem via the Karush-Kuhn-Tucker (KKT) conditions [7]. In order to obtain a tractable solution we make several simplifying working assumptions. When these are not satisfied, our estimator is not exactly ML and may suffer from mismatch. The proposed estimator is compared with the simple estimator for flat fading [8], with the too optimistic estimator of [4], and with the estimator of [5] for two typical channel MIPs under both the assumption of slow fading (constant channel) and fast fading. Computer simulations show that the proposed joint ML estimator exhibits good performances in all these conditions and it is fairly robust to mismatch of the assumed fading statistics.

2 Signal Model

The continuous-time complex baseband received signal is given by

$$y(t) = x(t;\theta) + v(t) \tag{1}$$

where $v(t)$ represents noise plus interference, modeled as a zero-mean complex circularly-symmetric Gaussian process with power spectral density I_0, and $x(t;\theta)$ is the received synchronization signal component, given by

$$x(t;\theta) = \sum_{m=0}^{M-1} \int h(t, t-\tau) s(\tau - \theta - mT)\, \mathrm{d}\tau \tag{2}$$

where θ is the slot timing, $s(t)$ is the bursty pilot waveform of duration T_s, $T \gg T_s$ is the period of repetition of $s(t)$, $h(t,\tau)$ is the time-varying multipath channel impulse response and M is the number of transmitted pilot bursts. Because of the slot periodicity, θ must be estimated modulo T. The channel is assumed to follow the wide-sense stationary uncorrelated scattering model (WSS-US [8]) with Rayleigh fading and multipath impulse response

$$h(t,\tau) = \sum_{p=0}^{P-1} c_p(t)\delta(\tau - \tau_p) \tag{3}$$

where $c_p(t)$ is the time-varying complex channel gain at delay τ_p. Since T_s is much shorter than T, we assume that the channel coherence time T_{coh} [8] satisfies $T_s < T_{\mathrm{coh}} \leq T$. This implies that the channel is almost constant during each m-th burst, but changes independently from burst to burst. Since T_s is very short[1] this condition is referred to as *fast fading*, and holds approximately for $\frac{1}{T} \leq B_d < \frac{1}{T_s}$, where B_d is the fading Doppler bandwidth. Moreover, we assume that the delays τ_p are constant over the whole observation window of duration MT.

[1] In the UMTS standard proposal [2, 3] the pilot waveform consists of a sequence of 256 chips convolved with a root-raised cosine chip-shaping pulse. Thus with good approximation we may say that $T_s \approx 270$ chip periods, whereas T can range from one slot (2560 chips) to one frame period (15 slots).

In practical systems [9] the period T ranges from one slot period to one frame period and the multipath delays vary at a much lower rate than the slot rate. Thus this assumption is satisfied. Subject to the above assumptions, $x(t;\theta)$ in (2) can be conveniently rewritten as

$$x(t;\theta) = \sum_{m=0}^{M-1} \sum_{p=0}^{P-1} c_{mp} s(t-\theta-\tau_p - mT) \tag{4}$$

where c_{mp} are complex zero-mean circularly-symmetric Gaussian mutually uncorrelated random variables such that $E\{c_{mp}c_{nq}^*\} = \sigma_p^2 \delta_{m,n}\delta_{p,q}$. The channel MIP is defined by the delays $\boldsymbol{\tau} = (\tau_0, \ldots, \tau_{P-1})$ and by the path variances $\boldsymbol{\sigma} = (\sigma_0^2, \ldots, \sigma_{P-1}^2)$. The pilot waveform is given by

$$s(t) = \sum_{n=0}^{N-1} s_n \psi(t - nT_c) \tag{5}$$

where T_c is the chip duration, s_n is a sequence of N chips known at the receiver and $\psi(t)$ is the chip-shaping pulse band-limited to $[-\frac{W_\psi}{2}, \frac{W_\psi}{2}]$ with $\frac{1}{T_c} \leq W_\psi \leq \frac{2}{T_c}$. In a digital receiver implementation the signal is low-pass filtered and sampled at a convenient rate $W > W_\psi$. Hence, baseband processing is performed in discrete time. We assume $W = n_c/T_c$, where $n_c > 1$ is the number of samples per chip. Let $Q = WT$ denote the number of samples per pilot repetition period, and define the discrete-time observed signal $\mathbf{y} = (y[0], \ldots, y[MQ-1])^T$. After a straightforward derivation, it is possible to write $\mathbf{y}$ in the compact form

$$\mathbf{y} = \mathbf{S}\mathbf{c} + \mathbf{v} \tag{6}$$

where $\mathbf{v} \triangleq (v[0], \ldots, v[MQ-1])^T$ is the vector of interference plus noise samples, $\mathbf{c} \triangleq (\mathbf{c}_0^T, \ldots, \mathbf{c}_{M-1}^T)^T$, with $\mathbf{c}_m \triangleq (c_{m0}, \ldots, c_{m(P-1)})^T$, contains all MP channel path coefficients over the M periods, and where $\mathbf{S}$ is the $MQ \times MP$ matrix whose $(mP+p)$-th column, for $m = 0, \ldots, M-1$ and $p = 0, \ldots, P-1$, is given by

$$\mathbf{s}_{mp} \triangleq (\underbrace{0, \ldots, 0}_{mQ}, s_p[0], \ldots, s_p[Q-1], \underbrace{0, \ldots, 0}_{(M-m-1)Q})^T \tag{7}$$

where

$$s_p[i] = \frac{1}{\sqrt{W}} \sum_{n=0}^{N-1} s_n \psi(i/W - \theta - \tau_p - nT_c) \tag{8}$$

Notice that the dependence of $\mathbf{y}$ on θ is *hidden* in the matrix $\mathbf{S}$. Since the columns of $\mathbf{S}$ are obtained by translating the same waveform, they have all the same square magnitude, equal to the energy E_s of the pilot waveform. Then, without loss of generality we can include the term E_s into the path variances σ_p^2, and consider $|s_{mp}|^2 = 1$ for all m, p and delay θ. Under our assumptions, $\mathbf{y}$ is a zero-mean complex circularly-symmetric Gaussian random vector with covariance matrix

$$\mathbf{R}_y = \mathbf{S}\mathbf{\Lambda}\mathbf{S}^H + I_0\mathbf{I}_{MQ} \tag{9}$$

where the superscript $(\cdot)^H$ denotes the Hermitian transpose, $\mathbf{I}_m$ indicates the $m \times m$ identity matrix and where $\mathbf{\Lambda}$ is the covariance matrix of $\mathbf{c}$, given by $\mathbf{\Lambda} = \mathbf{I}_M \otimes \mathbf{\Lambda}_c$ with $\mathbf{\Lambda}_c = \mathrm{diag}(\sigma_0^2, \ldots, \sigma_{P-1}^2)$ ($\otimes$ denotes Kronecker product [10]).

3 Maximum Likelihood Problem Formulation

The log-likelihood function for the parameters vector $\boldsymbol{\theta} = (\theta, \boldsymbol{\tau}, \boldsymbol{\sigma}, I_0)$ is immediately obtained as [11]

$$\mathcal{L}(\mathbf{y}|\boldsymbol{\theta}) = -\log\det(\mathbf{R}_y) - \mathbf{y}^H\mathbf{R}_y^{-1}\mathbf{y} \tag{10}$$

Notice that the number of channel paths P is also unknown and it is implicitly contained into the parameters $\boldsymbol{\tau}$ and $\boldsymbol{\sigma}$. By applying the matrix inversion lemma [10] to (9) we get

$$\mathbf{R}_y^{-1} = \frac{1}{I_0}\mathbf{I}_{MQ} - \frac{1}{I_0^2}\mathbf{S}\mathbf{\Lambda}^{1/2}\left(\mathbf{I}_{MP} + \frac{1}{I_0}\mathbf{\Lambda}^{1/2}\mathbf{S}^H\mathbf{S}\mathbf{\Lambda}^{1/2}\right)^{-1}\mathbf{\Lambda}^{1/2}\mathbf{S}^H \tag{11}$$

Now, we make the key working assumption that the columns of $\mathbf{S}$ are mutually orthogonal. In particular, this is approximately verified if the delays τ_p are sufficiently spaced apart (more than T_c) and if the sequence of chips s_n has a very peaky aperiodic autocorrelation function. In practice, the pilot sequence s_n is quite long ($N = 256$ in UMTS [9]) and paths spaced by less than one chip interval are substantially treated as a single path (they are not *resolvable* [8]), thus, our assumption is not very restrictive. If $\mathbf{S}$ has orthonormal columns, (11) reduces to

$$\mathbf{R}_y^{-1} = \tfrac{1}{I_0}\left(\mathbf{I}_{MQ} - \mathbf{S}\mathbf{\Sigma}\mathbf{S}^H\right) \tag{12}$$

where $\mathbf{\Sigma} = \mathbf{I}_M \otimes \mathrm{diag}\left(\frac{\xi_0}{1+\xi_0}, \ldots, \frac{\xi_{P-1}}{1+\xi_{P-1}}\right)$ and where we define the path average signal-to-noise ratio (SNR) $\xi_p = \sigma_p^2/I_0$, for $p = 0, \ldots, P-1$. Subject to the same assumption, the determinant of $\mathbf{R}_y$ is readily obtained as

$$\det(\mathbf{R}_y) = \left[\prod_{p=0}^{P-1}(1+\xi_p)\right]^M I_0^{MQ} \tag{13}$$

By using (12) and (13) in (10) and by defining the total received signal energy $E_y = |\mathbf{y}|^2$ and the vector of path average SNRs $\boldsymbol{\xi} = (\xi_0, \ldots, \xi_{P-1})$, we obtain the log-likelihood function in the form

$$\mathcal{L}(\mathbf{y}|\boldsymbol{\theta}) = \frac{1}{I_0}\left[\sum_{p=0}^{P-1}\frac{\xi_p}{1+\xi_p}X(\theta+\tau_p) - E_y\right] - MQ\log I_0 - M\sum_{p=0}^{P-1}\log(1+\xi_p) \tag{14}$$

where we define

$$X(\theta+\tau_p) \triangleq \sum_{m=0}^{M-1}|\mathbf{s}_{mp}^H\mathbf{y}|^2 \tag{15}$$

The term $X(\theta+\tau_p)$ is obtained by summing for $m = 0,\ldots,M-1$ the squared magnitude of the output of a discrete time filter with impulse response matched to the delayed pilot waveform $s(t-\theta-mT-\tau_p)$. Then, this is the output of a sort of *square-law diversity combiner* [8] collecting the signal energy over all the M pilot repetition periods, for a given guess of the timing θ and multipath component with delay τ_p. We shall refer to these terms as the *received path energies* at delay $\theta+\tau_p$. From the implementation point of view, $X(\theta+\tau_p)$ for all $p = 0,\ldots,P-1$ can be calculated by sampling at delays $\theta+\tau_p$ the output of the *same* filter matched to $s(t)$.

4 Approximated ML Estimation

In order to find the ML estimate of θ we should jointly estimate also the other unknown parameters $\boldsymbol{\tau},\boldsymbol{\xi}$ and I_0. For simplicity of exposition, we first assume that the delays $\boldsymbol{\tau}$ are known. Then, for any given value of θ we want to maximize the log-likelihood function with respect to $\boldsymbol{\xi}$ and I_0, i.e., we want to solve the constrained maximization problem

$$\begin{cases} \text{maximize} & \mathcal{L}(\mathbf{y}|\theta,\boldsymbol{\xi},\boldsymbol{\tau},I_0) \\ \text{subject to} & \boldsymbol{\xi}\geq\mathbf{0},\quad I_0\geq 0 \end{cases} \tag{16}$$

This can be solved by using the KKT method [7]. In this section we let $X_p \triangleq X(\theta+\tau_p)$ for the sake of notation simplicity.

We have the following:

Proposition 1. The solution of the maximization problem (16) is given by

$$\xi_p = \left[\frac{1}{\beta}X_p - 1\right]_+, \quad I_0 = \frac{\beta}{M}, \quad \beta = \frac{E_y - \sum_{p\in\mathcal{D}}X_p}{Q-D} \tag{17}$$

where $[\cdot]_+$ denotes positive part, and where we define the set of indexes $\mathcal{D} = \{p\in[0,P-1] : X_p > \beta\}$, of cardinality $|\mathcal{D}| = D$. Moreover, this solution exists and is unique for any set of received signal energies $\{X_p : p = 0, \ldots, P-1\}$, for all $Q > P$ and $M \geq 1$ and for all E_y such that $\sum_{p=0}^{P-1} X_p < E_y < \frac{Q}{P} \sum_{p=0}^{P-1} X_p$.

Proof. See appendix A. $\Diamond$

Solution (17) has the following intuitive interpretation: β acts as an adaptive threshold level. If $X_p > \beta$ then $\theta + \tau_p$ is a good candidate for being a true channel path.

Let π be the permutation of P indexes sorting the X_p's in non-decreasing order, i.e., such that $X_{\pi(0)} \geq X_{\pi(1)} \geq \cdots \geq X_{\pi(P-1)}$. Then, by substituting the solution of Proposition 1 into (14) we obtain the maximized log-likelihood function in the form

$$\overline{\mathcal{L}}(\mathbf{y}|\theta, \boldsymbol{\tau}) \stackrel{\triangle}{=} \max_{\boldsymbol{\xi}, I_0} \mathcal{L}(\mathbf{y}|\boldsymbol{\theta}) = -(Q-D) \log \frac{E_y - \sum_{p=0}^{D-1} X_{\pi(p)}}{Q-D} - \log \prod_{p=0}^{D-1} X_{\pi(p)} \tag{18}$$

The exact ML joint estimator of $\boldsymbol{\theta}$ is obtained by further maximizing $\overline{\mathcal{L}}(\mathbf{y}|\theta, \boldsymbol{\tau})$ with respect to all possible $\boldsymbol{\tau}$ and θ. While maximization with respect to θ involves searching in a one-dimensional real space, with complexity $\mathcal{O}(Q)$, the maximization with respect to $\boldsymbol{\tau}$ requires a search over P-dimensional real spaces for all $P = 1, 2, \ldots$, which is highly impractical.

In order to circumvent this hurdle, we restrict the search over a set of tentative delay vectors $\boldsymbol{\tau}$ selected according to some criteria. At the same time, the set of tentative $\boldsymbol{\tau}$ should enforce the condition that the delays are separated by more than T_c, which guarantees that the columns of $\mathbf{S}$ are approximately orthonormal. The selection of the tentative $\boldsymbol{\tau}$ is based on the following proposition and heuristic considerations.

Proposition 2. For a given fixed θ, let $\{\tau_p : p = 0, 1, \ldots\}$ be a sequence of delays such that the sequence of corresponding received energies $\{X_p : p = 0, 1, \ldots\}$ is non-increasing. Then, the sequence of log-likelihood functions $\{\overline{\mathcal{L}}(\mathbf{y}|\theta, \tau_0, \ldots, \tau_p) : p = 0, 1, \ldots\}$ is non-decreasing. Moreover, if for some $D \geq 1$ the condition

$$X_D \leq \beta_D \stackrel{\triangle}{=} \frac{E_y - \sum_{i=0}^{D-1} X_i}{Q-D} \tag{19}$$

holds, then $\max_{p \geq 0} \overline{\mathcal{L}}(\mathbf{y}|\theta, \tau_0, \ldots, \tau_p) = \overline{\mathcal{L}}(\mathbf{y}|\theta, \tau_0, \ldots, \tau_{D-1})$.

Proof. See appendix A. $\Diamond$

From Proposition 2 it follows that we can construct a set of "nested" tentative delay vectors of dimension $P = 1, 2, \ldots$ in sequence, by appending more and more delays corresponding to decreasing received energies. The maximum of the log-likelihood function over the (infinite) delay vector sequence is found when, for some (finite) integer D the condition (19) holds. Furthermore, we observe that:

1. The log-likelihood function increases with the received energy sum $A = \sum_{p=0}^{D-1} X_{\pi(p)}$ and decreases with the product $B = \prod_{p=0}^{D-1} X_{\pi(p)}$. For a given value of A, $\overline{\mathcal{L}}(\mathbf{y}|\theta, \boldsymbol{\tau})$ is maximized by a "peaky" distribution of the path energies X_p's. In fact, the product B is maximum for the uniform distribution $X_{\pi(0)} = \cdots = X_{\pi(D-1)} = A/D$ while it is small if the received energies are spread, i.e., if $X_{\pi(0)} \gg X_{\pi(1)} \gg \cdots \gg X_{\pi(D-1)}$ (notice that $X_{\pi(D-1)} > \beta > 0$, so B cannot be zero).

2. In practice, systems are designed to handle multipath channels up to a given maximum delay spread T_d (see e.g. [9]) (defined here as $T_d \stackrel{\triangle}{=} \max_p \tau_p - \min_p \tau_p$ [8]). Then, for a given θ, the channel paths are expected to be contained in a time window $[\theta, \theta + T_d]$, where T_d is *a priori* known.

Driven by the above two considerations, a meaningful choice of the sequence of delays $\{\tau_p : p = 0, 1, \ldots\}$ is to choose for each p the delay $\tau_p \in [0, T_d]$ for which X_p is maximum and τ_p has not appeared before in the sequence. Moreover, since the paths should be separated by at least one chip interval, we place around each chosen τ_p a *forbidden region* of size $2T_c$. The newly selected delays should not belong to the forbidden regions of already selected delays. The resulting approximated ML algorithm is given as follows.

Initial timing estimation. For all $\theta \in [0, T]$ we compute the value $\widehat{\mathcal{L}}(\mathbf{y}|\theta)$ given by the recursion: initialize $p = 0$, $\beta_0 = E_y/Q$, and the search interval $\mathcal{S}_0 = [0, T_d]$, then

1. Select $\tau_p = \arg\ \max_{\tau \in \mathcal{S}_p} X(\theta + \tau)$, and let $X_p = X(\theta + \tau_p)$.

2. If $X_p > \beta_p$, let $\beta_{p+1} = \frac{E_y - \sum_{i=0}^{p} X_i}{Q-p-1}$, let $\mathcal{S}_{p+1} = \mathcal{S}_p \setminus [\tau_p - T_c, \tau_p + T_c]$,[2] then increment p by 1 and go to step 1.

3. If $X_p \leq \beta_p$ or if $\mathcal{S}_{p+1}$ is empty, compute $\widehat{\mathcal{L}}(\mathbf{y}|\theta) = -(Q-p)\log\beta_p - \log\prod_{i=0}^{p-1} X_i$ and exit the recursion.

Finally, the estimated slot timing is given by $\widehat{\theta} = \theta_m + \widehat{\tau}_0$, where

$$\theta_m = \arg\ \max_{\theta \in [0,T]} \widehat{\mathcal{L}}(\mathbf{y}|\theta) \tag{20}$$

[2]The notation $\mathcal{A} \setminus \mathcal{B}$ denotes the complement of the set $\mathcal{B}$ with respect to the set $\mathcal{A}$.

and where $\widehat{\tau}_0$ is the estimated delay of the first path (also provided by the above algorithm) corresponding to θ_m. $\Diamond$

As far as implementation is concerned, some considerations are in order.

- Up to now, we considered the parameter θ as continuous. However, in a digital implementation, the search for the maximum of $\widehat{\mathcal{L}}(\mathbf{y}|\theta)$ is done on a discrete set of values. The most computationally demanding operation is the computation of the matched filter output $z_{mp}(\theta) = \mathbf{s}_{mp}^H \mathbf{y}$ involved in the calculation of $X(\theta + \tau_p)$ (see (15)). In most practical applications it is sufficient to acquire the slot timing with an error of less than one chip. Therefore, a very fine discretization of θ is not needed. In our numerical examples, we discretize θ with step $1/W$, so that we need just a matched filter operating at the signal sampling rate W, whose output is given by $z[i] = \sum_j y[j]s((j+i)/W)^*$. The receiver accumulates the squared matched filter outputs in a vector buffer $\mathbf{b} = (b[0], \ldots, b[Q-1])$ such that $b[i] = \sum_{m=0}^{M-1} |z[mQ+i]|^2$. The search of the delays and the search for the maximum in (20) is performed over the discrete values $\{t_k = k/W : k = 0, \ldots, Q-1\}$ by processing the data buffer $\mathbf{b}$.

- We have implicitly assumed that the pilot bursts fall approximately in the middle of the observation intervals $[mT, (m+1)T]$. However, the initial timing reference of the MT is arbitrary, and the pilot bursts may fall across the boundaries of the observation intervals. Since the slot timing is defined modulo T, in order to solve this problem it is sufficient to apply the estimation algorithm by treating the data buffer $\mathbf{b}$ as a *circular buffer*.

- The complexity of the proposed algorithm is linear in the observation size QM, as opposed to other timing algorithms based on least-squares (LS) [5] or subspace decomposition, which require matrix-vector multiplication or matrix eigen-analysis (see e.g. [12] and references therein).

- As byproducts of initial timing estimation, the proposed method also provides estimates for the interference plus noise power spectral density I_0, for the path SNRs ξ_p, for the path delays τ_p and for the number of paths P. These can be used to speed-up some terminal setup procedures [2, 3]. For example, the delays corresponding to the largest path SNRs can be used to initialize the fine delay search of a rake receiver, and the knowledge of I_0 can be exploited to initialize the power control loop.

5 Results

We compare the performances of our estimator, hereafter denoted by joint ML (JML), with the following estimators:

- The least computationally intensive estimator that selects as estimated delay the position of the maximum element in the buffer $\mathbf{b}$, in the sequel denoted as the "MAX" estimator.

- The approximated least-squares estimator (denoted by "LS") which is an approximation of the exact LS estimator that coincides with the ML estimator subject to the assumption of constant channel. In fact, when the channel is constant during the observation interval the ML slot timing estimator is given by [5, 6]

$$\hat{\theta} = \arg\max_{\theta} \bar{\mathbf{y}}^H \bar{\mathbf{S}} (\bar{\mathbf{S}}^H \bar{\mathbf{S}})^{-1} \bar{\mathbf{S}}^H \bar{\mathbf{y}} \tag{21}$$

  where $\bar{\mathbf{y}} = \frac{1}{M}\sum_{m=1}^{M} \mathbf{y}_m$, $\mathbf{y}_m = (y[(m-1)Q], \ldots, y[mQ-1])^T$, and where $\bar{\mathbf{S}}$ is the $Q \times L$ convolution matrix whose l-th column is given by $\bar{\mathbf{s}}_l = (s_l[0], \ldots, s_l[Q-1])^T$ where

$$s_l[i] = \frac{1}{\sqrt{W}} \sum_{n=0}^{N-1} s_n \psi((i-l-1)/W - nT_c - \theta)$$

  and L is the channel delay spread expressed in sample periods. From the implementation point of view the algorithm corresponds to filtering the received signal $\bar{\mathbf{y}}$ with a filter matched to the pilot waveform, windowing the filtered signal by the $L \times L$ matrix $(\bar{\mathbf{S}}^H \bar{\mathbf{S}})^{-1}$, taking the magnitude square of the output signal and finding the maximum. Even though the matrix $(\bar{\mathbf{S}}^H \bar{\mathbf{S}})^{-1}$ can be pre-computed this approach is extremely computational intensive since it requires the computation of L scalar products per output sample (i.e. for each possible θ). Therefore we consider the more practical approximated LS estimator based on the assumption $\bar{\mathbf{S}}^H \bar{\mathbf{S}} \approx \mathbf{I}$, also proposed in [5].

- The ML estimator with perfect knowledge of both the channel MIP and the interference power spectral density I_0 (denoted by "ML/known MIP") proposed in [4]. Since all the parameters but θ are known the log-likelihood function (14) for this estimator reduces to

$$\mathcal{L}(\mathbf{y}|\theta) = \sum_{p=0}^{P-1} \frac{\xi_p}{1+\xi_p} X(\theta + \tau_p) \tag{22}$$

Table 1: Channel MIPs

MIP	**Delays** (T_c)	**Variances** (dB)
CH1	$(0, 1, 2, 3)$	$(0, -3, -6, -9)$
CH2	$(0, 7.5)$	$(0, 0)$
CH3	$(0, 1.12, 1.34, 4.85, 5.35, 5.62)$	$(-3, 0, -3, -5, -7, -8)$
CH4	$(0, 7.5, 15)$	$(0, 0, 0)$

As performance measure we use the root-mean square error (RMSE) of the estimated slot timing normalized with respect to the chip interval (i.e., it is expressed in fraction of T_c) and the cumulative density function (CDF) $F_e(E)$ of the sum of the energies of the paths falling in the estimated window $W_e = [\hat{\theta}, \hat{\theta} + T_d]$, normalized with respect to the total channel energy. The CDF gives the probability that the fraction of the total channel energy "captured" inside W_e is below a given level E. Since in general the performance of the subsequent synchronization phase, involving BS identification, frame synchronization etc., depends on the fraction of captured channel energy (no matter which algorithm is used), the CDF is able to characterize the performance of the whole synchronization procedure, independently of the particular algorithm employed in the second phase.

In our examples the chip-shaping pulse is root-raised cosine with roll-off factor $\alpha = 0.22$. As pilot sequence, we used a PN sequence of length $N = 255$ and the UMTS primary synchronization sequence of length $N = 256$ defined in [13]. The receiver sampling rate is $W = 4/T_c$ and both the pilot repetition interval and the slot duration are equal to $T = 625T_c$, corresponding to $Q = 2500$ samples (in reality Q may be much larger but we were limited by the simulation time). In our simulations we considered four typical channel MIPs described in table 1 and denoted by CH1 [14], CH2, CH3, and CH4.

Figures 1–4 show the timing RMSE versus the pilot energy to interference plus noise ratio E_s/I_0 for channel MIPs CH1 and CH3 under both constant and fast fading conditions. We refer to fast fading conditions when the channel coherence time is much smaller than the period of repetition of the pilot (but still larger than the pilot duration). Notice that the "JML" and the "ML/known MIP" estimators are mismatched for constant channel, while the "LS" estimator is mismatched for fast fading. The number of accumulated bursts is fixed to $M = 10$.

The “LS” estimator performs very poorly in the presence of fast fading because is mismatched (see figures 2 and 4). Indeed in the presence of fast Rayleigh fading, as in our scenario, the amplitude of the averaged received signal $\bar{\mathbf{y}} = \frac{1}{M}\sum_m \mathbf{y}_m$ decreases as the number of accumulated bursts M increases. On the contrary the “JML” estimator shows good performances also in the presence of a constant channel despite it is mismatched in this case. Moreover it always performs closely to the “ML/known MIP” estimator.

Extensive simulations show that increasing M reduces the E_s/I_0 needed to achieve a given RMSE, without affecting the relative behaviors of the estimators.

Figures 5–8 show the CDF $F_e(E)$ of the fraction of channel energy captured in the window W_e for channels CH2 and CH4. In these figures the CDF of the “MAX” estimator shows abrupt transitions each corresponding to the loss of the channel energy associated with a path. For instance, in the case of channel MIP CH2, the transition occurs at $E = -3$ dB that corresponds to the loss of half the channel energy. This effect is due to a false lock of the “MAX” estimator on the the second path of the channel.

In the presence of fast fading the “JML” estimator generally outperforms all other estimators but the “ML/known MIP” that is based on non-realistic assumptions. In the presence of constant channel our estimator performs better than the “MAX” and the “LS” estimators for high E_s/I_0 (i.e. greater than 10 dB when $M = 10$) and it performs always close to the “ML/known MIP” estimator, even though it is mismatched.

Finally figures 9 and 10 compare the performances of the “JML” estimator in terms of RMSE versus E_s/I_0 for fast fading channel CH1 and CH3 when using the UMTS and the PN sequence, for $M = 10$ accumulations. Performances are worse with the UMTS sequence due to its worse acyclic autocorrelation properties.

6 Conclusions

Motivated by the initial BS acquisition procedure of UMTS, we considered the problem of slot timing estimation based on bursty pilot signals. Subject to some simplifying working assumptions, we derived a low-complexity algorithm based on joint ML estimation of the slot timing, of the multipath intensity profile and of the interference plus noise power spectral density and we solved the likelihood function maximization problem by using the KKT conditions. Comparisons with other algorithms such as the simple peak detection, the ideal algorithm that exploits perfect knowledge of the channel multipath intensity profile and an approximated least-squares are provided. The proposed algorithm shows good performances in all the considered conditions, especially for very sparse channels and under fast fading conditions. Moreover it shows significant robustness to mismatch namely to the presence of constant channels and non-ideal acyclic autocorrelation properties of the pilot sequence.

Appendix

A Proofs

A.1 Proof of Proposition 1

We restate the maximization problem (16) more in general as follows. For arbitrary P, let $\{X_p : p = 0, \ldots, P-1\}$ be a non-increasing non-negative sequence, let $Q > P$ and $M > 1$ be arbitrary integers and let $\sum_{p=0}^{P-1} X_p < E_y < \frac{Q}{P} \sum_{p=0}^{P-1} X_p$. We want to maximize the function

$$f(\boldsymbol{\xi}, I_0) = \frac{1}{I_0} \left[\sum_{p=0}^{P-1} \frac{\xi_p}{1+\xi_p} X_p - E_y \right] - MQ \log I_0 - M \log \prod_{p=0}^{P-1} X_p$$

subject to $\boldsymbol{\xi} \geq \mathbf{0}$ and $I_0 \geq 0$.

The KKT necessary conditions for a (local) maximum point of the objective function in the constraint set yield the system of inequalities

$$\begin{aligned} &\frac{\partial}{\partial \xi_i} f(\boldsymbol{\xi}, I_0) \leq 0, \qquad \text{for} \quad i = 0, \ldots, P-1 \\ &\frac{\partial}{\partial I_0} f(\boldsymbol{\xi}, I_0) \leq 0 \end{aligned} \tag{23}$$

where strict equality must hold if $\xi_i > 0$ or $I_0 > 0$.

By solving the inequality for ξ_i we obtain $\xi_i \geq X_i/(MI_0) - 1$. Since the RHS of the inequality for ξ_i is negative if $\xi_i = 0$ and $X_i/(MI_0) < 1$, we conclude that the solution is given by

$$\xi_i = \left[\frac{X_i}{\beta} - 1 \right]_+ \tag{24}$$

where we let $\beta = MI_0$. By substituting this into the inequality for I_0, we obtain

$$\beta = \frac{E_y - \sum_{p=0}^{D-1} X_p}{Q - D} \tag{25}$$

where $D \leq P$ is the minimum integer for which $X_D < \beta$. Notice that since $E_y > \sum_{p=0}^{P-1} X_p$, then $\beta > 0$ thus $I_0 > 0$ and (24), (25) satisfy the KKT necessary conditions.

Next, we prove that this solution exists and is unique. Finally, we prove that this is actually the *global* maximizer for our problem.

Existence. For $d = 1, \ldots, P$, let

$$\beta_d = \frac{E_y - \sum_{p=0}^{d-1} X_p}{Q - d} \tag{26}$$

and define the set $\mathcal{D}_d = \{X_p : X_p > \beta_d\}$. The solution (24), (25) exists if the equation $|\mathcal{D}_d| = d$ has a solution for some $D \in \{1, \ldots, P\}$.

By definition of β_d, we have that $X_0 > \beta_1$. If for $1 \leq p \leq P-1$ the condition $X_p > \beta_p$ is never verified, then the solution is obviously $D = 1$. Otherwise, there must exist some $1 \leq p \leq P-1$ such that $X_p > \beta_p$. Let d be the maximum of such indexes. We can write

$$\begin{aligned} X_d &> \beta_d \\ &= \frac{E_y - \sum_{p=0}^{d-1} X_p}{Q - d} \\ &= \frac{Q-d-1}{Q-d-1}\left(\frac{E_y - \sum_{p=0}^{d-1} X_p}{Q-d} + \frac{X_d}{Q-d} - \frac{X_d}{Q-d}\right) \\ &= \frac{Q-d-1}{Q-d}\beta_{d+1} + \frac{X_d}{Q-d} \end{aligned} \tag{27}$$

which implies $X_d > \beta_{d+1}$. Since by construction $d = \max\{p : X_p > \beta_p\}$, then $X_{d+1} \leq \beta_{d+1}$, which implies that $D = d + 1$ is a solution.

Uniqueness. Suppose that there exist $1 \leq D < D' \leq P$ such that $|\mathcal{D}_D| = D$ and $|\mathcal{D}_{D'}| = D'$, i.e.,

$$\begin{cases} X_p > \beta_D & \text{for } 0 \leq p \leq D-1 \\ X_p \leq \beta_D & \text{for } D \leq p \leq P-1 \end{cases} \tag{28}$$

and

$$\begin{cases} X_p > \beta_{D'} & \text{for } 0 \leq i \leq D'-1 \\ X_p \leq \beta_{D'} & \text{for } D' \leq i \leq P-1 \end{cases} \tag{29}$$

By following the same steps of (27) we can show that

$$\beta_d > X_d \Rightarrow \beta_{d+1} > X_d \tag{30}$$

Then, starting from (28) we can write the chain of inequalities $\beta_D \geq X_D \Rightarrow \beta_{D+1} \geq X_D \geq X_{D+1} \Rightarrow \beta_{D+2} \geq X_{D+1} \geq X_{D+2} \Rightarrow \cdots \Rightarrow \beta_{D'} \geq X_{D'-1}$, which contradicts (29). We conclude that the solution must be unique.

Global maximum. The function $f(\boldsymbol{\xi}, I_0)$ is not concave in $\mathbb{R}_+^{P+1}$. However, it is continuous on the (convex) constraint $\mathbb{R}_+^{P+1}$ and $f(\boldsymbol{\xi}, I_0) \to -\infty$ if any of its variables grows without bound. By continuity, the global maximum of $f(\boldsymbol{\xi}, I_0)$ is finite and the maximizer must have all finite components, therefore it must satisfy the necessary KKT conditions. Since the solution (24), (25) exists and is unique, it must be the global maximizer.

A.2 Proof of Proposition 2

Let $\{X_p : p = 0, 1, \ldots\}$ denote a sequence of non-increasing path energies. If for some D the condition (19) holds, then $\beta_D \geq X_p$ for all $p \geq D$ and, by Proposition 1, the function $\mathcal{L}(\mathbf{y}|\theta, \xi_0, \ldots, \xi_p, \tau_0, \ldots, \tau_p, I_0)$ for all $p \geq D$ is maximized with respect to the ξ_i's and I_0 by letting $\beta = \beta_D = (E_y - \sum_{i=0}^{D-1} X_i)/(Q - D)$, $I_0 = \beta/M$ and $\xi_i = [X_i/\beta - 1]_+$. Since β_D is determined only by the X_i's for $i = 0, \ldots, D-1$, by adding more delays to the sequence $\tau_0, \ldots, \tau_{D-1}$ with received energy $X_p \leq \beta_D$ is not going to affect neither the value of β_D nor the value of the maximum $\overline{\mathcal{L}}(\mathbf{y}|\theta, \tau_1, \ldots, \tau_p)$.

Therefore, the only situation that must be taken into account is when, for given p, $\beta_p < X_{p-1}$ and we add a delay τ_p with received energy $X_p > \beta_p$. From the proof of Proposition 1 we have that $X_p > \beta_p \Rightarrow X_p > \beta_{p+1}$, i.e., $\mathcal{L}(\mathbf{y}|\xi_0, \ldots, \xi_p, \tau_0, \ldots, \tau_p, I_0)$ is actually maximized by all $\xi_i > 0$, implying that $\overline{\mathcal{L}}(\mathbf{y}|\theta, \tau_0, \ldots, \tau_{p-1}) \neq \overline{\mathcal{L}}(\mathbf{y}|\theta, \tau_0, \ldots, \tau_p)$. Proposition 2 is proved by showing that

$$\overline{\mathcal{L}}(\mathbf{y}|\theta, \tau_0, \ldots, \tau_{p-1}) \leq \overline{\mathcal{L}}(\mathbf{y}|\theta, \tau_0, \ldots, \tau_p)$$

By using the fact that

$$\beta_{p+1} = \frac{Q-p}{Q-p-1}\left(\beta_p - \frac{X_p}{Q-p}\right)$$

and by letting $Q - p = k$ we have

$$\begin{aligned}
&\overline{\mathcal{L}}(\mathbf{y}|\theta, \tau_0, \ldots, \tau_p) - \overline{\mathcal{L}}(\mathbf{y}|\theta, \tau_0, \ldots, \tau_{p-1}) = \\
&\quad = -(k-1)\log\beta_{p+1} - \log\prod_{i=0}^{p} X_i + k\log\beta_p + \log\prod_{i=0}^{p-1} X_i \\
&\quad = -k\log\left[\frac{k}{k-1}\left(1 - \frac{X_p}{k\beta_p}\right)\right] + \log\left[\frac{k}{k-1}\left(\frac{\beta_p}{X_p} - \frac{1}{k}\right)\right] \\
&\quad = (k-1)\log\frac{1-1/k}{\beta_p/X_p - 1/k} + k\log\frac{X_p}{\beta_p} \\
&\quad > 0
\end{aligned}$$

since the arguments of both logarithms in the third line above are larger than 1. This concludes the proof.

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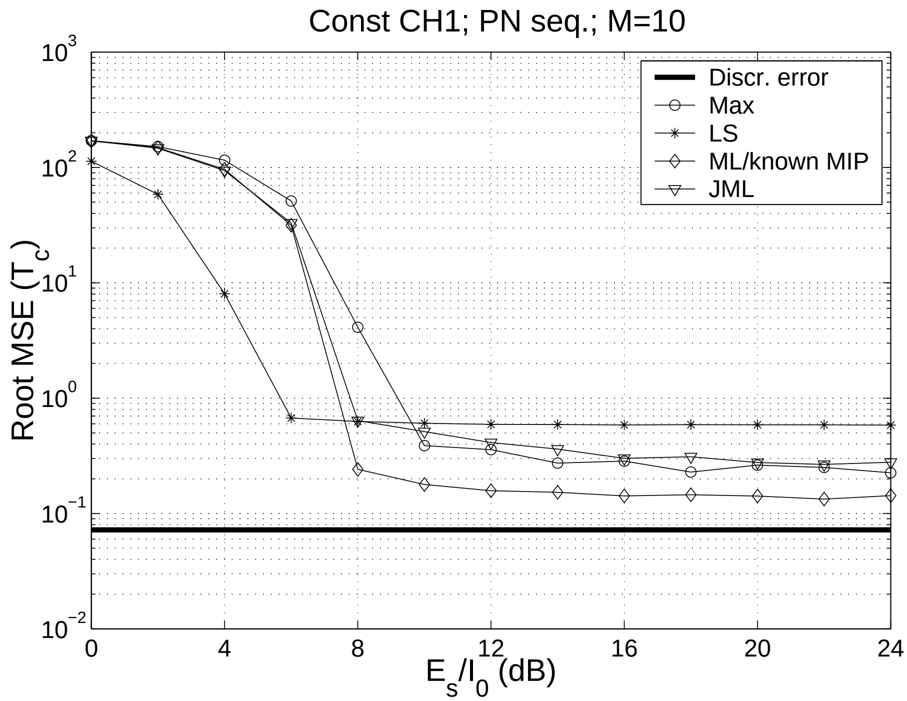

Figure 1: RMSE with constant CH1 vs. E_s/I_0 for $M = 10$ accumulated bursts

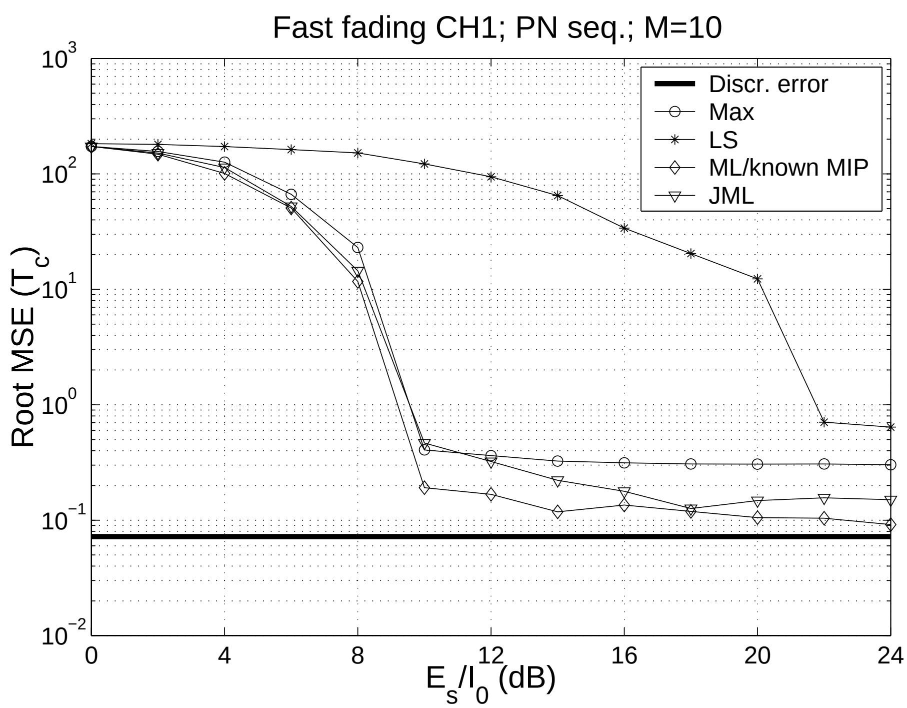

Figure 2: RMSE with fast fading CH1 vs. E_s/I_0 for $M = 10$ accumulated bursts

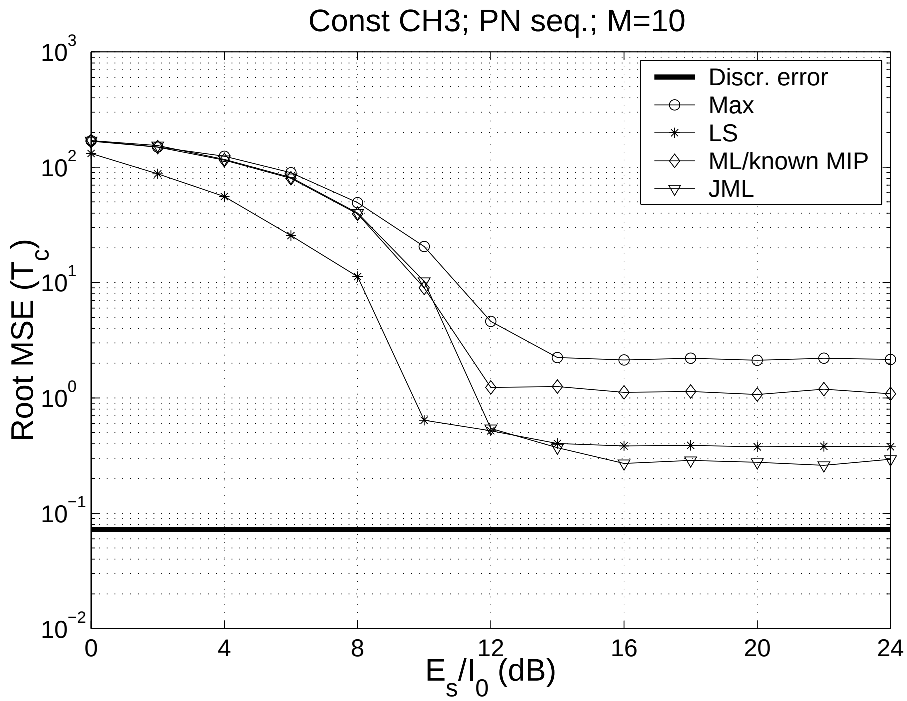

Figure 3: RMSE with constant CH3 vs. E_s/I_0 for $M = 10$ accumulated bursts

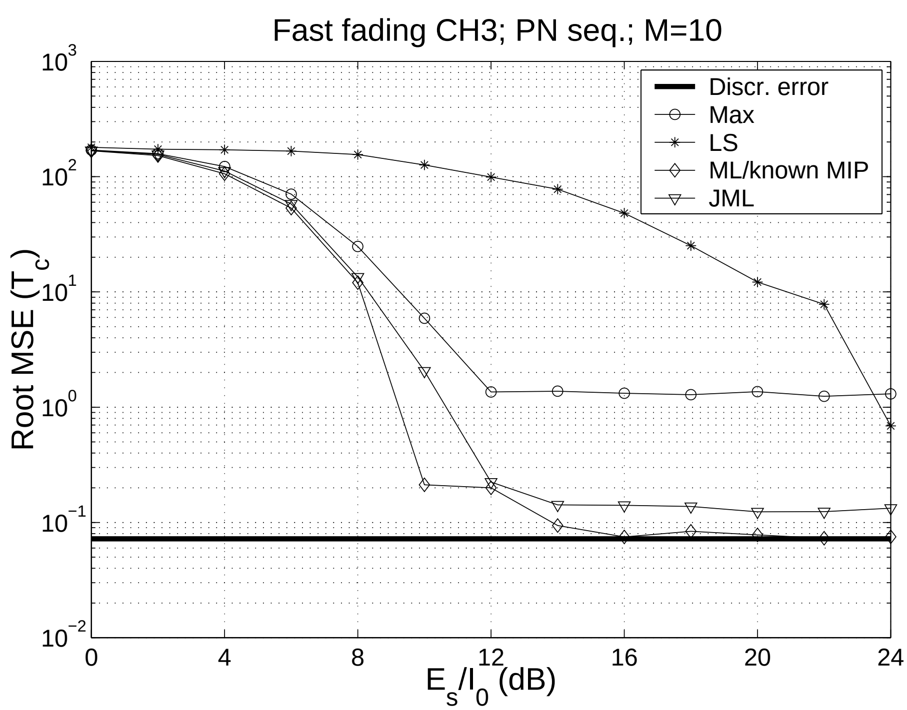

Figure 4: RMSE with fast fading CH3 vs. E_s/I_0 for $M = 10$ accumulated bursts

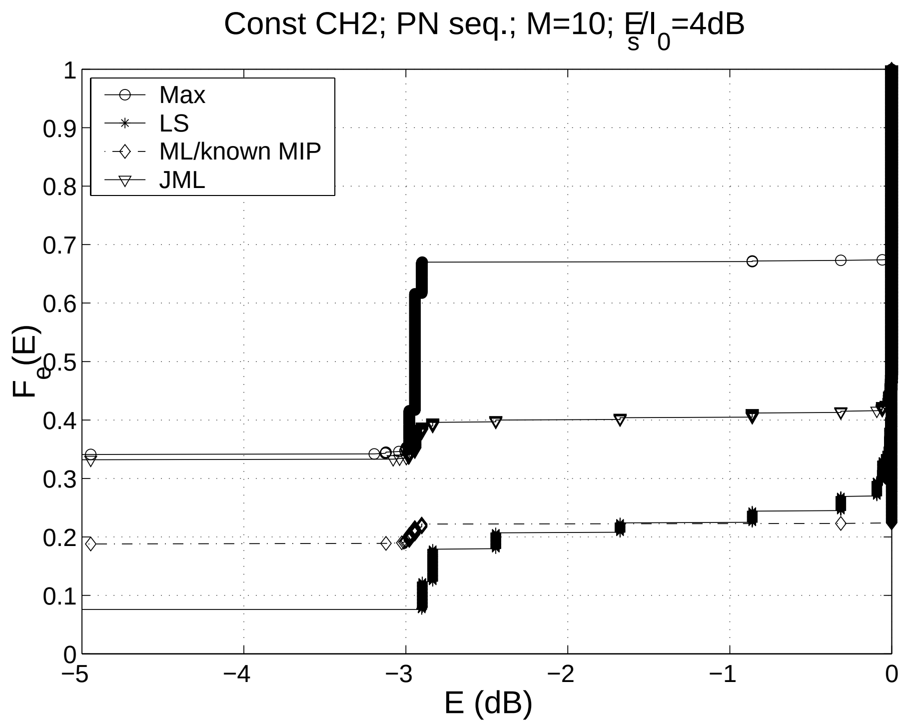

Figure 5: CDF with constant CH2 for $E_s/I_0 = 4$ dB and $M = 10$ accumulated bursts

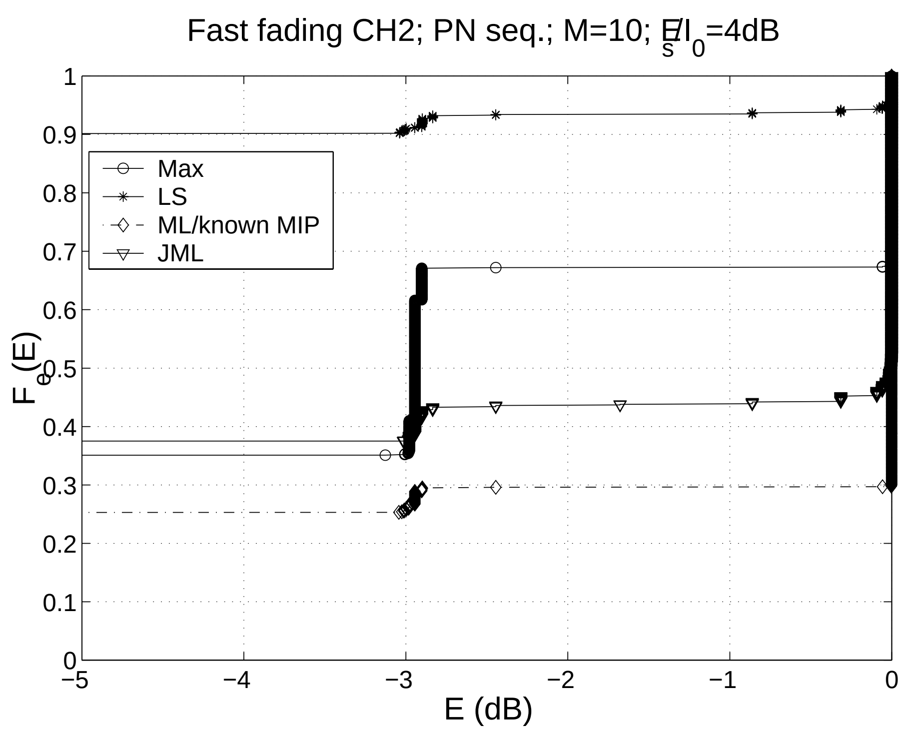

Figure 6: CDF with fast fading CH2 for $E_s/I_0 = 4$ dB and $M = 10$ accumulated bursts

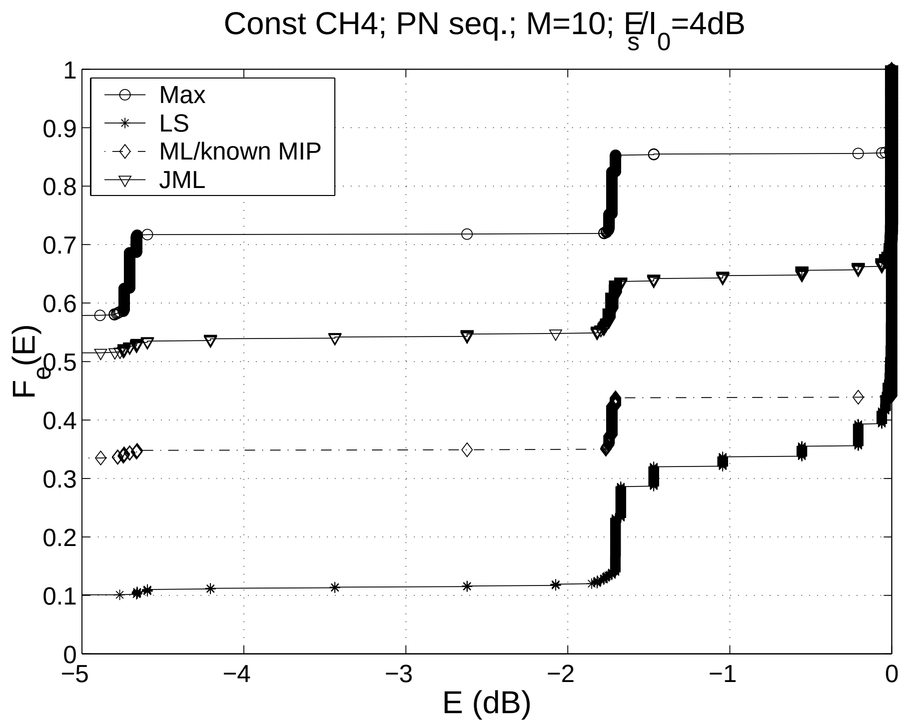

Figure 7: CDF with constant CH4 for $E_s/I_0 = 4$ dB and $M = 10$ accumulated bursts

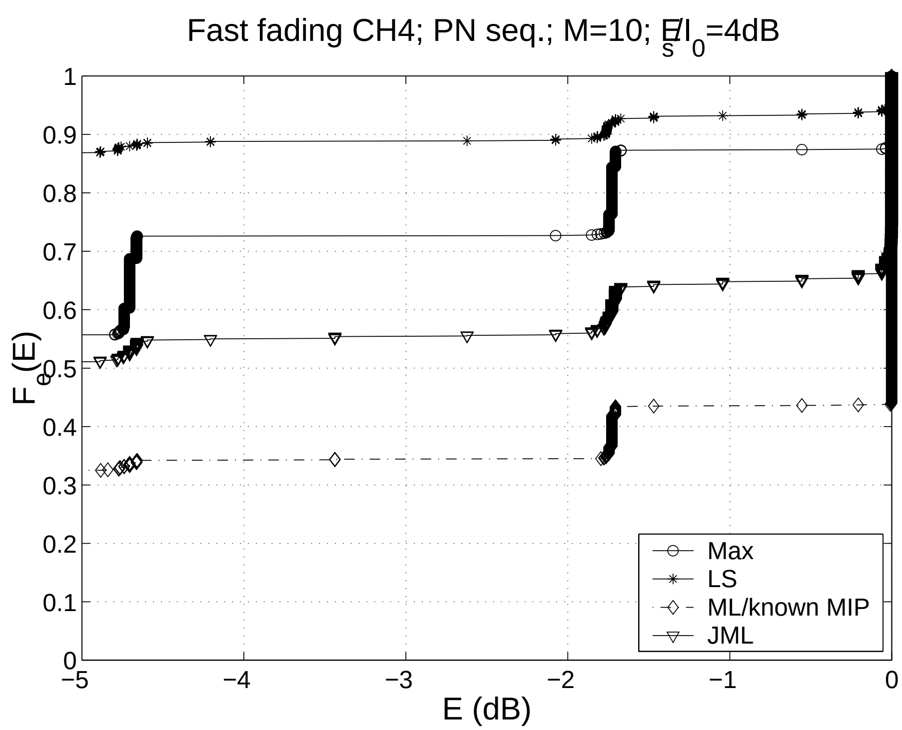

Figure 8: CDF with fast fading CH4 for $E_s/I_0 = 4$ dB and $M = 10$ accumulated bursts

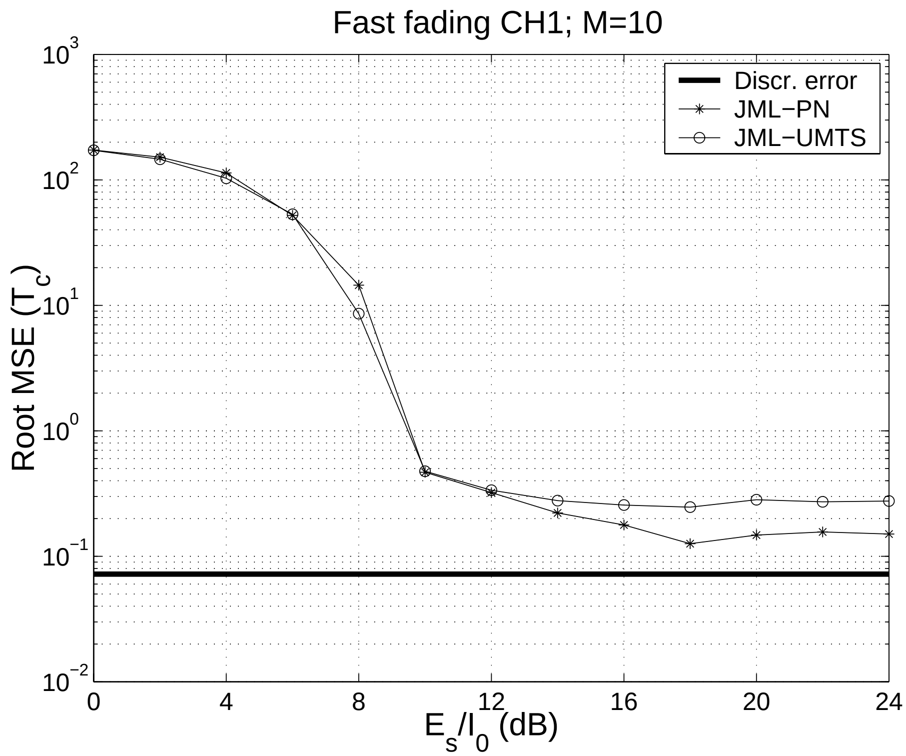

Figure 9: RMSE with fast fading CH1 vs. E_s/I_0 for UMTS and PN sequences and $M = 10$ accumulated bursts

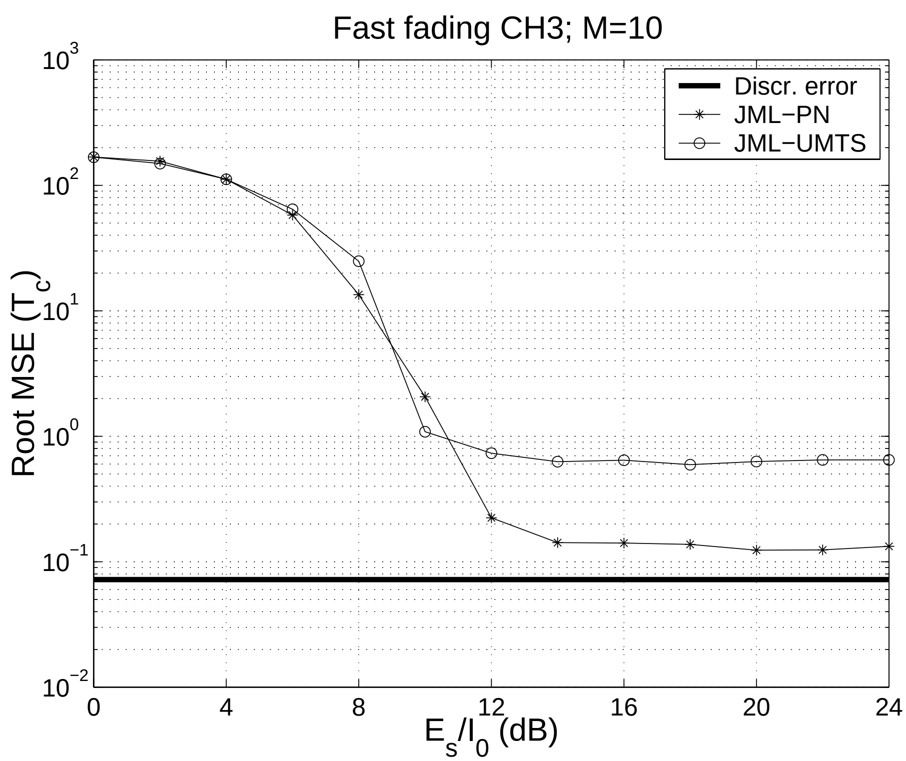

Figure 10: RMSE with fast fading CH3 vs. E_s/I_0 for UMTS and PN sequences and $M = 10$ accumulated bursts