Placement Delivery Array Design for Coded Caching Scheme in Partially Cooperative Device-to-Device Networks

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***Abstract*—Device-to-device (D2D) communication is a promising technique for next-generation wireless cellular communication systems. This paper studies coded caching in a partially cooperative D2D network, where only a subset of users transmit, while all request files. In all existing partially cooperative D2D coded caching schemes, each file must be divided into subfiles, referred to as the subpacketization level, which grows exponentially with the total number of users. For practical implementations, it is desirable to have schemes with reduced subpacketization levels. We propose an array known as a partially cooperative D2D placement delivery array (PC-DPDA), which describes the placement and delivery phases of a partially cooperative D2D coded caching scheme. Low subpacketization level schemes can be obtained by constructing appropriate PC-DPDAs. The placement delivery array (PDA) was proposed as a tool for designing coded caching schemes for broadcast networks. A wide range of PDA classes has been constructed in the literature. We propose a construction of a PC-DPDA from any given PDA, thereby obtaining several classes of PC-DPDAs. We show that an existing partially cooperative D2D coded caching scheme, which has an advantage over other known schemes in terms of operating memory regime, corresponds to a class of PC-DPDAs.**

***Index Terms*—D2D communication, coded caching, placement delivery array, partially cooperative network, subpacketization.**

I. Introduction

Device-to-device (D2D) communication is a key enabler of the Internet of Things (IoT) in future 5G and beyond wireless networks [1], [2]. Cache-aided D2D networks have been widely studied due to their importance in both cellular and IoT systems. Inspired by the coded caching scheme of Maddah-Ali and Niesen [3] (referred to as the MAN scheme) for an error-free broadcast network, Ji, Caire, and Molisch [4] introduced coded caching for wireless D2D networks (referred to as the JCM scheme). A D2D coded caching scheme operates in two phases: a *placement phase* followed by a *delivery phase*. In the placement phase, a central server places content in all user caches. In the delivery phase, the central server is not present, and the demands of all users are served through inter-user coded multicast transmissions.

Both the MAN scheme and the JCM scheme require each file to be divided into subfiles, referred to as the *subpacketization level*, which grows exponentially with the number of users for a given memory. This demands the file size to be very large when the number of users is large. Even if the number of users is less, any practical scheme requires header information for each subfile to enable decoding [5]. The placement delivery array (PDA) [6] and its D2D counterpart, the DPDA [7], were introduced to achieve such schemes in broadcast and D2D networks, respectively. Many PDA-based low-subpacketization schemes have since been proposed, including those via strong edge coloring of bipartite graphs [8], concatenating constructions [9], combinations of strong edge colorings [10], row index matrices and direct product sets [11], combinatorial designs [12], [13], proper orthogonal arrays [14], and Cartesian products [15]. In [7], a method to construct DPDAs from PDAs was given, with further DPDA constructions in [16]–[18].

Most existing D2D coded caching schemes, including the JCM scheme and those based on DPDAs, assume all users participate in transmission during delivery. In practice, however, some users may act selfishly—avoiding transmission to conserve energy or protect privacy—while still requesting files [19]. Such users are called *selfish users*, and a D2D network containing selfish users is termed a *partially cooperative D2D network* (PC-D2D network). Tebbi and Sung [20] first studied coded caching in this setting. A (K, S, N) PC-D2D network consists of K cache-aided users, S of whom are selfish, and a central server with a library of N files. Coded caching schemes for a PC-D2D network were also discussed in [21]–[23]. The scheme in [21] improves the transmission load compared to the deterministic caching scheme in [20]. Subsequently, Guan *et al.* [22] proposed three partially cooperative D2D schemes (Schemes A, B, and C). Among them, Scheme A achieves a lower transmission load than both [20] and [21]; however, all three schemes ([20], [21], and Scheme A) are restricted to higher memory regimes, i.e., $M \geq \frac{N}{K}(S+1)$. In contrast, the scheme in [23] is applicable to all feasible memory regimes. Furthermore, Schemes B and C in [22] require prior knowledge of the selfish users already at the placement phase.

Existing partially cooperative D2D coded caching schemes require subpacketization levels that grow exponentially with the number of users, given fixed memory. For practical implementation, schemes with reduced subpacketization are desirable. To address this, we introduce a combinatorial structure called the *partially cooperative D2D placement delivery array (PC-DPDA)* and present corresponding constructions. The main contributions of this paper are:

- We introduce an array called *partially cooperative D2D placement delivery array (PC-DPDA)*, which describes the placement and delivery phases of a partially cooperative D2D coded caching scheme. Low subpacketization level schemes can be obtained by constructing appropriate PC-DPDAs.

• We propose a construction of a PC-DPDA from any given PDA. Since several classes of PDAs have been constructed in the literature, one can obtain several classes of PC-DPDAs, which give low subpacketization level schemes. The existing partially cooperative D2D coded caching scheme recently proposed in [23], which has an advantage over other known schemes in terms of operating memory regime, corresponds to a class of PC-DPDAs obtained from the class of PDAs representing the MAN scheme.

Notations: For any positive integer n and any integer $m < n$, $[n]$ denotes the set $\{1, 2, ..., n\}$ and $[m : n]$ denotes the set $\{m, m+1, ..., n\}$. For sets $\mathcal{A}$ and $\mathcal{B}$, $\mathcal{A}\backslash\mathcal{B}$ denotes the elements in $\mathcal{A}$ but not in $\mathcal{B}$.

II. System Model

A (K, S, N) PC-D2D network consists of a central server with a library of N files $\{W_n : n \in [N]\}$ each of size B symbols over a finite field $\mathbb{F}_q$ [1], and $K \leq N$ cache-aided users $\{U_k : k \in [K]\}$, among which S users are selfish. The partially cooperative D2D coded caching scheme operates in two phases.

• **Placement phase**: During this phase, the server populates the caches of the users with file contents. For every $k \in [K]$, the cache of user U_k can store up to MB symbols, and the content store is denoted by $\mathcal{Z}_k$. During the placement phase, the later demands of the users are unknown. We assume that only the number of selfish users, and not their identities, is known before the placement phase.

• **Delivery phase**: During this phase, each user requests a file, with the set of requested files represented by a demand vector $\vec{d} = (d_1, \dots, d_K)$. Even though all users request a file, only the $(K - S)$ non-selfish users participate in the data transmission. Let $\mathcal{U}_\mathcal{K} = \bigcup_k \{U_k : k \in \mathcal{K}, \mathcal{K} \subseteq [K], |\mathcal{K}| = K - S\}$ be the set of non-selfish users. For a given demand vector $\vec{d}$, each non-selfish user $U_k \in \mathcal{U}_\mathcal{K}$ transmits a coded message $X_{k,\vec{d}}$ constructed from its cache content. The worst-case transmission load is then defined as $R \triangleq \max_{\vec{d}} \left(\sum_{k:U_k \in \mathcal{U}\backslash\mathcal{U}_\mathcal{S}} |X_{k,\vec{d}}| \right) / B$, where $|X_{k,\vec{d}}|$ denotes the size of the transmission by user U_k.

Additionally, the number of subfiles into which each file is partitioned is called the subpacketization level. Our goal is to design schemes with low subpacketization.

Remark 1. *Since any set of $K - S$ users must be able to satisfy the demands of all users, the entire file library should be recoverable from the caches of any $K - S$ users. Consequently, the cache memory must satisfy $M \geq \frac{N}{K-S}$.*

III. Partially cooperative D2D placement delivery array (PC-DPDA)

In this section, we first introduce a combinatorial structure called a partially cooperative D2D placement delivery array (PC-DPDA). Then, we show that corresponding to any PC-DPDA, there exists a partially cooperative D2D coded caching scheme.

Definition 1 (Partially cooperative D2D placement delivery array (PC-DPDA))**.** *For positive integers K, S, F, F', Z and S' such that $S < K$ and $Z \leq F' \leq F$, an $F \times K$ array $\mathbf{Q} = (q_{j,k})$, $j \in [F]$ and $k \in [K]$ whose entries are in the set $\{\star\} \cup \mathcal{S}$, where $\mathcal{S} = \{s^{(k_s)} : s \in \mathbb{Z}^+, k_s \in [K]\}$, k_s denote the unique column index associated with s and $|\mathcal{S}| = S'$, is called a (K, S, F, F', Z, S') PC-DPDA if it satisfies the following conditions:*

C1. The symbol $\star$ appears Z times in each column.

C2. Each element in $\mathcal{S}$ occurs at least once in the array.

C3. For any two distinct entries q_{j_1,k_1} and q_{j_2,k_2}, $q_{j_1,k_1} = q_{j_2,k_2} = s^{(k_s)}$ only if (a) $j_1 \neq j_2$, $k_1 \neq k_2$, i.e., they lie in distinct rows and distinct columns, and (b) $q_{j_1,k_2} = q_{j_2,k_1} = \star$.

C4. If $q_{j,k} = s^{(k_s)}$, then $q_{j,k_s} = \star$.

C5. For any $\mathcal{K} \subseteq [K] : |\mathcal{K}| = K - S$, there exist at least $F' - Z$ entries $s^{(k_s)} \in \mathcal{S}$ in every column of $\mathbf{Q}$ such that $k_s \in \mathcal{K}$.

When $S = 0$ and $F = F'$ in Definition 1 of PC-DPDA, it reduces to the DPDA defined in [7]. Next, we show that corresponding to any (K, S, F, F', Z, S') PC-DPDA $\mathbf{Q}$, there exists a partially cooperative D2D coded caching scheme, as stated in the following theorem.

Theorem 1. *Given a (K, S, F, F', Z, S') PC-DPDA, a (K, S, N) partially cooperative D2D coded caching scheme with subpacketization level F' and memory ratio $\frac{M}{N} = \frac{Z}{F'}$ can be obtained using Algorithm 1. For any demand vector $\vec{d}$, the demands of all users are met with a transmission load $R = \frac{\max_{\mathcal{K}}\{|\mathcal{S}'_\mathcal{K}|\}}{F'}$, where $\mathcal{S}'_\mathcal{K} = \bigcup_s \{s^{(k_s)} : k_s \in \mathcal{K}, \mathcal{K} \subseteq [K], |\mathcal{K}| = K - S\}$.*

The proof of Theorem 1 is given in *Appendix A*. An example illustrating Theorem 1 is provided next.

Example 1. *It can be verified that the array $\mathbf{Q}$ (given in the next page) is a $(5, 2, 20, 14, 8, 30)$ PC-DPDA. Based on this PC-DPDA, one can obtain a coded caching scheme for a $(5, 2, N)$ PC-D2D network, using Algorithm 1, as follows.*

• ***Placement phase:*** *From line 2 of Algorithm 1, each file is divided into 14 packets, i.e., $W_n = \{W_{n,1}, W_{n,2}, \dots, W_{n,14}\}$ for every $n \in [N]$. By line 3 of Algorithm 1, these subfiles are then encoded using a $[20, 14]$ MDS code. The 20 coded subfiles of file W_n are denoted by $\{Y_{n,1}, Y_{n,2}, \dots, Y_{n,20}\}$. The 20 rows of $\mathbf{Q}$ represent these 20 coded subfiles. By lines $4-6$ of Algorithm 1, the coded subfiles placed in the caches are:*

$$\mathcal{Z}_1 = \{Y_{n,1}, Y_{n,2}, Y_{n,3}, Y_{n,4}, Y_{n,8}, Y_{n,12}, Y_{n,16}, Y_{n,20}\},$$
$$\mathcal{Z}_2 = \{Y_{n,4}, Y_{n,5}, Y_{n,6}, Y_{n,7}, Y_{n,8}, Y_{n,11}, Y_{n,15}, Y_{n,19}\},$$
$$\mathcal{Z}_3 = \{Y_{n,3}, Y_{n,7}, Y_{n,9}, Y_{n,10}, Y_{n,11}, Y_{n,12}, Y_{n,14}, Y_{n,18}\},$$
$$\mathcal{Z}_4 = \{Y_{n,2}, Y_{n,6}, Y_{n,10}, Y_{n,13}, Y_{n,14}, Y_{n,15}, Y_{n,16}, Y_{n,17}\},$$
$$\mathcal{Z}_5 = \{Y_{n,1}, Y_{n,5}, Y_{n,9}, Y_{n,13}, Y_{n,17}, Y_{n,18}, Y_{n,19}, Y_{n,20}\}.$$

[1]We assume that q is large enough such that all MDS codes considered in this work exist over $\mathbb{F}_q$

$$
\mathbf{Q} = \begin{array}{c} \\ 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15\\16\\17\\18\\19\\20 \end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{pmatrix}
\star & 1^{(1)} & 4^{(1)} & 6^{(1)} & \star \\
\star & 2^{(1)} & 5^{(1)} & \star & 6^{(1)} \\
\star & 3^{(1)} & \star & 5^{(1)} & 4^{(1)} \\
\star & \star & 3^{(1)} & 2^{(1)} & 1^{(1)} \\
1^{(2)} & \star & 4^{(2)} & 6^{(2)} & \star \\
2^{(2)} & \star & 5^{(2)} & \star & 6^{(2)} \\
3^{(2)} & \star & \star & 5^{(2)} & 4^{(2)} \\
\star & \star & 3^{(2)} & 2^{(2)} & 1^{(2)} \\
1^{(3)} & 4^{(3)} & \star & 6^{(3)} & \star \\
2^{(3)} & 5^{(3)} & \star & \star & 6^{(3)} \\
3^{(3)} & \star & \star & 5^{(3)} & 4^{(3)} \\
\star & 3^{(3)} & \star & 2^{(3)} & 1^{(3)} \\
1^{(4)} & 4^{(4)} & 6^{(4)} & \star & \star \\
2^{(4)} & 5^{(4)} & \star & \star & 6^{(4)} \\
3^{(4)} & \star & 5^{(4)} & \star & 4^{(4)} \\
\star & 3^{(4)} & 2^{(4)} & \star & 1^{(4)} \\
1^{(5)} & 4^{(5)} & 6^{(5)} & \star & \star \\
2^{(5)} & 5^{(5)} & \star & 6^{(5)} & \star \\
3^{(5)} & \star & 5^{(5)} & 4^{(5)} & \star \\
\star & 3^{(5)} & 2^{(5)} & 1^{(5)} & \star
\end{pmatrix}
\end{array}
$$

Therefore, we have $M/N = Z/F' = 8/14 = 4/7$.

• ***Delivery phase**: Let U_2 and U_5 be the selfish users. Thus, $\mathcal{U}_{\{1,3,4\}} = \{U_1, U_3, U_4\}$ is the set of three transmitting users. Let the demand vector $\vec{d} = (1, 2, 3, 4, 5)$. By lines 10 to 12 of Algorithm 1, corresponding to the elements $\{1^{(1)}, 2^{(1)}, 3^{(1)}, 4^{(1)}, 5^{(1)}, 6^{(1)}\}$ in $\mathbf{Q}$, user U_1 does the transmissions $Y_{2,1} \oplus Y_{5,4}$, $Y_{2,2} \oplus Y_{4,4}$, $Y_{2,3} \oplus Y_{3,4}$, $Y_{3,1} \oplus Y_{5,3}$, $Y_{3,2} \oplus Y_{4,3}$ and $Y_{4,1} \oplus Y_{5,2}$, respectively. Similarly, corresponding to the elements $\{1^{(3)}, 2^{(3)}, 3^{(3)}, 4^{(3)}, 5^{(3)}, 6^{(3)}\}$ in $\mathbf{Q}$, U_3 transmits $Y_{1,9} \oplus Y_{5,12}$, $Y_{1,10} \oplus Y_{4,12}$, $Y_{1,11} \oplus Y_{2,12}$, $Y_{2,9} \oplus Y_{5,11}$, $Y_{2,10} \oplus Y_{4,11}$ and $Y_{4,9} \oplus Y_{5,10}$, respectively. And, corresponding to the elements $\{1^{(4)}, 2^{(4)}, 3^{(4)}, 4^{(4)}, 5^{(4)}, 6^{(4)}\}$ in $\mathbf{Q}$, U_4 transmits $Y_{1,13} \oplus Y_{5,16}$, $Y_{1,14} \oplus Y_{3,16}$, $Y_{1,15} \oplus Y_{2,16}$, $Y_{2,13} \oplus Y_{5,15}$, $Y_{2,14} \oplus Y_{3,15}$ and $Y_{3,13} \oplus Y_{5,14}$, respectively.*

Now, let us consider a selfish user U_2 and check how it gets the demanded file W_2. It already has 8 coded subfiles of W_2 in its cache, given by $\{Y_{2,4}, Y_{2,5}, Y_{2,6}, Y_{2,7}, Y_{2,8}, Y_{2,11}, Y_{2,15}, Y_{2,19}\}$. From the transmissions $Y_{2,1} \oplus Y_{5,4}$, $Y_{2,2} \oplus Y_{4,4}$, $Y_{2,3} \oplus Y_{3,4}$, $Y_{1,11} \oplus Y_{2,12}$, $Y_{2,9} \oplus Y_{5,11}$, $Y_{2,10} \oplus Y_{4,11}$, $Y_{1,15} \oplus Y_{2,16}$, $Y_{2,13} \oplus Y_{5,15}$ and $Y_{2,14} \oplus Y_{3,15}$, user U_2 gets the coded subfiles $Y_{2,1}$, $Y_{2,2}$, $Y_{2,3}$, $Y_{2,12}$, $Y_{2,9}$, $Y_{2,10}$, $Y_{2,16}$, $Y_{2,13}$ and $Y_{2,14}$, respectively. That is, user U_2 receives 9 coded subfiles of W_2 from the transmissions. Together with the 8 coded subfiles already cached, U_2 has a total of 17 coded subfiles of W_2. Since only $F' = 14$ distinct coded subfiles are required for decoding, U_2 can recover W_2. Similarly, for a non-selfish user such as U_1, the transmissions provide 6 additional coded subfiles of W_1, and together with the 8 cached subfiles, U_1 obtains exactly 14 distinct coded subfiles, which is sufficient to decode W_1.

In the sequel, we propose a novel construction of PC-DPDAs from PDAs. Before getting into the details of the construction, we review the definition of a PDA.

Definition 2 (Placement Delivery Array (PDA) [6]). *For positive integers K, F, Z and Σ, an $F \times K$ array $\mathbf{P} = (p_{j,k})$, $j \in [F]$ and $k \in [K]$, composed of a specific symbol $\star$ and Σ positive integers $[\Sigma]$, is called a (K, F, Z, Σ) PDA if it satisfies the following conditions:*
C1. The symbol $\star$ appears Z times in each column.
C2. Each integer occurs at least once in the array.
C3. For any two distinct entries p_{j_1,k_1} and p_{j_2,k_2}, $p_{j_1,k_1} = p_{j_2,k_2} = \sigma$ is an integer only if (a) $j_1 \neq j_2$, $k_1 \neq k_2$, i.e., they lie in distinct rows and distinct columns, and (b) $p_{j_1,k_2} = p_{j_2,k_1} = \star$.

Algorithm 1 Partially cooperative D2D coded caching scheme based on a (K, S, F, F', Z, S') PC-DPDA $\mathbf{Q}$

1: **procedure** PLACEMENT($\mathbf{Q}, \mathcal{W}$)
2: Divide each file $W_n \in \mathcal{W}$ into F' subfiles: $W_n = \{W_{n,i} : i \in [F']\}$.
3: Encode the F' subfiles of each file $W_n, n \in [N]$, into F coded subfiles using an $[F, F']$ MDS code with generator matrix $\mathbf{G}$ of size $F' \times F$:
$$\big[Y_{n,j} : j \in [F]\big] = \mathbf{G}\big[W_{n,i} : i \in [F']\big].$$
4: **for** $k \in [K]$ **do**
5: $\mathcal{Z}_k \leftarrow \{Y_{n,j} : q_{j,k} = \star, \, n \in [N], j \in [F]\}$
6: **end for**
7: **end procedure**
8: **procedure** DELIVERY($\mathbf{Q}, \mathcal{W}, S, \vec{d}$)
9: Let $\mathcal{U}_{\mathcal{K}} = \bigcup_k \{U_k : k \in \mathcal{K}, \, \mathcal{K} \subseteq [K], \, |\mathcal{K}| = K - S\}$ be the set of transmitting users.
10: Let $\mathcal{S}'_{\mathcal{K}} = \bigcup_s \{s^{(k_s)} : k_s \in \mathcal{K}\}$.
11: **for** $s^{(k_s)} \in \mathcal{S}'_{\mathcal{K}}$ **do**
12: User U_{k_s} transmits:
$$\bigoplus_{q_{j,k} = s^{(k_s)}, \, j \in [F], k \in [K]} Y_{d_k, j}.$$
13: **end for**
14: **end procedure**

Since several classes of PDAs have been developed in the literature (as noted in the introduction), this construction yields multiple classes of PC-DPDAs. From these, low-subpacketization coded caching schemes can be derived for PC-D2D networks.

Construction 1. Let $\mathbf{P} = (p_{j,k})$, $j \in [F_1]$ and $k \in [K_1]$ be a (K_1, F_1, Z_1, S_1) PDA. Let $\mathbf{v} = (\star)_{K' \times 1}$ denote the K_1-length column vector whose entries are all $\star$. For the given PDA $\mathbf{P}$, we define an array $\mathbf{P}^{(k_s)}$ obtained by replacing each non-$\star$ entry $p_{j,k}$, where $p_{j,k} \in [S'], j \in [F_1], k \in [K_1]$, with $p_{j,k}^{(k_s)}$, where $k_s \in \mathbb{Z}^+$. Let $\mathbf{p}_k^{(k_s)} : k \in [K_1]$ denote the k^{th} column of $\mathbf{P}^{(k_s)}$. Then, for each $k_s \in [K_1 + 1]$, we construct an array $\mathbf{Q}^{(k_s)} = \Big[\mathbf{q}_1^{(k_s)}, \mathbf{q}_2^{(k_s)}, \ldots, \mathbf{q}_k^{(k_s)}, \ldots, \mathbf{q}_{K_1}^{(k_s)}, \mathbf{q}_{K_1+1}^{(k_s)}\Big]$, where $\mathbf{q}_k^{(k_s)}$ denotes the k^{th} column of $\mathbf{Q}^{(k_s)}$, $k \in [K_1 + 1]$, which is defined as follows.

$$\mathbf{q}_k^{(k_s)} = \begin{cases} \mathbf{p}_k^{(k_s)}, & \text{if } k < k_s, \\ \mathbf{v}, & \text{if } k = k_s, \\ \mathbf{p}_{k-1}^{(k_s)}, & \text{if } k > k_s. \end{cases} \tag{1}$$

Now, we construct an array $\mathbf{Q}$ by vertically concatenating the K_1+1 arrays $\mathbf{Q}^{(k_s)} : k_s \in [K_1+1]$ in the increasing order of k_s, i.e.,

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}^{(1)} \\ \mathbf{Q}^{(2)} \\ \vdots \\ \mathbf{Q}^{(K_1+1)} \end{bmatrix}. \tag{2}$$

This array $\mathbf{Q}$ is in fact a $(K_1+1, S \leq K_1, (K_1+1)F_1, (K_1+1-S)F_1+SZ_1, F_1+K_1Z_1, (K_1+1)S_1)$ PC-DPDA, proof of which is given in Appendix B.

Construction 1 results in the following theorem.

Theorem 2. *Given a (K_1, F_1, Z_1, S_1) PDA, there exist a $(K_1+1, S \leq K_1, (K_1+1)F_1, (K_1+1-S)F_1+SZ_1, K_1Z_1+F_1, (K_1+1)S_1)$ PC-DPDA which gives a (K_1+1, S, N) partially cooperative D2D coded caching scheme with subpacketization level $F' = (K_1+1-S)F_1+SZ_1$ and achieving the memory-load pair $(M, R) =$*

$$\left(\frac{N(K_1Z_1+F_1)}{(K_1+1-S)F_1+SZ_1}, \frac{(K_1+1-S)S_1}{(K_1+1-S)F_1+SZ_1} \right).$$

Proof: Given a (K_1, F_1, Z_1, S_1) PDA, using Construction 1, we obtained a $(K_1+1, S \leq K_1, (K_1+1)F_1, (K_1+1-S)F_1+SZ_1, F_1+K_1Z_1, (K_1+1)S_1)$ PC-DPDA. From this PC-DPDA, Theorem 1 gives a (K_1+1, S, N) partially cooperative D2D coded caching scheme with subpacketization level $F' = (K_1+1-S)F_1+SZ_1$, memory ratio $\frac{M}{N} = \frac{Z}{F'} = \frac{K_1Z_1+F_1}{(K_1+1-S)F_1+SZ_1}$, and transmission load $R = \frac{\max\limits_{\mathcal{K}}\{|\mathcal{S}'_{\mathcal{K}}|\}}{F'}$, where $\mathcal{S}'_{\mathcal{K}} = \bigcup\limits_{s}\{s^{(k_s)} : k_s \in \mathcal{K}, \mathcal{K} \subseteq [K], |\mathcal{K}| = K_1+1-S\}$. From Construction 1, we know that $\bigcup\limits_{s}\{s^{(k_s)}\}$ for a given k_s is S'. Therefore, and since $|\mathcal{K}| = K_1+1-S$, we have, $\max\limits_{\mathcal{K}}\{|\mathcal{S}'_{\mathcal{K}}|\} = (K_1+1-S)S'$. Thus, $R = \frac{(K_1+1-S)S_1}{(K_1+1-S)F_1+SZ_1}$. ■

The following example illustrates Construction 1.

Example 2.

$$\mathbf{P} = \begin{pmatrix} 1 & 4 & 6 & \star \\ 2 & 5 & \star & 6 \\ 3 & \star & 5 & 4 \\ \star & 3 & 2 & 1 \end{pmatrix} \text{ is a } (4,4,1,6) \text{ PDA.}$$

From this PDA, using (1), one can obtain the following 5 arrays $\mathbf{Q}^{(k_s)}$, $k_s \in [5]$. That is, $\mathbf{Q}^{(1)} = \begin{pmatrix} \star & 1^{(1)} & 4^{(1)} & 6^{(1)} & \star \\ \star & 2^{(1)} & 5^{(1)} & \star & 6^{(1)} \\ \star & 3^{(1)} & \star & 5^{(1)} & 4^{(1)} \\ \star & \star & 3^{(1)} & 2^{(1)} & 1^{(1)} \end{pmatrix}$,

$$\mathbf{Q}^{(2)} = \begin{pmatrix} 1^{(2)} & \star & 4^{(2)} & 6^{(2)} & \star \\ 2^{(2)} & \star & 5^{(2)} & \star & 6^{(2)} \\ 3^{(2)} & \star & \star & 5^{(2)} & 4^{(2)} \\ \star & \star & 3^{(2)} & 2^{(2)} & 1^{(2)} \end{pmatrix}, \mathbf{Q}^{(3)} = \begin{pmatrix} 1^{(3)} & 4^{(3)} & \star & 6^{(3)} & \star \\ 2^{(3)} & 5^{(3)} & \star & \star & 6^{(3)} \\ 3^{(3)} & \star & \star & 5^{(3)} & 4^{(3)} \\ \star & 3^{(3)} & \star & 2^{(3)} & 1^{(3)} \end{pmatrix},$$

$$\mathbf{Q}^{(4)} = \begin{pmatrix} 1^{(4)} & 4^{(4)} & 6^{(4)} & \star & \star \\ 2^{(4)} & 5^{(4)} & \star & \star & 6^{(4)} \\ 3^{(4)} & \star & 5^{(4)} & \star & 4^{(4)} \\ \star & 3^{(4)} & 2^{(4)} & \star & 1^{(4)} \end{pmatrix}, \mathbf{Q}^{(5)} = \begin{pmatrix} 1^{(5)} & 4^{(5)} & 6^{(5)} & \star & \star \\ 2^{(5)} & 5^{(5)} & \star & 6^{(5)} & \star \\ 3^{(5)} & \star & 5^{(5)} & 4^{(5)} & \star \\ \star & 3^{(5)} & 2^{(5)} & 1^{(5)} & \star \end{pmatrix}.$$

By vertically concatenating these arrays as given in (2), we obtain the $(5, 2, 20, 14, 8, 30)$ PC-DPDA $\mathbf{Q}$ in Example 1.

IV. Performance Analysis

In this section, we analyze the performance of the schemes obtained from the PC-DPDA constructions in Section III. We first demonstrate that the partially cooperative D2D coded caching scheme proposed in [23], which outperforms existing schemes in terms of operating memory regime, can be represented as a special class of PC-DPDAs.

In [6], the authors showed that, for any $K_1 \in \mathbb{Z}^+$ and $t \in [0 : K_1]$, there exist a $\left(K_1, \binom{K_1}{t}, \binom{K_1-1}{t-1}, \binom{K_1}{t+1}\right)$ PDA, which corresponds to the MAN scheme. Given this class of PDA, using Theorem 2, one can obtain a $(K_1+1, S \leq K_1, (K_1+1)\binom{K_1}{t}, (K_1+1-S)\binom{K_1}{t}+S\binom{K_1-1}{t-1}, K_1\binom{K_1-1}{t-1}+\binom{K_1}{t}, (K_1+1)\binom{K_1}{t_1+1})$ PC-DPDA. For any $K_1 \in \mathbb{Z}^+$ and $t \in [0 : K_1]$, this PC-DPDA gives a (K_1+1, S, N) partially cooperative D2D coded caching scheme with subpacketization level $F' = (K_1+1-S)\binom{K_1}{t} + S\binom{K_1-1}{t-1}$, memory ratio $\frac{M}{N} = \frac{Z}{F'} = \frac{K_1\binom{K_1-1}{t-1}+\binom{K_1}{t}}{(K_1+1-S)\binom{K_1}{t}+S\binom{K_1-1}{t-1}}$ and transmission load $R = \frac{(K_1+1-S)\binom{K_1}{t+1}}{(K_1+1-S)\binom{K_1}{t}+S\binom{K_1-1}{t-1}}$. Upon substituting $K_1+1 = K$, we can rewrite the previous statement as follows. For any $K \in \mathbb{Z}^+$ and $t \in [0 : K-1]$, there exist a (K, S, N) partially cooperative D2D coded caching scheme with subpacketization level $F' = (K-S)\binom{K-1}{t} + S\binom{K-2}{t-1}$, memory ratio $\frac{M}{N} = \frac{(K-1)\binom{K-2}{t-1}+\binom{K-1}{t}}{(K-S)\binom{K-1}{t}+S\binom{K-2}{t-1}} = \frac{(t+1)(K-1)}{(K-S)(K-1)+tS}$, and transmission load $R = \frac{(K-S)\binom{K-1}{t+1}}{(K_1+1-S)\binom{K_1}{t}+S\binom{K_1-1}{t-1}} = \frac{(K-S)(K-1)}{(K-S)(K-1)+tS}\frac{K-t-1}{t+1}$. This is exactly the scheme proposed in [23].

Next, we show the advantage of schemes obtained from PC-DPDAs in terms of subpacketization level over existing schemes. In Theorem 3 of [15], a PDA construction is proposed using the Cartesian product method for any positive integers q, z, and m with $z < q$. By choosing the parameters $m = 5, q = 11$ and $z \in [1 : 10]$ in PDA in [15], $n = 11, i = 2$ and $j \in [2 : 9]$ in PDA in [13], the parameters $n = 11$ and $i = 2, 9$ in the first variant MAN PDA in [24], and using Theorem 2 with $S = 10$, we obtain coded caching schemes for a $(56, 10, 56)$ PC-D2D network, at various memory points. Schemes at other memory points can be obtained by memory sharing. The load and subpacketization levels of these schemes, along with those in [20]–[23], are plotted in Fig. 1 and Fig. 2, respectively. Thus, using different classes of PDAs, partially cooperative D2D coded caching schemes with different subpacketization levels can be obtained. For a given number of users and cache size, as the subpacketization level decreases, the coded multicasting opportunities reduce, and hence the transmission load increases. Thus, as shown in the figures, the gain in subpacketization is inevitably achieved at the expense of an increased transmission load. Consequently, the selection of a PDA family can be guided by the system's tolerance to this trade-off between subpacketization and transmission load.

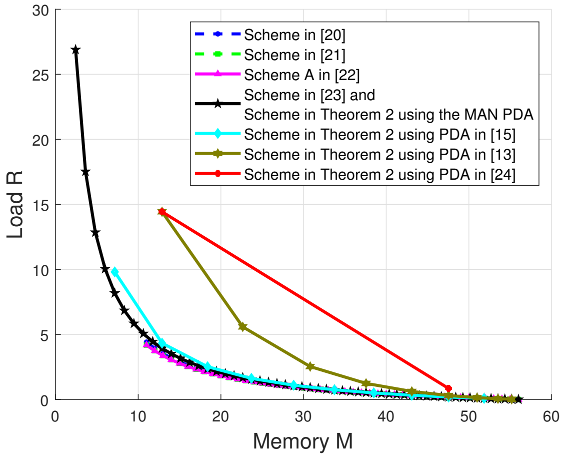

Fig. 1: The transmission loads as a function of memory of schemes in [20], [21], [22], [23] and the proposed scheme in Theorem 2 using various PDAs.

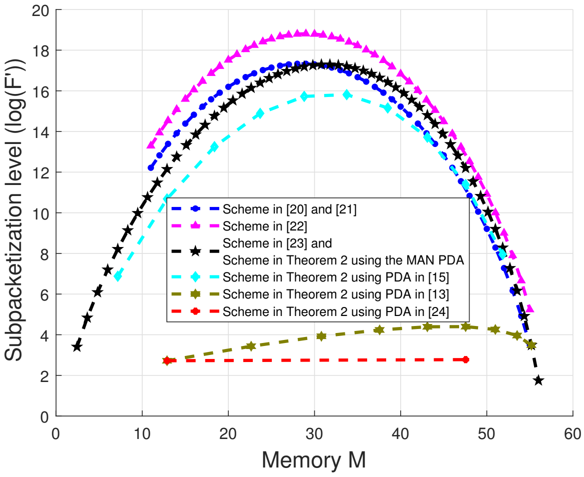

Fig. 2: The subpacketization levels as a function of memory of schemes in [20], [21], [22], [23] and the proposed scheme in Theorem 2 using various PDAs.

V. Conclusion

In this work, we introduced the partially cooperative D2D placement delivery array, from which coded caching schemes for partially cooperative D2D networks can be obtained. We further proposed a method to construct PC-DPDAs from any PDA, enabling the design of low subpacketization schemes and subsuming the scheme in [23] as a special case. A natural direction for future work is to establish lower bounds on the subpacketization needed to achieve a given transmission load.

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Appendix A
Proof of Theorem 1

We show that from a given (K, S, F, F', Z, S') PC-DPDA $\mathbf{Q}$, Algorithm 1 gives a (K, S, N) partially cooperative D2D coded caching scheme. The scheme operates in two phases.

• *Placement phase:* From lines 1–3 of Algorithm 1, in the placement phase each file is first divided into F' subfiles, i.e., $W_n = \{W_{n,i} : i \in [F']\}, \forall n \in [N]$, giving a subpacketization level of F'. These F' subfiles are then encoded into F coded subfiles using an $[F, F']$ MDS code with generator matrix $\mathbf{G}$ of order $F' \times F$, i.e., $[Y_{n,j} : j \in [F]] = \mathbf{G}\,[W_{n,i} : i \in [F']]$. The j^{th} row of $\mathbf{Q}$ corresponds to the j^{th} coded subfile of all files, while the k^{th} column corresponds to user U_k. From line 5 of Algorithm 1, the cache of user U_k is $\mathcal{Z}_k = \{Y_{n,j} : p_{j,k} = \star, n \in [N], j \in [F]\}$. Thus, $p_{j,k} = \star$ implies that U_k stores $Y_{n,j}$ for all $n \in [N]$. By condition *C2* of Definition 1, each column of $\mathbf{Q}$ has Z stars, and since each coded subfile has $\frac{1}{F'}$ of a file size, the memory ratio is $\frac{M}{N} = \frac{Z}{F'}$.

• *Delivery phase:* Let $\mathcal{U}_{\mathcal{K}} = \bigcup_{k}\{U_k : k \in \mathcal{K}, \mathcal{K} \subseteq [K], |\mathcal{K}| = K - S\}$ be the set of transmitting users and let $\mathcal{S}'_{\mathcal{K}} = \bigcup_{s}\{s^{(k_s)} : k_s \in \mathcal{K}\}$. Let the demand vector $\vec{d} = (d_1, d_2, \ldots, d_K)$. By lines 10 to 12 of Algorithm 1, user U_{k_s}, $k_s \in \mathcal{K}$ makes coded transmissions corresponding to every element $s^{(k_s)}$ given by $\bigoplus_{p_{j,k} = s^{(k_s)}, j \in [F], k \in [K]} Y_{d_k,j}$. By condition *C4* of Definition 1, if $p_{j,k} = s^{(k_s)}$, then $p_{j,k_s} = \star$. That means, user U_{k_s} has access to every $Y_{d_k,j}$ for which $p_{j,k} = s^{(k_s)}$, and therefore it can construct the transmitted coded message. Since each user $U_{k_s} : k_s \in \mathcal{K}$ do transmissions corresponding to all elements $s^{(k_s)} : s \in \mathbb{Z}^+$, we have the transmission load $R = \frac{\max_{\mathcal{K}}\{|\mathcal{S}'_{\mathcal{K}}|\}}{F'}$.

Next, we show that every user can decode their demanded file. An $[F, F']$ MDS code guarantees that any F' out of F coded symbols suffice to recover the original F' subfiles. Hence, any user U_k can reconstruct its demand W_{d_k} from F' distinct coded subfiles of W_{d_k}. By condition *C1*, each user already stores Z coded subfiles of its demand in its cache. Consider a transmission $\bigoplus_{p_{j,k} = s^{(k_s)}, j \in [F], k \in [K]} Y_{d_k,j}$ by user U_{k_s}. If the symbol $s^{(k_s)}$ appears g times in $\mathbf{Q}$, then by condition *C3* this transmission provides one coded subfile of the demand to g users. Moreover, by condition *C5*, for any set $\mathcal{K} \subseteq [K]$ with $|\mathcal{K}| = K - S$, each column of $\mathbf{Q}$ contains at least $F' - Z$ entries $s^{(k_s)}$ with $k_s \in \mathcal{K}$. Thus, from the transmissions corresponding to these $F' - Z$ entries in each column, each user U_k gets $F' - Z$ additional coded subfiles of W_{d_k}. In total, each user has $Z + (F' - Z) = F'$ coded subfiles, ensuring that W_{d_k} can be decoded.

Remark 2. *The objective of incorporating an MDS code in the proposed scheme is not to provide error correction capability, and therefore, the minimum distance of the code is irrelevant in this context. Rather, we require the property that, for a linear $[n, k]$ MDS code with generator matrix $\mathbf{G}$, every*

$k \times k$ submatrix of $\mathbf{G}$ is full-rank. This guarantees that the k information symbols can be reconstructed from any subset of k coded symbols, which is the property exploited in our construction.

Appendix B
Proof that $\mathbf{Q}$ in Construction 1 is a PC-DPDA

By (2), $\mathbf{Q}$ is obtained by vertically concatenating the K_1+1 arrays $\mathbf{Q}^{(k_s)} : k_s \in [K_1+1]$. By (1), each array $\mathbf{Q}^{(k_s)}$ has F_1 rows and K_1+1 columns. Therefore, $\mathbf{Q}$ has $K = (K_1+1)$ columns and $F = (K_1+1)F_1$ rows. Since $\mathbf{P}$ is a (K_1, F_1, Z_1, S_1) PDA, each vector $p_k^{(k_s)}$ consist of Z_1 $\star$s. Now consider a given column $\mathbf{q}_k$, where $k \in [K_1+1]$, of $\mathbf{Q}$. Then, by (2),

$$\mathbf{q}_k = \begin{bmatrix} \mathbf{q}_k^{(1)} \\ \mathbf{q}_k^{(2)} \\ \vdots \\ \mathbf{q}_k^{(K_1+1)} \end{bmatrix}.$$

From (1), it is clear that for $j = k_s$, $\mathbf{q}_k^{(j)} = \mathbf{v}$ which consist of F_1 number of $\star$s, and for all $j \neq k_s, j \in [K_1+1]$, $\mathbf{q}_k^{(j)} = p_i^{(j)}$ for some $i \in [K_1]$ which consists of Z_1 number of $\star$s. Therefore, the number of $\star$s in a given column of $\mathbf{Q}$ is $Z = F_1 + K_1 Z_1$. Thus, $C1$ of the PC-DPDA definition holds. $C2$ is obvious by construction.

From (1), all non-$\star$ entries in $\mathbf{Q}^{(k_s)}$ belong to the set $\{s^{(k_s)} : s \in [S']\}$. Hence, for any two arrays $\mathbf{Q}^{(i)}$ and $\mathbf{Q}^{(j)}$ with $i \neq j$, the set of non-$\star$ entries of $\mathbf{Q}^{(i)}$ and $\mathbf{Q}^{(j)}$ are disjoint. Now consider two elements q_{j_1,k_1} and q_{j_2,k_2} such that $q_{j_1,k_1} = q_{j_2,k_2} = s^{(k_s)}$. Then,$q_{j_1,k_1}$ and q_{j_2,k_2} are entries of the array $\mathbf{Q}^{(k_s)}$. Since $\mathbf{P}$ is a PDA, $\mathbf{P}^{(k_s)}$ is also a PDA, and $\mathbf{Q}^{(k_s)}$ is equivalent to $[\mathbf{P}^{(k_s)} \;\; \mathbf{v}]$ up to column permutation. Therefore, we must have $j_1 \neq j_2$, $k_1 \neq k_2$, and $q_{j_1,k_2} = q_{j_2,k_1} = \star$. Thus, condition $C3$ of the PC-DPDA definition holds.

As shown before, a non-$\star$ entry $q_{j,k} = s^{(k_s)} : s \in [S']$ appears in exactly one array $\mathbf{Q}^{(k_s)}$. By (1), for the array $\mathbf{Q}^{(k_s)}$, we have $\mathbf{q}_{(k_s)}^{(k_s)} = \mathbf{v}$. Therefore, for any $q_{j,k} = s^{(k_s)}$, we have $q_{j,k_s} = \star$. Thus, $C4$ of the PC-DPDA definition holds.

By (1) and (2), for any column index $k \in [K_1+1]$, the column $\mathbf{q}_k$ of $\mathbf{Q}$ contains exactly $F_1 - Z_1$ distinct non-$\star$ entries $q_{j,k} = s^{(k_s)} : s \in [S'], j \in [F]$, for each $k_s \in [K_1+1] \backslash \{k\}$. Choose an $S \leq K_1$ and consider any $\mathcal{K} \subseteq [K_1+1]$ such that $|\mathcal{K}| = K_1 + 1 - S$. Thus, for any column $\mathbf{q}_k$ such that $k \in \mathcal{K}$, the number of distinct non-$\star$ entries $q_{j,k} = s^{(k_s)} : k_s \in \mathcal{K}$ is $(|\mathcal{K}|-1)(F_1 - Z_1) = (K_1 - S)(F_1 - Z_1)$. Also, for any column $\mathbf{q}_k$ such that $k \notin \mathcal{K}$, this number is $(|\mathcal{K}|)(F_1 - Z_1) = (K_1 + 1 - S)(F_1 - Z_1) > (K_1 - S)(F_1 - Z_1)$. Therefore, by choosing $F' = (K_1 - S)(F_1 - Z_1) + Z = (K_1 - S)(F_1 - Z_1) + F_1 + K_1 Z_1 = (K_1 + 1 - S)F_1 + Z_1 S$, we ensure that for any $\mathcal{K} \subseteq [K] : |\mathcal{K}| = K - S$, every column of $\mathbf{Q}$ contains at least $F' - Z$ entries $s^{(k_s)} \in \mathcal{S}$ such that $k_s \in \mathcal{K}$. Thus, $C5$ of the PC-DPDA definition holds. Therefore, the array $\mathbf{Q}$ is a $(K_1+1, S \leq K_1, (K_1+1)F_1, (K_1+1-S)F_1 + SZ_1, K_1 Z_1 + F_1, (K_1+1)S_1)$ PC-DPDA. ∎

References

[1] M. N. Tehrani, M. Uysal, and H. Yanikomeroglu, "Device-to-device communication in 5G cellular networks: challenges, solutions, and future directions," *IEEE Commun. Mag.*, vol. 52, no. 5, pp. 86-92, May 2014.

[2] X. Shen, "Device-to-device communication in 5G cellular networks," *IEEE Network*, vol. 29, no. 2, pp. 2-3, Mar.-Apr. 2015.

[3] M. A. Maddah-Ali and U. Niesen, "Fundamental Limits of Caching," *IEEE Trans. Inf. Theory*, vol. 60, no. 5, pp. 2856-2867, May 2014.

[4] M. Ji, G. Caire, and A. F. Molisch, "Fundamental Limits of Caching in Wireless D2D Networks," *IEEE Trans. Inf. Theory*, vol. 62, no. 2, pp. 849-869, Feb. 2016.

[5] L. Tang and A. Ramamoorthy, "Coded Caching Schemes With Reduced Subpacketization From Linear Block Codes," *IEEE Trans. Inf. Theory,* vol. 64, no. 4, pp. 3099-3120, Apr. 2018.

[6] Q. Yan, M. Cheng, X. Tang, and Q. Chen, "On the Placement Delivery Array Design for Centralized Coded Caching Scheme," *IEEE Trans. Inf. Theory,* vol. 63, no. 9, pp. 5821-5833, Sep. 2017.

[7] J. Wang, M. Cheng, Q. Yan, and X. Tang, "Placement Delivery Array Design for Coded Caching Scheme in D2D Networks," *IEEE Trans. Commun.,* vol. 67, no. 5, pp. 3388-3395, May 2019.

[8] Q. Yan, X. Tang, Q. Chen, and M. Cheng, "Placement Delivery Array Design Through Strong Edge Coloring of Bipartite Graphs," *IEEE Commun. Lett.,* vol. 22, no. 2, pp. 236-239, Feb. 2018.

[9] X. Zhong, M. Cheng, and J. Jiang, "Placement Delivery Array Based on Concatenating Construction," *IEEE Commun. Lett.,* vol. 24, no. 6, pp. 1216-1220, Jun. 2020.

[10] J. Michel and Q. Wang, "Placement Delivery Arrays From Combinations of Strong Edge Colorings," *IEEE Trans. Commun.,* vol. 68, no. 10, pp. 5953-5964, Oct. 2020.

[11] M. Cheng, J. Wang, X. Zhong, and Q. Wang, "A Framework of Constructing Placement Delivery Arrays for Centralized Coded Caching," *IEEE Trans. Inf. Theory,* vol. 67, no. 11, pp. 7121-7131, Nov. 2021.

[12] J. Li and Y. Chang, "Placement Delivery Arrays Based on Combinatorial Designs," *IEEE Commun. Lett.,* vol. 26, no. 2, pp. 296-300, Feb. 2022.

[13] Rashid Ummer N. T. and B. S. Rajan, "Hierarchical Coded Caching With Low Subpacketization and Coding Delay Using Combinatorial t-Designs," *IEEE Internet of Things J.*, vol. 12, no. 12, pp. 18791-18805, Jun. 2025.

[14] X. Wu, M. Cheng, C. Li, and L. Chen, "Design of Placement Delivery Arrays for Coded Caching With Small Subpacketizations and Flexible Memory Sizes," *IEEE Trans. Commun.,* vol. 70, no. 11, pp. 7089-7104, Nov. 2022.

[15] J. Wang, M. Cheng, K. Wan, and G. Caire, "Placement Delivery Array Construction via Cartesian Product for Coded Caching," *IEEE Trans. Inf. Theory,* vol. 69, no. 12, pp. 7602-7626, Dec. 2023.

[16] J. Li and Y. Chang, "New Constructions of D2D Placement Delivery Arrays," *IEEE Commun. Lett.,* vol. 27, no. 1, pp. 85-89, Jan. 2023.

[17] Rashid Ummer N. T. and B. S. Rajan, "D2D Coded Caching from Two Classes of Optimal DPDAs Using Cross Resolvable Designs," in *IEEE Wireless Commun. Networking Conf. (WCNC)*, 2025, pp. 1-6.

[18] Rashid Ummer N.T., and B. S. Rajan. "D2D Coded Caching Schemes for Multiaccess Networks with Combinatorial Access Topology," Available on *arXiv: 2501.10756 [cs.IT],* Jan 2025.

[19] C. Gao, H. Zhang, X. Chen, Y. Li, D. Jin, and S. Chen, "Impact of Selfishness in Device-to-Device Communication Underlying Cellular Networks," *IEEE Trans. Veh. Technol.*, vol. 66, no. 10, pp. 9338-9349, Oct. 2017.

[20] A. Tebbi and C. W. Sung, "Coded caching in partially cooperative D2D communication networks," in *Int. Congr. Ultra Modern Telecommun. Control Syst. Workshops (ICUMT)*, Munich, Germany, 2017, pp. 148-153.

[21] A. Phatak and M. K. Varanasi, "An improved coded caching scheme for partially cooperative D2D networks," in *IEEE Int. Symp. Inf. Theory (ISIT)*, 2022, pp. 1121-1126.

[22] W. Guan, K. Huang, X. Xie, J. Zhang, K. Cai, and X. Huang, "New Results on Coded Caching in Partially Cooperative D2D Networks," in *IEEE Int. Symp. Inf. Theory (ISIT)*, 2024, pp. 1556-1561.

[23] Rashid Ummer N.T., K. K. K. Namboodiri, and B. S. Rajan, "A Novel Coded Caching Scheme for Partially Cooperative Device-to-Device Networks," Available on *arXiv: 2509.02532 [cs.IT],* Sep 2025.

[24] M. Cheng, J. Jiang, X. Tang, and Q. Yan, "Some Variant of Known Coded Caching Schemes With Good Performance," *IEEE Trans. Commun.,* vol. 68, no. 3, pp. 1370-1377, Mar. 2020.